Section 1: Stars

Question 1

a) A binary star has two equal components of apparent magnitude 1. In a telescope which cannot resolve the system into two components, what is the magnitude of the single object seen by this telescope? (You may take $\log_{10}(2) = 0.3$.)

b) Show that a homogeneous star with pressure due to a perfect gas with radiation can be modelled as a polytrope of index 3 if the ratio of gas to total pressure β is constant throughout the star.

c) Consider a perfect monatomic gas with $\gamma = 5/3$ and no radiation pressure. Show that under adiabatic changes we have

$$\nabla_{ad} = \frac{\mathrm{d}\ln T}{\mathrm{d}\ln P} = \frac{\gamma - 1}{\gamma} = 0.4.$$

d) Explain the difference between an "adiabatic" gas and an "isothermal" gas. You should use mostly words but can refer to equations as well if you like.

Question 2

a) A star initially has a convective envelope of mass M_e. During the first dredge-up event it extends inward to mix with a mass M_B of material which has undergone complete hydrogen burning by the CNO cycles. Let the initial mass-fractions in the envelope be X for hydrogen, Y for helium, X_{12} for C^{12}, X_{14} for N^{14} and X_{16} for O^{16}.

   i) Calculate the new abundances after the dredge-up, as well as the changes ΔX, ΔY, $\Delta \mathrm{C}^{12}$, $\Delta \mathrm{N}^{14}$ and $\Delta \mathrm{O}^{16}$.

   ii) Does the metallicity $Z = 1 - X - Y$ change as a result of this mixing? Prove your assertion.

b) The Lane-Emden equation can be written as

$$\frac{\mathrm{d}}{\mathrm{d}\xi}\left(\xi^2 \frac{\mathrm{d}\theta}{\mathrm{d}\xi}\right) = -\xi^2\theta^n$$

Derive the solution for $n = 0$, applying the appropriate boundary conditions, and hence determine the scaled radius ξ_1 of the resulting model.

Question 3

Consider the following set of fictional nuclear reactions:

$$\mathrm{A} + \mathrm{A} = \mathrm{B} + \gamma \qquad \text{rate} = r_{AA} \tag{1}$$
$$\mathrm{A} + \mathrm{B} = \mathrm{C} + \gamma \qquad \text{rate} = r_{AB} \tag{2}$$
$$\mathrm{B} + \mathrm{C} = \mathrm{D} + \mathrm{A} \qquad \text{rate} = r_{BC} \tag{3}$$

a) What is the mass number of species D relative to that of species A?

b) Determine expressions for the rates r_{AA}, r_{AB} and r_{BC}.

c) Determine differential equations for the number abundances n_A, n_B and n_C for species A, B and C, respectively.

d) Suppose that species C comes into equilibrium. Determine an expression for the equilibrium abundance of species C in terms of species A: $(n_C/n_A)_{eq}$.

e) By assuming species C is in equilibrium, rewrite the differential equations without any reference to species C.

Section 2: Galaxies

Question 4

a) Describe, with the aid of an annotated sketch, the shape of the spectrum of continuum radio emission from the lobes of a powerful extragalactic radiosource. Include in your discussion the optically thick and optically thin regions of the spectrum, and describe how the shape of the spectrum is modified by radiation losses as the radiosource ages.

b) In galaxy clasification, what do the terms SBb and E3 mean?

c) Describe the Faber – Jackson for elliptical galaxies. Sketch this.

d) Explain how it is possible to identify neutral hydrogen (HI) in the interstellar medium.

e) Describe two methods used to determine the masses of black holes in galactic nuclei.

Question 5

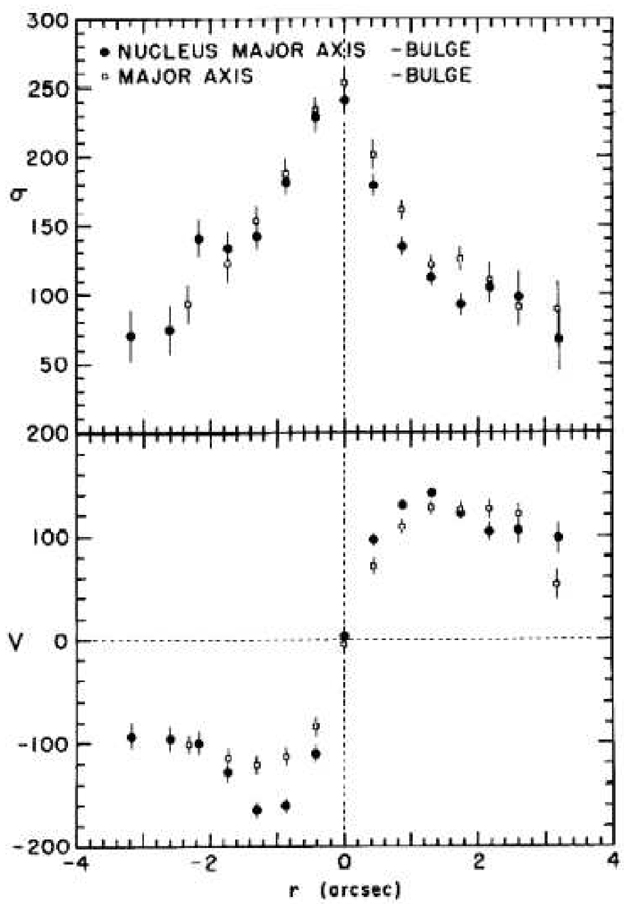
a) Using the formula for the apparent projected velocity $\beta_{app}c$ of material in a jet with velocity βc at an angle θ to the line of sight

$$\beta_{app} = \frac{\beta \sin\theta}{1 - \beta\cos\theta}$$

show that for any particular jet, the minimum apparent speed is:

$$\beta_{min} = \sqrt{\frac{\beta_{app}^2}{1 + \beta_{app}^2}}$$

b) The following image shows the stellar velocity dispersion and rotational velocities of stars in M31.



i) Describe what the graphs tell you about the inner region of M31

ii) Estimate the dynamical mass of the central black hole of M31 using the rotational velocity measured at 1 arcsec (3.7 pc) distance from the center.

Question 6

a) What is the physical significance of the Eddington luminosity, L_{Edd} Derive the expression for pure hydrogen plasma accreting onto an object of mass M , where σ_T is the Thomson cross-section, clearly stating any assumptions made.

b) Assume that the emission from a geometrically-thin, optically-thick accretion disc is dominated by a component from its innermost part. Take this to be a uniform circular ring at radius $r = 6GM/c^2$, where M is the black hole mass, G is the gravitational constant and c is the speed of light.

If the luminosity of a ring is dL_{ring}, then the energy radiated by the ring in time t is $dE = dL_{\mathrm{ring}}\ t$. $\dot{M}$ is the accretion mass rate through the ring. Derive an approximate expression for the black-body temperature of radiation from the inner ring.

c) Estimate the temperatures for black holes of $10M_{\mathrm{sun}}$ and $\dot{M} = 1.6 \times 10^{-10}\ M_{\mathrm{sun}}/yr$.

Estimate the temperatures for black holes of $10^8 M_{\mathrm{sun}}$ and $\dot{M} = 1.6 \times 10^{-1}\ M_{\mathrm{sun}}/yr$.

d) Comment on your results in the context of the observed spectra of X-ray binaries and active galaxies.

USEFUL FORMULAE

$$\int_{-\infty}^{+\infty} e^{-\alpha u^2} du = \sqrt{\frac{\pi}{\alpha}}$$

$$\Re = \text{Universal Gas Constant} = 8.314 \times 10^7 \text{erg/K/g}$$

$$G = 6.67 \times 10^{-8} \text{cm}^3/\text{g}/\text{s}^2$$

$$\mu_o = 4\pi \times 10^{-7} N A^{-2}$$

$$k = \text{Boltzmann's constant} = m_u \Re = 1.38 \times 10^{-16} \text{erg/K}$$

$$a = \text{radiation density constant} = 7.57 \times 10^{-15} \text{erg/cm}^3/\text{K}^4$$

$$c = \text{speed of light} = 2.99 \times 10^{10} \text{cm/s}$$

$$\sigma = ac/4 = 5.67 \times 10^{-8} \ \ \text{Watts/m}^2/\text{K}^4$$

$$1 \text{ Gauss} = 10^{-4} \text{ Tesla}$$

$$1 \text{ erg} = 10^{-7} \text{ J}$$

$$1 \text{ parsec} = 3 \times 10^{18} \text{ cm}$$

$$R_\odot = 6.96 \times 10^{10} \text{ cm}$$

$$M_\odot = 1.98 \times 10^{33} \text{ g}$$

$$L_\odot = 3.86 \times 10^{33} \text{ erg/sec}$$

$$1 \text{ year} = 3.156 \times 10^7 \text{ s}$$

$$\begin{aligned}
m_u = \text{atomic mass unit} &= 1.66053 \times 10^{-24} \text{g} \\
\text{mass of H nucleus} &= 1.0073\, m_u \\
\text{mass of H}_2 \text{ molecule} &= 2.0160\, m_u \\
\text{mass of He}^4 \text{ nucleus} &= 4.0014\, m_u \\
\text{mass of He}^4 \text{ atom} &= 4.0028\, m_u \\
\text{mass of C}^{12} \text{ nucleus} &= 12.0000\, m_u
\end{aligned}$$

For a perfect monatomic gas :

$$\frac{c_P}{c_V} = \gamma = 5/3$$

$$\frac{\Re}{\mu} = c_P - c_V$$

$$u = c_V T = \frac{3}{2}\frac{\Re}{\mu} T$$

For radiation :

$$P = \frac{1}{3} a T^4$$

$$u = \frac{aT^4}{\rho}$$

$$m_1 - m_2 = -2.5 \log \frac{F_1}{F_2}$$

$$M_{BOL} = -2.5 \log \left(\frac{L}{L_\odot} \right) + 4.72$$

$$m - M = 5 \log d - 5 \quad \text{without extinction}$$

$$m - M = 5 \log d - 5 + A \quad \text{with extinction of } A \text{ magnitudes}$$

Reaction rate between species i and j is

$$r_{ij} = \frac{n_i n_j}{1 + \delta_{ij}} \sigma_{ij} \text{ reactions per second}$$

Eddington luminosity:

$$L_{\rm edd} = 1.3 \times 10^{38} \frac{M}{M_\odot} \text{ erg/s}$$

Some useful trigonometric formulae:

$$\sin \phi = \sqrt{\frac{1}{1 + \cot^2 \phi}}$$

$$\cos \phi = \sqrt{\frac{\cot^2 \phi}{1 + \cot^2 \phi}}$$

EQUATIONS OF STELLAR STRUCTURE:

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\rho \frac{GM(r)}{r^2}$$

$$\frac{\mathrm{d}L}{\mathrm{d}r} = 4\pi r^2 \rho \epsilon$$

$$\frac{\mathrm{d}M}{\mathrm{d}r} = 4\pi r^2 \rho$$

$$\frac{\mathrm{d}T}{\mathrm{d}r} = -\frac{3\kappa\rho}{16\pi a c r^2} \frac{L(r)}{T^3}$$

(if radiative)

End of Exam