Stars Lab 3: Hydrogen Burning

1 Introduction

Now we will look at nucleosynthesis via hydrogen burning. The details depend on the temperature, mostly, with some dependence on density of course. You will use various codes written to integrate different networks and then plot the results. You will find it useful to be able to use `gnuplot` to plot some extra results.

2 The PP Chains

Move to the directory containing the pp chain codes. Your tutor will show you how, but its probably as simple as `cd ASP3012-Lab3` after you have "un tar'd" the file `ASP3012-Lab3.tgz`.

We will first run the `pp` code which generates many output files. But we need to compile it first. Simply type: `f95 -w -o pp pp.f` to create the executable file `pp`.

Run `pp` and enter the following values:

- For T use 10 million K (1e7)
- For ρ use 100 g/cc
- Let it burn for 1e10 years

The code will use a solar composition to start with. It then requires a "root" name for the output files: I suggest `test`. This means that the output files will start with `test` and then the code will append other details to the name as we will see below. So for now, when asked for the "output root name" enter `test`.

Now look at the output summary on the screen (you may need to make your window wide enough for all the data to fits in the window.) The output lists some details of the equation of state, which you can ignore. It prints out the stop time (in seconds) and the burning temperature and density. Then we see the seven most common species in the initial mixture (H, He^4 etc).

Then the integrations start and the output is:

- step number
- time (in years)
- the species controlling the time-step (ie the one changing the most at this time)
- the time-step (in years)
- the three largest abundances

From the output you can see that H is not fully destroyed even after 1e10 years. There are two reasons for this. Firstly, 10 million degrees is not very hot really. Secondly, this code only includes the pp chains. Other H burning reactions are *not* included. So we rely on pp only to burn the H. And that is going to take a higher temperature or a longer time. After 10^{10} years we still have a hydrogen mass fraction of 0.5794.

But we will continue for now. Lets look at the files created by the `pp` code. Do a `ls` to see the files in the directory. You will see a few which start with `test` which is the root name for the run we just made. Alternatively, just list these file by typing `ls test*`

There are four files present:

<div align="center">

`test-0.dat` `test-1.dat` `test-2.dat` `testsummary.dat`

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1. `test-0.dat` You can look at this file by typing `less test-0.dat`. Use the arrow keys to move up and down in the file. Use `q` to quit. This file contains info from the equation of state. Its not very interesting for us, since we are burning at constant T and ρ.
2. `test-1.dat` You can look at this file by typing `less test-1.dat`. This one is also of little interest to us, as we are burning in constant conditions.
3. `test-2.dat` This is the one we want. It shows the time (in seconds!) as well as the mass fraction of various species.
4. `testsummary.dat` This contains just a brief summary - just the burning and final conditions, basically.

Run the code `plotpp` to see the time variation of the abundances. (Compile it by typing `make plotpp`.) This reads the data from the files described above. Enter the following parameters:

- For the root name enter `test` again
- We can plot either log(X) or (X); for now enter 0 for the linear scale
- Similarly we can plot t or $\log(t)$. Enter 0 for t

The resulting graph is not very useful! H decreases a little and He increases a little. Everything else is at a very low value and is not easily seen. So lets rerun using log(X) instead of X.

Note that the program lets you alter the viewing window by specifying xmin, xmax, ymin and ymax. This will be useful later, but for now enter 0 to end the program (you need to hit ENTER first, to return from the graphics window).

Now re-run but choose log(X). Use a linear scale for t. Now we can see the values of the other species. Note that D^2 drops very quickly to something like 10^{-18}, and He^3 settles at its equilibrium value of about $10^{-3.5}$ for this T and ρ. Note that Li^7 and Be^7 are also present at very low abundance only. B^8 is not present on this scale. You can alter ymin to determine the value of B^8 if you like.

Q1 What is the approximate value of the B^8 abundance for this case?

It might be useful to rerun `plotpp` using a log scale for time also. This shows you how quickly some things change.

A final tool. There is a shell script called `clean` in your directory. You can use this to remove the files generated by `pp` so that you can use the same name again without having to keep track of hundreds of names. Feel free to keep a few runs but when you do want to clean them up, use clean in the following way: simply type `clean root` and the files with root as the name will all be deleted. Type `clean test` now to see how it works.

(In fact this script removes all files with names `root*.dat`; so if you have run files with root names `case1`, `case2` and `case3` then `clean case` will remove them all at once! If you only want to remove, say, `case2` then be sure to type `clean case2`.)

2.1 Deuterium

In lectures we argued that the reaction rates were such that the first thing to burn was D^2. Lets investigate this.

Q2 Investigate the time for D^2 equilibrium by giving a table of this time vs T, for T between 1 and 10 million K. Keep the density at 100 g/cc. Also give the equilibrium value of D^2. You can get these values from the output files or by plotting using `plotpp`. Ask your tutor to show you how to plot these using `gnuplot`. It might be wise to keep these different cases for use in the following questions as well. Give them a name that makes sense. (eg casei for T=i million K).

Now you can see why we said in lectures, that at typical H burning temperatures, D^2 reaches equilibrium in seconds!

2.2 Helium-3

We saw that the fate of the He^3 nucleus determines the path of the pp reactions. For convenience these are shown in Figure 1 below.

Lets have a look at the timescale for He^3 to reach equilibrium. We will see that the timescale for He^3 to burn is much longer than for D^2 so we will go for higher temperatures here. We will also need to burn for longer. One example is shown in Figure 2 for a temperature of 5 million K. Be careful when looking for the He^3 equilibrium abundance. Note that D^2 rapidly burns, after about $10^{-3.5}$ years. This causes a slight rise in the He^3 abundance, as seen in the graph. But it is not yet in equilibrium. In fact, H is hardly burned at all yet. We see that H does not really start to deplete until nearly 10^{10} years. Just prior to that we see an increase in the He^3 abundance. It is this peak value that we are interested in. As the graph shows, the He^3 then decreases in step with the H abundance (and if we look at the ratio He^3/H^1 we would see a constant value). But to keep things simple, lets just look at the He^3 abundance rather than the ratio.
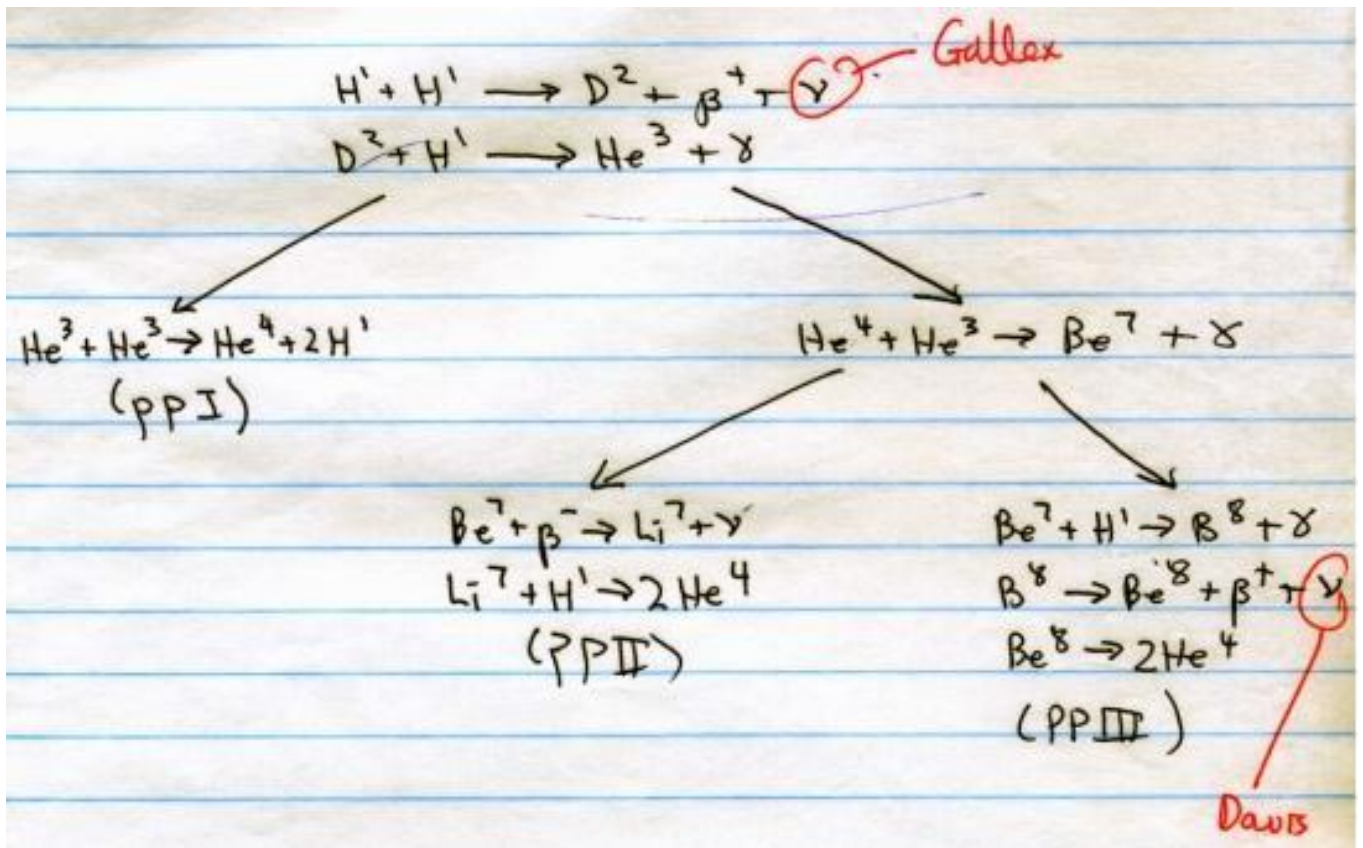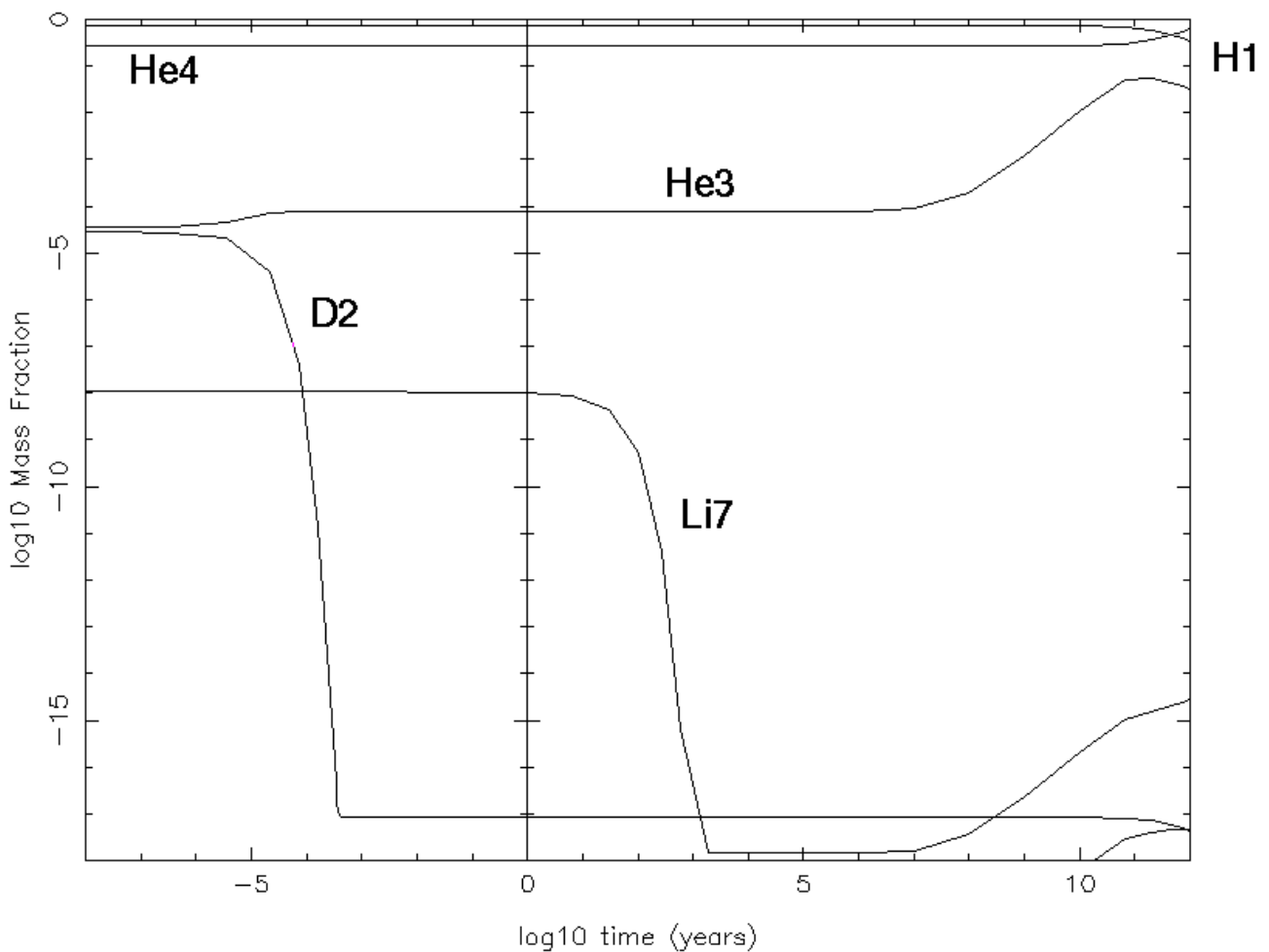
Figure 1: The pp Chains

Figure 2: Results from pp burning at 5 million K

3

Q3 Investigate the time for He^3 equilibrium by giving a table of this time vs T, for T = 6, 8, 10, 12.5, 15, 17.5, 20, 25 and 30 million K. Keep the density at 100 g/cc and let it burn for 10^{11} years. Also give the approximate equilibrium value of He^3. It would be best to plot these using `gnuplot`.

2.3 Lithium-7

Q4a If we allow up to 10^{10} years for burning, at what temperature is Li^7 destroyed?

You can see why Li is considered to be a very "fragile" element! It is very easily destroyed.

Q4b At a typical H-burning temperature of 10 or 15 million K, how long does it take to destroy almost all of the Li^7?

3 CNO Cycling

We saw in lectures that as the temperature of burning increases then we switch from pp burning to CNO cycling. The approximate temperature at which this switch occurs is 13 million K (see Figure 3 below). Lets now look at these cycles using a similar code to that used for the pp chains.
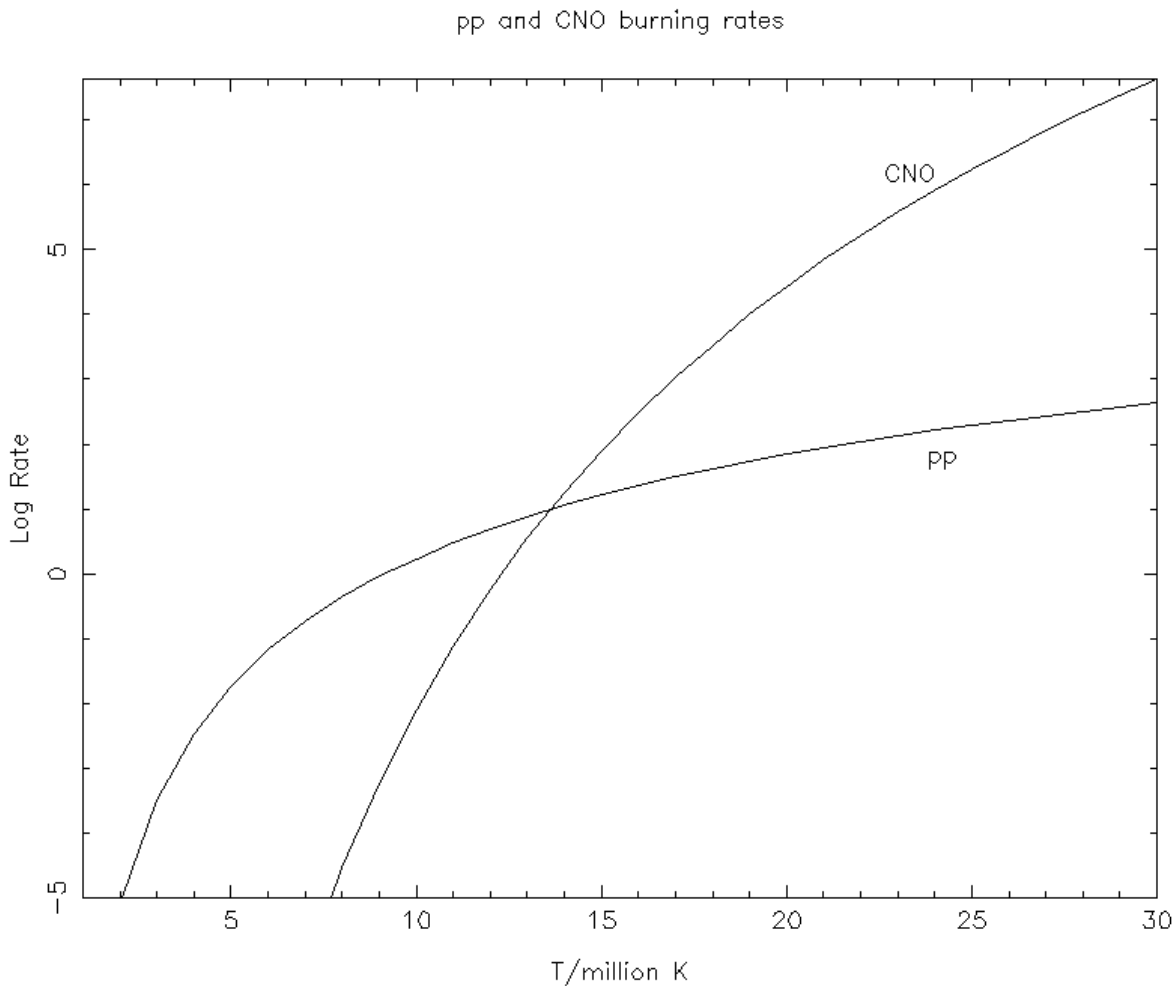
Figure 3: Relative rates for pp and CNO reactions

Compile the code `cno` which we will use now. Simply type `f95 -w -o cno cno.f` to do this. This code burns H by the CNO cycles only. There are *no pp chains operating!* This is not realistic but its good for learning how the CNO cycles work. We will later put them all together. Inputs and outputs to this code are analogous to the `pp` code you have been using.

The files that are written out are entirely analogous to the case of the pp chains, except there is an extra file here as there are more species. The ones you most likely will want to look at are `rootname2.dat` and `rootname3.dat`.

3.1 The CN Cycle

Run the code `cno` and enter 20 million degrees for the temperature and 100 g/cc for the density. Burn for 10 billion years. Then plot the results using `plotcn`. First you compile it by typing `make plotcn`. From now on we will use a convention of plotting unstable species with dotted lines. You will notice that these species never really acquire high concentrations. This is because they decay so rapidly!

Have a look at the case you just ran. The two unstable species, N^{13} and O^{15} are never very abundant.

Q5a What is the half-life of N^{13} and O^{15}? You can get this from my chart of the nuclides webpage — `http://yoyo.cc.monash.edu.au/~simcam/ton/`

You will notice that nothing happens until a time of about ten years. Zoom in on the graph, by setting the lower limit of the abundances to -8 (assuming you are using logs) and xmin=0 (again, in logs).

Q5b What are the first species to change significantly, and how do they change? What reactions cause that (see the figure of the CNO cycles below).

Note that the early burning of C^{12} into C^{13} (via the temporary existence of the unstable N^{13}!) produces an equilibrium ratio of carbon isotopes which has the value near 3.5.

Q5c For this case, what is the equilibrium value of C^{12}/C^{13}?

At the same time that we see the destruction of C^{12} and the production of C^{13} we see the growth of N^{14}. Recall that this is a main feature of the CN cycles - almost all species end up as N^{14} because the reaction destroying N^{14} is the slowest.

Clearly in this case the CN cycle comes into equilibrium by a time of a few $\times 10^4$ years. There is a second increase in N^{14} at a time of about 10^8 years, but that is due to the ON cycle, which we will discuss below.

Q6a Run a case with T of 30 million K and a density of 100 g/cc. How long does it take for the CN cycle to come to equilibrium now? What is the duration of H burning?

Q6b How about 50 million degrees? Use $\rho = 100 g/cc$ still. What is the duration of H burning?

Q6c Try 100 million. Keep ρ at 100g/cc. What is the duration of H burning?

Q6d What do you notice about the abundance of the unstable species as the temperature increases?

3.2 The ON Cycle

We saw that the CNO cycles can be divided into the CN and ON cycles because they operate on different timescales. The CN cycle comes into equilibrium long before the ON cycle. Lets have a look at the same cases we just ran but now we will look at the species taking part in the ON cycle.

Run the code `ploton` (you need to compile it first, with `make ploton`) and enter the 20 million degree case you previously ran. Zoom in as you did before: with $\log(t)$ between 0 and 10, and log of abundances between -6 and 0. For orientation I have included the carbon isotopes, as we can use these as a diagnostic of the CN cycle. Note that there is a second rise in N^{14} which occurs at the same time as the destruction of O^{16}. This signifies the operation of the ON cycle. Note also that the ratio O^{16}/O^{17} is approximately constant at a value of about 200. Thus the ON cycle produces some significant amounts of O^{17}. On the other hand O^{18} is significantly destroyed.

> **Q7** Using the cases run for a density of 100g/cc and temperatures of 20, 30, 50 and 100 million K, give a table (or a graph) of the approximate time when the ON cycle comes to equilibrium.

There is another routine you can use for investigating the CNO cycle. This is `plotcno-select`, which allows you to specify which species you want to plot. You can specify `all` to get all of the species of interest to the CNO cycles, including the unstable species that are only important in the hot CNO cycles (at T exceeding 100 million K). This enables you to look at things in an uncluttered way.

As an example, use `plotcno-select` to look at the ratio of N^{14}/N^{15} when the CN and ON cycles are operating.

> **Q8** What are typical values of the N isotope ratio at various temperatures, when the CN and ON cycles are operating in equilibrium?

4 Advanced Burning

For those of you who want to have some fun there is one more thing you can do. There is a code `Hburn` which contains a fully general network of 489 isotopes and 5939 reactions!

SINs Group, Centre for Stellar and Planetary Astrophysics, Monash University
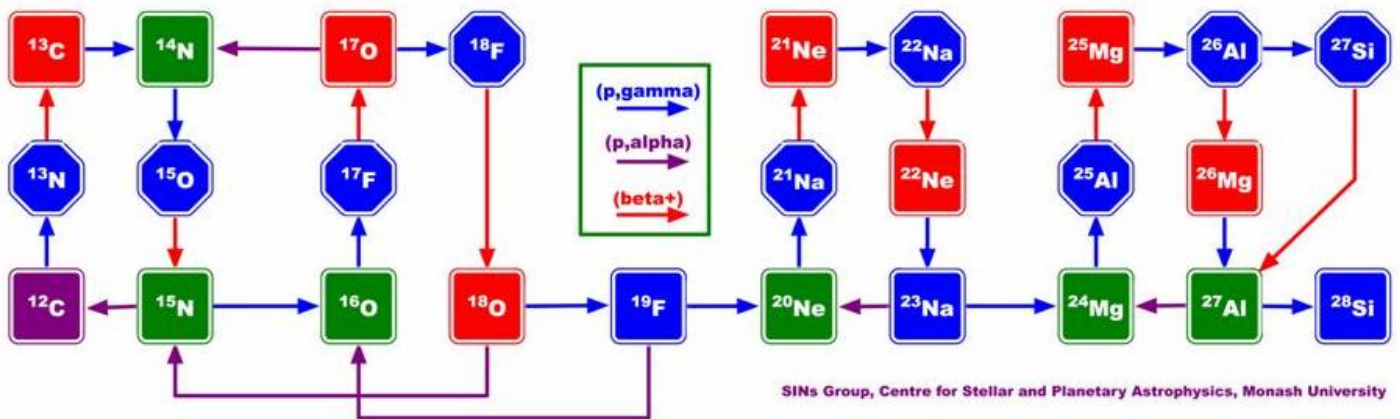
Compile and run the code `Hburn` and enter the usual temperature and density and integration time. This code includes all pp and CNO reactions at the same time (as well as many more!) You will see that there are a lot of files made now, as there are 489 isotopes to store results for. In fact, there are some 66 data files — `rootname01.dat` to `rootname63.dat` as well as `rootnamedecayed.dat`, `rootnamefinal.dat`, and `rootnamesummary.dat`.

If you choose high enough temperatures, the code will continue through to He burning, but it stops there.

To plot the results there are some new routines.

- `plotnena.f` Specifically designed to plot those species figuring in the Ne-Na chain.

- `plotmgal.f` Specifically designed to plot those species figuring in the Mg-Al chain.

- `plothotCNO.f` Specifically designed to plot those species figuring in the hot CNO cycles.

- `plotHburn.f` This allows you to specify whatever you want to plot. If you specify `all` you will get a wonderful mess showing 45 species!

Feel free to explore with these codes. There is a lot of fascinating stuff here. Here is one more question, for **Bonus Marks!**

Run `Hburn` with T=200 million K and $\rho = 100$g/cc. Integrate for 10^{10} years. Plot the results with `plotHburn` and specify the following isotopes: H^1, He^4, C^{12} and O^{16}. Use logs to plot and set ymin to be -3 and ymax to be 0.

Q9 What are the ultimate products of He burning in this case?