ASP3012: Stars and Galaxies
Stars Exercise Sheet 6

Q1. A binary is composed of two stars A and B. Star A has a mass of $1.5M_\odot$ and has $X = 0.7$, $Y = 0.28$ and $Z = 0.02$. Star B is in fact a captured star of $0.5M_\odot$ with a different composition: $X = 0.65$, $Y = 0.30$ and $Z = 0.05$. If the two stars merge without any nuclear burning or mass-loss, what is the composition (in X, Y and Z) of the final merged object?

Q2. A $1M_\odot$ star has $X = 0.7$ and $Y = 0.25$ on the ZAMS. It ascends the giant branch and when it suffers the first dredge-up it has a H-exhuasted core-mass of $0.45M_\odot$. After dredge-up the core mass is reduced to $0.4M_\odot$.

a) What is the envelope composition of X, Y and Z.

b) If the initial Z is comprised of 60% CNO in the ratios C^{12}:N^{14}:O^{16} = 3:1:6 then what are the final abundances of these species in the envelope after first dredge-up?

Q3. A particular branching in the s-process occurs at the fictional unstable isotope Stuff, with chemical symbol Sf. At a neutron flux of $N_n = 1\times10^{10}$ $cm^{-3}s^{-1}$ we find that 30% of the incoming neutrons are captured by Sf^m and the rest beta-decay to another species. After 30 seconds how many of the heavier isotope of Sf has been made and what is its atomic mass?

Q4. A branching ratio operating at an unstable nucleus within the s-process is given by the formula

$$f = \frac{\lambda_n}{\lambda_n + \lambda_\beta}$$

where

$$\lambda_n = N_n \langle \sigma_A \rangle v_{th}$$

and

$$\lambda_\beta = \ln 2/t_{1/2}$$

where v_{th} is the thermal velocity at the prevailing temperature, and $t_{1/2}$ is the half-life of the unstable nucleus to beta-decay. Here $\langle \sigma_A \rangle$ is the appropriately averaged cross-section for the unstable species of mass A.

a) Assuming a cross-section of 12 mbarn and half-life of 10 days, plot the branching ratio f as a function of the neutron density between $N_n = 10^5$ and 10^{13} cm^{-3}. You may assume a thermal velocity of 3×10^8 cm/sec.

b) Repeat but for half-lives of various values between 100 and 100,000 days.

Q5.

a) A region of a star has $X(C^{12}) = 0.3$. What is the mole fraction of C^{12}?

b) In a star we have a region where A(Fe)=6.45. What is ϵ(Fe)?

c) What is the value of [Fe/H] for the star in part (b)? (You may use $A(Fe)_\odot$=7.45.)

d) A star has [Si/Fe] = −2. What is the value of n(Si)/n(Fe) in this star relative to the Solar value?

Q6. Suppose that the initial mass function is given by the Salpeter law:

$$\frac{dN}{dm} = kM^{-2.3}$$

for some constant k. The IMF is observed to turn over steeply below $M = 0.1M_\odot$, and no stars are found to be more massive than $100M_\odot$.

a) Calculate what fraction of all stars go through an AGB phase (ie have masses between 1 and $10M_\odot$)
b) Calculate what fraction of stars die as supernovae (ie what fraction have masses between 20 and $50M_\odot$).

Q7. Suppose that the yield of N^{14} per star depends on the mass of the star (in solar units) according to:

$$
\begin{aligned}
y(\mathrm{N}^{14}) =& 0, \quad \text{if } m < 4 \\
=& 0.2m, \quad \text{if } 4 \le m \le 7 \\
=& 0, \quad \text{if } 7 < m < 20 \\
=& 0.1(m-10), \quad \text{if } 20 \le m \le 50
\end{aligned}
$$

Suppose all stars between 4 and $7M_\odot$ become AGB stars and all stars between 20 anbd $50M_\odot$ become supernovae. WHich kind of stars, AGB or supernovae, produce the most N^{14}?

Q8. An unstable element decays by emitting positrons. At any time the probability of decay is proportional to the instantaneous number N of atoms present. Show that the formula governing the decay is

$$\frac{dN}{dt} = e^{-t/\tau}$$

where the timescale τ is related to the half-life $t_{1/2}$ via

$$t_{1/2} = \tau \ln 2.$$

Q9. The isotope Tc^{99} has a half-life of 700,000 years. It has recently been produced in large amounts in a particular star. An alien civilization is observing this star and has noticed the appearance of Tc. Given that their instruments can detect Tc at an abundance as low of 0.001 of its present value. for how long will they be able to observe Tc in this star?

Q10. We believe that Tc^{99} is produced by the s-process in AGB stars undergoing thermal pulses every 10,000 years.
a) How low will the abundance fall (from its peak) before it is produced again in the next thermal pulse?
b) If our detectors fail to see Tc when it reaches 1% of its maximum value, what is the minimum required half-life for it to be visible throughout the AGB phase.