ASP3012: Stars Assignment

Helium burning occurs via the "triple alpha" reaction: $3\mathrm{He}^4 \rightarrow \mathrm{C}^{12}$ which occurs at the rate $\lambda_{3\alpha}(\mathrm{He}^4)^3$, where He^4 denotes the number abundance of He^4. But once there is some C^{12} present then we can get $\mathrm{C}^{12}(\alpha,\gamma)\mathrm{O}^{16}$ which occurs at the rate $\lambda_{\alpha 12}\mathrm{He}^4\mathrm{C}^{12}$. Furthermore, once some O^{16} has been produced then we also get $\mathrm{O}^{16}(\alpha,\gamma)\mathrm{Ne}^{20}$ which occurs at the rate $\lambda_{\alpha 16}\mathrm{He}^4\mathrm{O}^{16}$.

a) Write down the differential equations for the He^4, C^{12}, O^{16} and Ne^{20} abundances as a function of time.

b) Let

$$R_{12} = \frac{\lambda_{\alpha 12}}{\lambda_{3\alpha}\mathrm{He}^4(0)}$$
$$R_{16} = \frac{\lambda_{\alpha 16}}{\lambda_{3\alpha}\mathrm{He}^4(0)}$$

where $\mathrm{He}^4(0)$ is the initial abundance of He^4. Define the four dimensionless variables:

$$x = \frac{\mathrm{He}^4}{\mathrm{He}^4(0)} \qquad u = \frac{\mathrm{C}^{12}}{\mathrm{He}^4(0)} \qquad v = \frac{\mathrm{O}^{16}}{\mathrm{He}^4(0)} \qquad w = \frac{\mathrm{Ne}^{20}}{\mathrm{He}^4(0)}$$

Show that the differential equations become:

$$\frac{du}{dx} = \frac{1 - R_{12}u/x^2}{-3 - R_{12}u/x^2 - R_{16}v/x^2}$$
$$\frac{dv}{dx} = \frac{R_{12}u/x^2 - R_{16}v/x^2}{-3 - R_{12}u/x^2 - R_{16}v/x^2}$$
$$\frac{dw}{dx} = \frac{R_{16}v/x^2}{-3 - R_{12}u/x^2 - R_{16}v/x^2}$$

c) Solve these equations numerically for the four cases:
   i) $R_{12} = 0.1$, $R_{16} = 0.1$;
   ii) $R_{12} = 1.0$, $R_{16} = 0.1$;
   iii) $R_{12} = 0.1$, $R_{16} = 1.0$;
   iv) $R_{12} = 1.0$, $R_{16} = 1.0$.

Present your results as graphs of u, v and w against x (with x varying from 1.0 on the left down to 0.0 on the right).

d) Is there some consistency check you can use to ensure that you have an accurate solution?

Hand your Solutions to me or Stuart Heap by Fri Oct 7.

JCL: 10-September-2011