On the Numerical Treatment and Dependence of the Third Dredge-Up Phenomenon

C. A. Frost and J. C. Lattanzio
Department of Mathematics, Monash University, Vic. 3168, Australia

ABSTRACT

We present results of an investigation into the behaviour of the base of the convective envelope of models of AGB stars during third dredge-up. We find that the extent, and even presence, of third dredge-up depends critically on the treatment of convection within a stellar structure calculation.

Subject headings: Convection — methods: numerical — stars: AGB — stars: evolution

1. Introduction

Determining the position of convective boundaries in stellar models is a long-standing problem. Formally, a boundary is defined (from the Schwarzschild criterion) to be located where the adiabatic and radiative temperature gradients, ∇_{ad} and ∇_{rad}, are equal. However, this merely tells us where the acceleration of the convective elements goes to zero. The region between this and where the velocity reaches zero is called the 'overshoot' region. Much effort has been made to modify the commonly used Mixing Length Theory (MLT) of convection to extend beyond the formal boundary in order to calculate the extent of the overshoot region (e.g., Shaviv and Salpeter 1973, Maeder 1975, Bressan et al., 1981, Langer 1986). These techniques were shown by Renzini (1987) to produce non-physical situations, and he concluded that only a new theory of convection could resolve the problem.

Although several new convective theories and modifications to the MLT have been proposed (Canuto and Mazzitelli 1991, Forestini et al., 1991, Gehmeyr and Winkler 1992, Lydon et al., 1992, Grossman et al., 1993, Singh et al., 1994, Singh et al., 1995), we use the standard MLT for our work. We shall look at phenomenological techniques to try to better approximate the true boundary. This is of paramount importance in models of thermally-pulsing Asymptotic Giant Branch (AGB) stars during the third dredge-up phase. AGB stars consist of a degenerate carbon-oxygen core, surrounded by a helium-rich region, above which lies a hydrogen-rich convective envelope. Following thermal pulses of the helium-burning shell, the convective envelope moves inwards in mass, penetrating the hydrogen-exhausted regions. This is known as third dredge-up. As convection moves inwards, a chemical discontinuity develops at the boundary. The radiative temperature gradient is defined as

$$\nabla_{rad} = \frac{3}{16\pi acG}\frac{\kappa lp}{mT^4}$$

where κ, p, l, m, and T are opacity, pressure, luminosity, mass, and temperature respectively. The chemical discontinuity associated with third dredge-up results in a discontinuity in κ, which likewise causes a discontinuity in ∇_{rad}. This last discontinuity then inhibits further growth in the convective envelope as large readjustments of temperature and other state variables must occur in order to raise $\frac{\nabla_{rad}}{\nabla_{ad}}$ in the radiative zone above unity. However, any mixing beyond the abundance discontinuity encourages further penetration as each mass element obtains $\frac{\nabla_{rad}}{\nabla_{ad}}$ well above unity when mixed, and thus the discontinuity is propagated.

That third dredge-up does occur is well-supported by observations of Carbon stars on the AGB. Codes which do not include some method of mixing beyond the Schwarschild boundary (though this may depend on the mixing algorithm; see below), may not see third dredge-up and hence not produce Carbon star models. In addition, unless $\frac{\nabla_{rad}}{\nabla_{ad}}$ approaches unity at the convective boundary, we have the unstable situation of a finite acceleration at the base of the envelope. Clearly, the inclusion of some form of extra mixing is a physical necessity.

One method, employed by Boothroyd and Sackmann (1988), extends the convective zone until a point is reached where, even if mixed into the convective zone, it would remain radiative. A different method is used by Lattanzio (1986), who calculates the ratio of the temperature gradients, $\frac{\nabla_{rad}}{\nabla_{ad}}$, and linearly extrapolates the ratio from the last two convective mesh points to the first radiative point. If the extrapolated value is greater than unity, then that point is included in the convective zone. If less than unity, the point remains radiative. This scheme allows at most one radiative point to be added to the convective zone per iteration, and is the method used in this work[1]. It tries to ensure that the last convective point is close to neutral buoyancy (i.e. $\frac{\nabla_{rad}}{\nabla_{ad}}$ =1). This is distinct from that which is usually called 'overshoot' in the literature, meaning the extension of the mixed region by finite velocity at the boundary (even though the acceleration is zero).

2. Behaviour of $\frac{\nabla_{rad}}{\nabla_{ad}}$ During Third Dredge-Up

To study the evolution of AGB stars, we use a version of the Mount Stromlo Stellar Structure Program (Wood and Zarro 1981) with updated radiative opacities (Rogers and Iglesias 1992). Conductive opacities are calculated from a program supplied by McDonald (1992) which uses opacities from various sources (Hubbard and Lampe 1969, Iben 1975, Itoh et al., 1983, Itoh et al., 1984, Mitake et al., 1984, Raikh and Yakovlev 1982). The change in entropy of the convective envelope during third dredge-up is treated using Wood's method (1981). The code begins with a model and uses the rate of composition change to step forward in time and make an initial estimate of a new model, and then iterates on the state variables until the equations of stellar

[1]As this procedure is used at each iteration, it permits the growth of the convective zone to be determined by the demand for neutrality in a way which is almost (but not quite) independent of space and time discretisation. An analysis of the dependence on mesh-spacing and time-stepping is not reported here.

structure are satisfied. The model is then deemed to have converged, and a new model calculated. Convective boundaries are re-calculated for each iteration according to the current conditions, and abundances are re-calculated from the previous converged model to fit the new convective zones. Thus the code does not retain memory of the abundances from the previous iteration (nor should it, as iterations are not time-steps); this is an important point in the discussion below. Extension of convection beyond the Schwarzschild boundary is allowed for, using the method described above.

In the past it was found to be not always possible to converge a model if the convective boundaries were allowed to change with each iteration. To overcome this problem, we find it necessary to (a) set a maximum value on the number of iterations allowed before changing the time-step (we choose 60 as the maximum), and (b) allow the convective boundaries to change for the first 40 of these iterations only (Lattanzio 1986). The boundaries are then held at whatever position they occupied on the 40^{th} iteration. In general, a model would converge within the first 10 iterations. Note that convergence difficulties are often found in stellar evolution calculations, especially for AGB models. They are usually not reported in publications until they become so severe as to prohibit further evolution (e.g., Boothroyd and Sackmann 1992).

A question naturally arises: Why is it so difficult to converge the models during third dredge-up? We look closely at the behaviour of $\frac{\nabla_{rad}}{\nabla_{ad}}$ with each iteration of a stellar model undergoing convergence difficulties during third dredge-up, and find that the convective envelope initially moves steadily inwards as expected (Fig. 1(a)). However, $\frac{\nabla_{rad}}{\nabla_{ad}}$ begins to decrease just above the base of the envelope (Fig. 1(b)) and eventually drops below unity so that the convective envelope splits into two zones (Fig. 1(c)). During the following iterations the envelope retreats (Figs 1(d)-(e)) until it has returned to (nearly) the conditions of the earlier iterations (Fig. 1(f)).

This behaviour may be understood in terms of the method used for calculating abundances. Abundances are always calculated from the previous model, and no memory is preserved of the abundance profile of the previous iteration. Once the convective envelope splits, the second zone (located just below the convective envelope) reverts to its original pre-mixed abundances (ie helium-rich and not hydrogen-rich), and is now radiatively stable. Similarly, as the envelope retreats, the matter previously engulfed by the envelope also reverts to its pre-mixed abundances. The model now returns to its original conditions and abundances and the entire process begins again (Fig. 2). We recognise this behaviour to be reminiscent of semi-convection of the convective core during core helium-burning (Castellani et al., 1971). Once caught in this cycle the code will not converge unless the convective boundaries are fixed in some way.

To overcome this problem, which we refer to as 'envelope breathing', we decided to mix down to the 'split' on subsequent iterations regardless of how the temperature gradients alter afterwards. We allow convection to extend even further downwards, if required by the temperature gradients, but abundances are always mixed to the point of splitting even if convection formally retreats. Thus we implicitly treat the iterative process as time-dependent, which is now reasonable as the convective mixing is the only source of change in the stellar structure equations after a few iterations.

Fig. 3 shows the mass of the H-exhausted core versus time for the first 6 thermal pulses of four $5M_\odot$ models with Z=0.004. These models have exactly the same starting model and conditions, but case A (solid) includes the method of dealing with 'envelope breathing' as outlined above, and case B (dashed) does not. Both models allow the convective boundaries to change during the first 40 iterations as this is still necessary on occasion for convergence, even in case A. Both cases experience dredge-up during the second pulse, but it is larger in case A and grows more quickly during subsequent dredge-up events. By holding the convective boundaries at the 40^{th} iteration, the choice of location of the base of the envelope in the converged model is not well determined and would lie somewhere between the minimum and maximum extent. Naturally we would expect to see less dredge-up in this instance, case B, than in case A.

3. The Numerical Dependency of Dredge-Up

By treating convective overshoot phenomenologically rather than physically, we need to understand how the depth of dredge-up is affected by numerical treatment, and which treatment yields results that are physically reasonable and accurate. The latter is probably best tested by comparison of models with stellar observations and is beyond the scope of this letter.

We already see that a simple method of dealing with 'envelope breathing' allows the convective envelope to penetrate further inwards than would otherwise be calculated. We investigate two further cases, also shown in Fig. 3. Cases C (dot-dash) and D (dotted) do not include the method for handling 'envelope breathing'. Case C does not attempt to force convective neutrality at a convective boundary, according to Lattanzio's method as discussed above, but simply uses the Scwharzschild criterion. Case D does attempt to force neutrality, but calculates abundances at a different stage to that usually used. Instead of determining abundances after each iteration the abundance profiles are held as they were in the last model (changes due to nuclear burning are allowed) until the current model has converged. Abundances are then mixed for the converged model. This method is used in some stellar codes.

The differences between cases A and B have already been discussed. We first obtain dredge-up for case C during the 5^{th} pulse (cf the 2^{nd} pulse for cases A and B). In subsequent pulses, the extent of dredge-up grows, but more slowly. Case D sees negligible dredge-up (or none at all), despite the inclusion of the convective neutrality algorithm. When mixing is not calculated concurrently, the boundaries can change little from model to model and the convective neutrality algorithm allows at most one mesh point to be mixed into the convective envelope per model. This is clearly inadequate in case D.

4. Discussion

The motivation for calculating the extent of dredge-up has been to obtain more accurate estimates of those elements whose surface abundances (e.g., carbon) are affected by this phenomonen. We note that simple, and reasonable, differences in computational details between codes could explain the differences in third dredge-up reported in the literature by various authors. Further, from Fig. 3, it is clear that dredge-up affects the structure and strength of thermal pulses (Sackmann 1977, Paczyński 1977). Those models which experience deep dredge-up have longer interpulse periods and smaller core masses than those which do not. Much of the evolution of stars during the TP-AGB phase is thought to be controlled by the size of the H-exhausted core, and a fuller understanding of the effects of deep dredge-up is needed. This will be addressed in a later paper.

5. Acknowledgements

We would like to thank Forrest Rogers and Carlos Iglesias for making the OPAL opacity tables available ahead of publication. We would also like to thank Peter Wood for many helpful discussions. CAF gratefully acknowledges the support of an Australian Postgraduate Award.

REFERENCES

Boothroyd, A. I., and Sackmann, I.-J. 1988, ApJ, 328, 653

Boothroyd, A. I., and Sackmann, I.-J. 1992, ApJ, 393, L21

Bressan, A. G., Bertelli, G., and Chiosi, C. 1981, A&A, 102, 25

Canuto, V. M., and Mazzitelli, I. 1991, ApJ, 370, 295

Castellani, V., Giannone, P., and Renzini, A. 1971, Ap&SS, 10, 340

Forestini, M., Lumer, E., and Arnould, M. 1991, A&A, 252, 127

Gehmeyr, M., and Winkler, K.-H, A. 1992, A&A, 253, 92

Grossman, S. A., Narayan, R., and Arnett, D. 1993, ApJ, 407, 284

Hubbard, W. B., and Lampe, M. 1969, ApJS, 18, 297

Iben, I. Jr. 1975, ApJ, 196, 525

Itoh, N., Mitake, S., Iyetomi, H., and Ichimaru, S. 1983, ApJ, 273, 774

Itoh, N., Kohyama, Y., Matsumoto, N., and Seki, M. 1984, ApJ, 285, 758

Langer, N. 1986, A&A, 164, 45

Lattanzio, J. C. 1986, ApJ, 311, 708

Lydon, T. J., Fox, P. A., and Sofia, S. 1992, ApJ, 397, 701

Maeder, A. 1975, A&A, 40, 303

McDonald, J. 1992, private communication

Mitake, S., Ichimaru, S., and Itoh, N. 1984, ApJ, 277, 375

Paczyński, B. 1977, ApJ, 214, 812

Raikh, M. E., and Yakovlev, D. G. 1982, Ap&SS, 87, 193

Renzini, A. 1987, A&A, 188, 49

Rogers, F. J., and Iglesias, C. A. 1992, ApJS, 79, 507

Sackmann, I.-J. 1977, ApJ, 212, 159

Shaviv, G., and Salpeter, E. E. 1973, ApJ, 184, 191

Singh, H. P., Roxburgh, I. W., and Chan, K. L. 1994, A&A, 281, L73

Singh, H. P., Roxburgh, I. W., and Chan, K. L. 1995, A&A, 295, 703

Wood, P. R. 1981, ApJ, 248, 311

Wood, P. R., and Zarro, D. M. 1981, ApJ, 247, 247

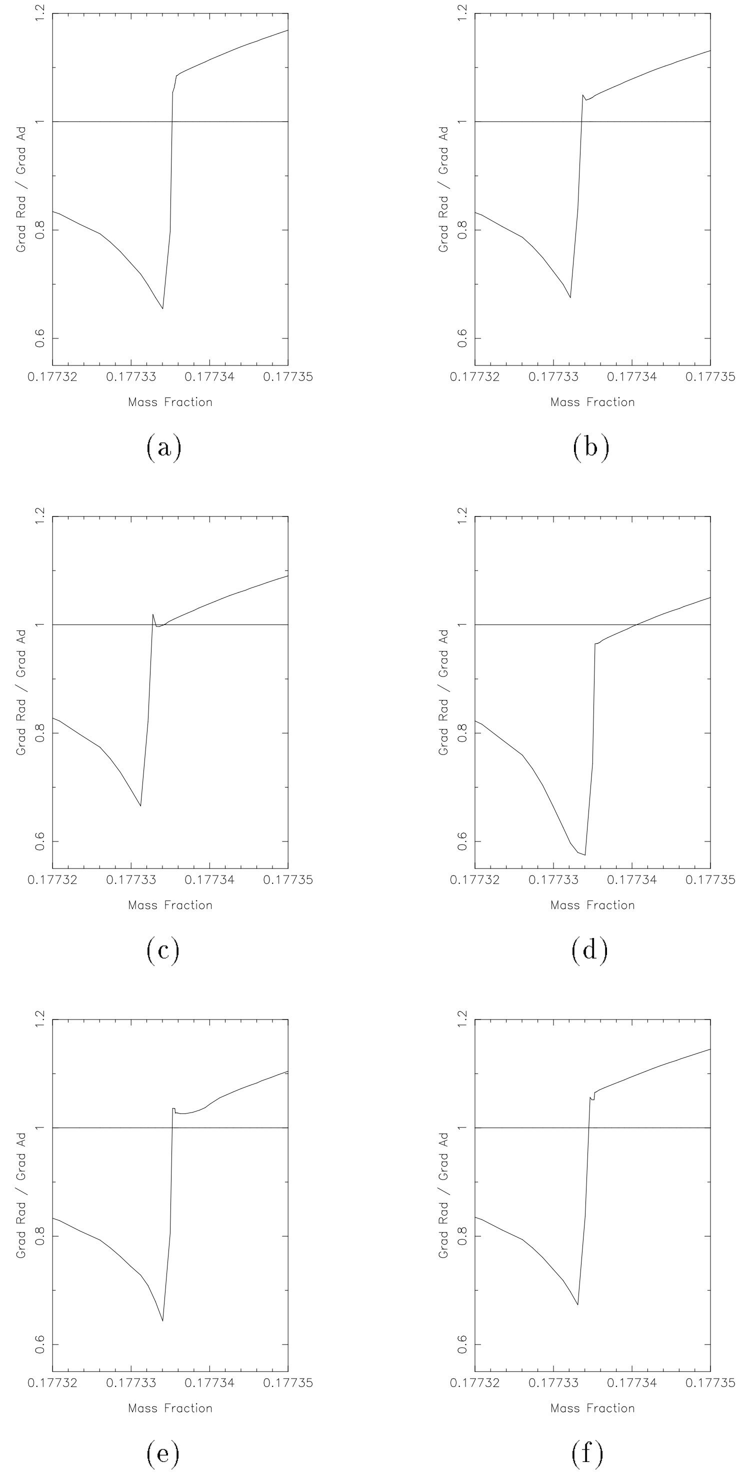

Fig. 1.— $\frac{\nabla_{rad}}{\nabla_{ad}}$ versus mass for a $5M_{\odot}$ model with Z=0.004 during third dredge-up, showing the discontinuity at the base of the convective envelope. The line at $\frac{\nabla_{rad}}{\nabla_{ad}} = 1$ marks convective neutrality according to the Schwarzschild criterion. (a) 3^{rd} iteration. (b) 5^{th} iteration, in which a drop in $\frac{\nabla_{rad}}{\nabla_{ad}}$ just above the base of the envelope is clearly visible. (c) 6^{th} iteration. The dip in $\frac{\nabla_{rad}}{\nabla_{ad}}$ has dropped below unity, producing a small radiative zone. (d) 7^{th} iteration, in which the convective envelope has retreated. (e) 8^{th} iteration. (f) 9^{th} iteration.

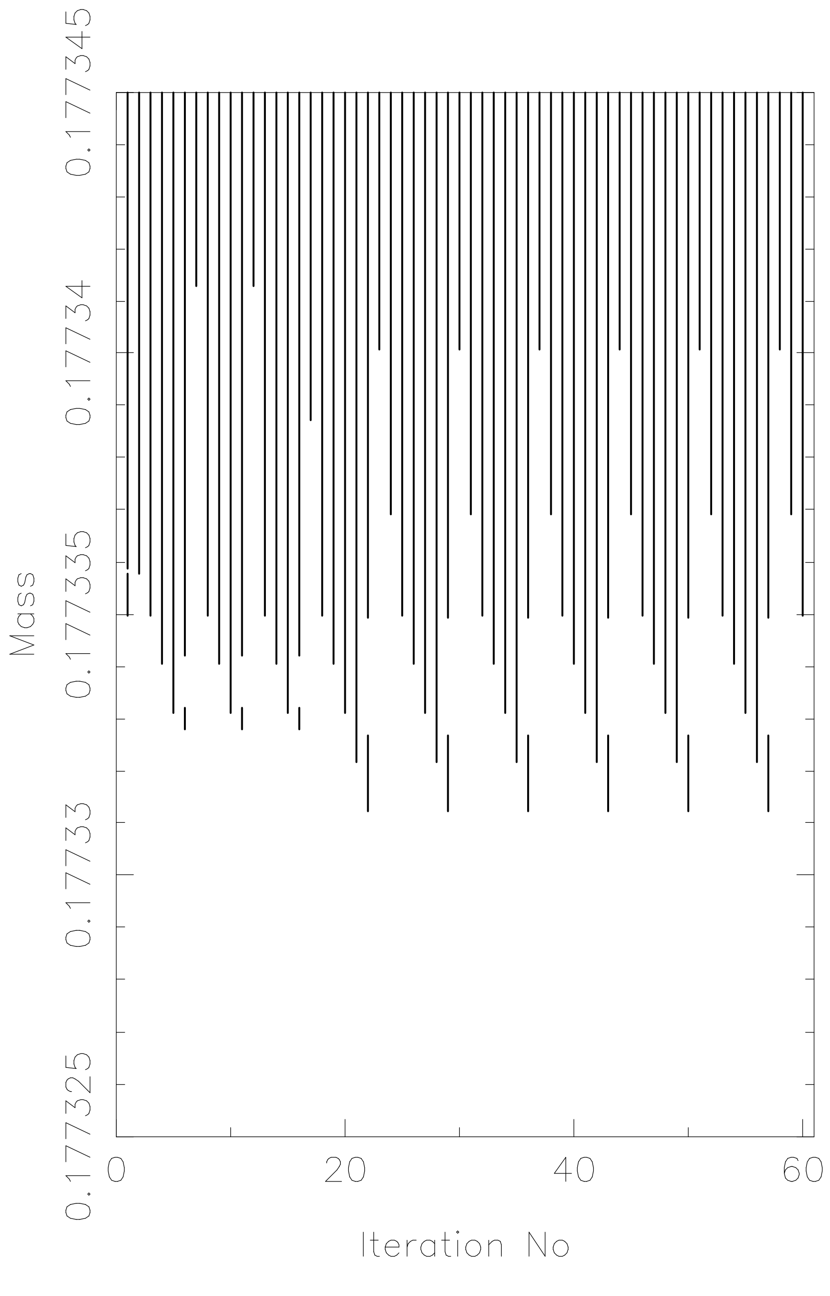

Fig. 2.— Diagram showing convection at the base of the envelope as a function of iteration number, for the model in Fig. 1. The progression and retreat of the envelope are clearly seen, together with the splitting of the envelope. Two separate cycles are evident; the first occurring within the first fifteen iterations, repeating every six iterations, and the second occurring for later iterations, repeating every eight iterations. The difference lies in a new convergence criterion being introduced after the fifteenth iteration, to try and help convergence. This further demonstrates the sensitivity of calculations to numerical technique.

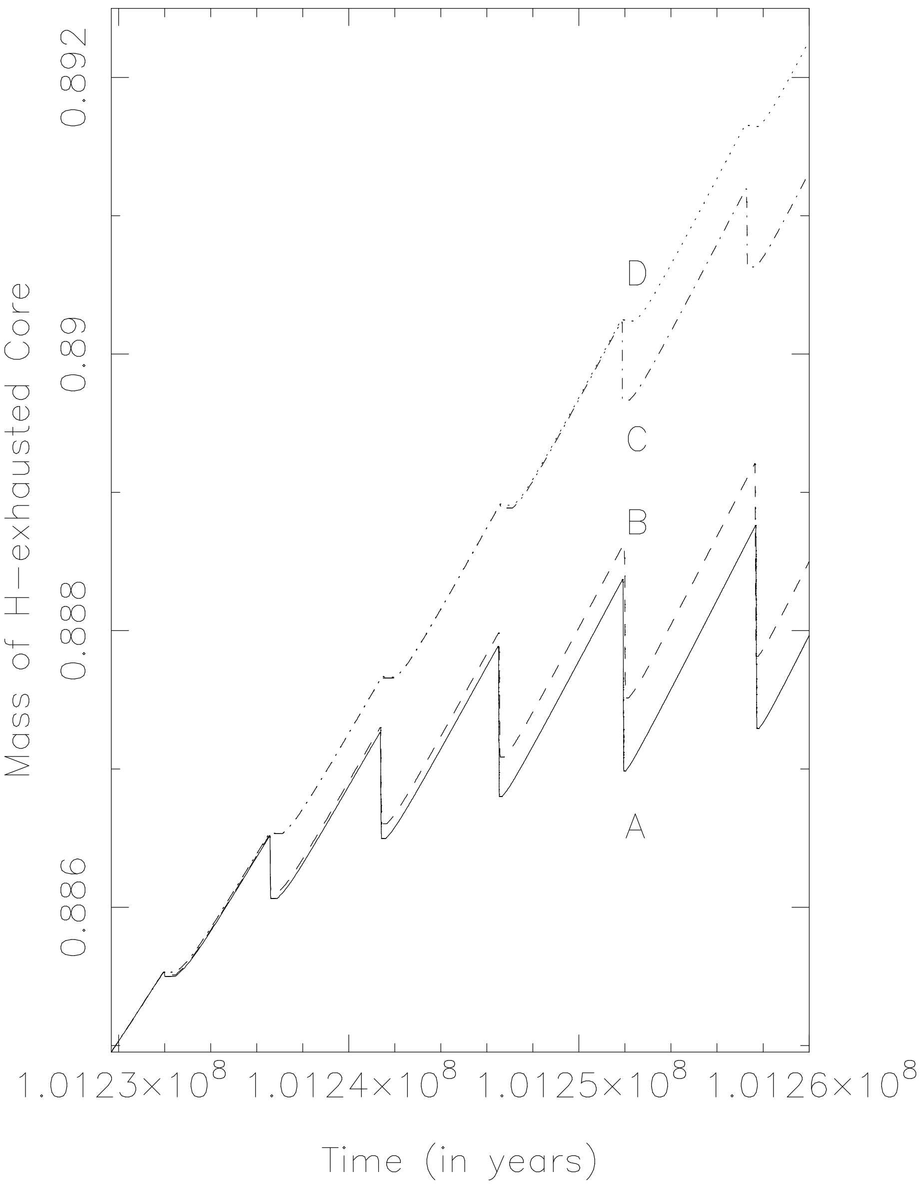

Fig. 3.— Mass of the H-exhausted core verus time for four $5M_{\odot}$ models withe Z=0.004. The first 6 thermal pulses are shown. Case A (solid) included the metod of extending the envelope to the "split". Case B (dashed) did not. Case C (dot-dashed) did not allow the convective neutrality algorithm to include additional mesh points into the envelope. In case D (dotted), mixing was performed after each model had converged, so that at most one mesh point per model would be added to the convective envelope by the convective neutrality algorithm. This was the only case that did not experience third dredge-up.