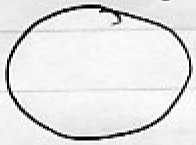
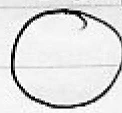
A $1.5M_\odot$



$X=0.7$
$Y=0.28$
$Z=0.02$

B $0.5M_\odot$



$X=0.65$
$Y=0.30$
$Z=0.05$

Final merged object

final H mass = initial H mass

$\cancel{1.5M_\odot \times X}$ =

$$2XM_{\odot} = 1.5M_{\odot}\times 0.7 + 0.5M_{\odot}\times 0.65$$

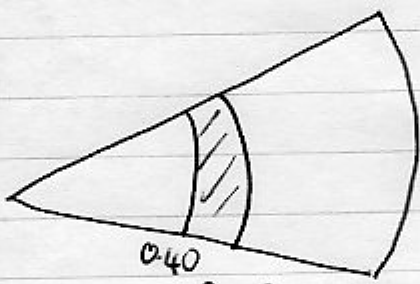
$$X = \frac{1.5\times 0.7 + 0.5\times 0.65}{2} = 0.6875$$

$$Y = \frac{1.5\times 0.28 + 0.5\times 0.3}{2} = 0.285$$

$$Z = \frac{1.5\times 0.02 + 0.5\times 0.05}{2} = 0.0275$$

$\Sigma = 1.0$ ✓

a)




$X^i = 0.7$
$Y^i = 0.25$
$\therefore Z^i = 0.05$

In burned region
$X^b = 0$
$Y^b = 0.95$
$Z^b = 0.05$

Conservation of H: $(1.0-0.4)X^f = (1.0-0.45)\times 0.7 + 0$

$\cancel{X^f = \left(\frac{m-0.45}{m-0.4}\right)}\times 0.7$

$$X^f = \frac{0.55}{0.6}\times 0.7 = 0.641\dot{6}$$

$$0.6\,Y^f = 0.55\times 0.25 + 0.05\times 0.95$$

$$Y^f = 0.185/0.6 = 0.308\dot{3}$$

$$Z^f = Z^i = 0.05$$

$X^f + Y^f + Z^f = 1$ ✓

b) $Z_{CNO} = 0.6Z = 0.6 \times 0.05 = 0.03$

$C^{12} : N^{14} : O^{16} = 3 : 1 : 6$

$\therefore C^{12} = \frac{3}{10}\, C+N+O = \frac{3}{10} \times 0.03 = 0.009$

$N^{14} = \frac{1}{10}\, C+N+O = \frac{1}{10} \times 0.03 = 0.003$

$O^{16} = \frac{6}{10}\, C+N+O = \frac{6}{10} \times 0.03 = 0.018$

$\Sigma = 0.03$ ✓

In burned region:

$C^{12}_b = 0 \qquad O^{16}_b = 0 \qquad N^{14}_b = 0.03$

$\therefore \quad 0.6\, C^{12}_f = 0.55\, C^{12}_i + 0.05 \times C^{12}_b$

$C^{12}_f = \frac{0.55}{0.60} \times 0.009 = 0.00825$

$\Rightarrow O^{16}_f = \frac{0.55}{0.60} \times 0.018 = 0.01650$

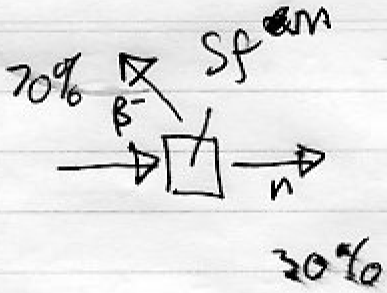
& $0.6\, N^{14}_f = 0.55\, N^{14}_i + 0.05\, N^{14}_b$

$N^{14}_f = \frac{1}{0.6}(0.55 \times 0.003 + 0.05 \times 0.03)$

$= 0.00525$

Check $C^{12}_f + N^{14}_f + O^{16}_f = 0.03$ ✓

Q3




$Sf^{m}(n,\gamma)\,Sf^{m+1}$ $\quad\therefore$ mass is $m+1$

$N = 30\% \times 1\times10^{10} \times 30$

$= 9\times10^{10}$ nuclei

Q4 See graph.

NB. As $t_{1/2} \uparrow$ the chance of n-capture increases so required $N_n$ for a given $f$ decreases

Q5 a) $X(^{12}C) = 0.3$ $\quad\therefore\ Y(^{12}C) = \frac{X(^{12}C)}{12} = \frac{0.3}{12} = 0.025$

b) $A(X) = \varepsilon(X) + 12$

$\varepsilon(X) = \log(n_X/n_H)$

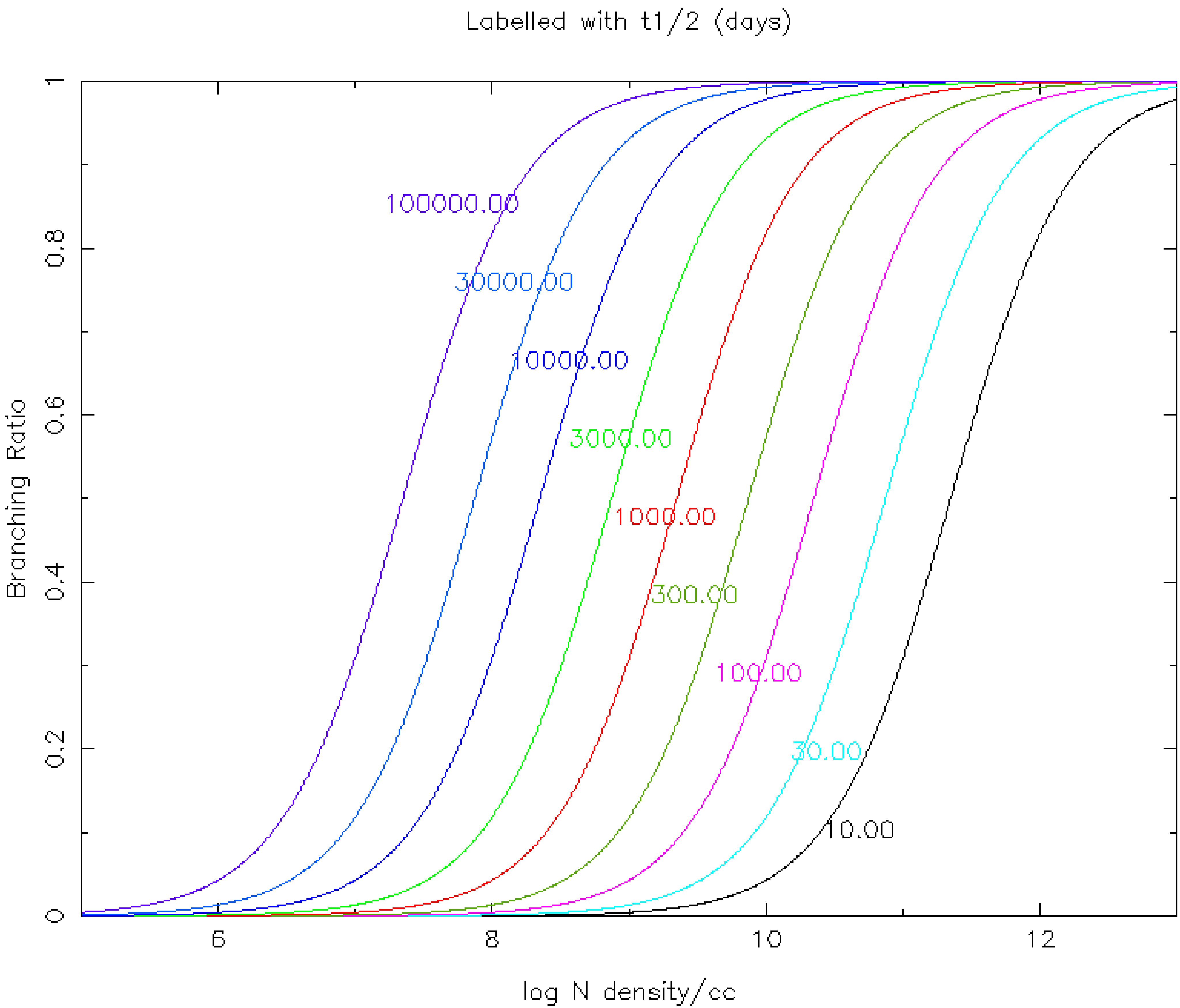
$A(Fe) = 6.45 \quad\therefore\ \varepsilon(Fe) = 6.45 - 12$

$= -5.55$

c) If $A(Fe)_\odot = 7.45$ then $[Fe/H] = -1.0$

$(7.45 - 6.45)$

d) 0.01


Labelled with t1/2 (days)
100000.00
30000.00
10000.00
3000.00
1000.00
300.00
100.00
30.00
10.00
Branching Ratio
0
0.2
0.4
0.6
0.8
1
6
8
10
12
log N density/cc

Q6 $\quad M_{min} = 0.1 M_\odot \quad M_{max} = 100 M_\odot$

a) $N = \int_{0.1}^{100} dN = \int_{0.1}^{100} \frac{dN}{dm} . dm$

$$= k \int_{0.1}^{100} m^{-2.3} dm$$

$$= \frac{-k}{1.3} m^{-1.3} \Big]_{0.1}^{100}$$

$$= -\frac{k}{1.3} \left( 100^{-1.3} - 0.1^{-1.3} \right)$$

$$= \frac{-k}{1.3} \left( 2.511 \times 10^{-3} - 19.95 \right)$$

$$= 15.346 k$$

So $\frac{N(1-10)}{N} = \frac{k \int_{1}^{10} m^{-2.3} dm}{15.346 k}$

$$= \frac{-\frac{1}{1.3} \left( 10^{-1.3} - 1^{-1.3} \right)}{15.346}$$

$$= \frac{1}{1.3 \times 15.346} \left( 5.01 \times 10^{-2} - 1 \right)$$

$$= 0.0476$$

b) $\frac{N(20-50)}{N} = \frac{\cancel{k} \int_{20}^{50} m^{-2.3} dm}{15.346 \cancel{k}}$

$$= \frac{-\frac{1}{1.3} \left( 50^{-1.3} - 20^{-1.3} \right)}{15.346}$$

$$= 0.00071$$

Q7 $$\text{Total } N^{14} = \int_0^{\infty} y(m)\frac{dN}{dm}\cdot dm$$

$$\text{mass } N^{14}(AGB) = \int_4^7 0.2m.\,m^{-2.3}\,dm$$

$$= 0.2\int_4^7 m^{-1.3}\,dm$$

$$= -\frac{0.2}{0.3}\,m^{-0.3}\Big|_4^7 = \frac{-2}{3}\left(7^{-0.3} - 4^{-0.3}\right)$$

$$= 0.068$$

$$\text{mass } N^{14}(SN) = \int_{20}^{50} 0.1(m-10)\,m^{-2.3}\,dm$$

$$= 0.1\int_{20}^{50} m^{-1.3}\,dm - \int_{20}^{50} m^{-2.3}\,dm$$

$$= \frac{-0.1}{0.3}\left[50^{-0.3} - 20^{-0.3}\right] + \frac{1}{1.3}\left[50^{-1.3} - 20^{-1.3}\right]$$

$$= 0.0218$$

Q8 $$\frac{dN}{dt} \propto -N \quad\Rightarrow\quad \frac{dN}{dt} = -kN$$

$$\frac{dN}{N} = -k\,dt$$

$$\ln N = -kt + c$$

At $t=0$ $N=N_0$:

$$\ln N_0 = c$$

$$\therefore \ln N = -kt + \ln N_0$$

$$N = N_0 e^{-kt} = N_0 e^{-t/\tau}$$

At $t = t_{1/2}$ $N = \frac{N_0}{2}$

$$\frac{\cancel{N_0}}{2} = \cancel{N_0} e^{-t_{1/2}/\tau}$$

$$2 = e^{t_{1/2}/\tau}$$

$$\ln 2 = t_{1/2}/\tau$$

$$\therefore t_{1/2} = \tau \ln 2$$

Q9 $\frac{N}{\cancel{N_0}} = N_0 e^{-t/\tau}$

$\tau = t_{1/2}/\ln 2$

$= \cancel{0.639/700000}$ ($t \approx y$)

$= \cancel{9.13 \times 10^{-7}}$

$\tau = 1,010,101$

$N = N_0/1000$ $t = ?$

$$\frac{\cancel{N_0}}{1000} = \cancel{N_0} e^{-t/\tau}$$

$$e^{t/\tau} = 1000$$

$$t = \tau \ln(1000)$$

$$= 1.01 \times 10^6 y \times 6.9077$$

$= 6.977$ million years

Q10 a) $N = N_0 e^{-10^4/1.01\times10^6} = N_0 \times 0.9901$

ie about 1%

b) Want $N = \frac{N_0}{100}$ to be just visible.

It decays to this value in 10,000 years.

ie $\frac{\cancel{N_0}}{100} = \cancel{N_0} e^{-10^4/\tau}$

$$100 = e^{10^4/\tau}$$

$$10^4/\tau = \ln(100)$$

$\tau = 10^4/\ln(100) = 2171$ years

$\therefore t_{1/2} = 1504$ years