School of Physics and Astronomy
February 2016

Lab Exercises for Low Mass Stars

1. Isochrones for the globular cluster M4

One common use of stellar models is to determine ages for stellar populations. This is usually done through fitting evolutionary tracks to an observed colour-magnitude diagram (CMD, or HR diagram, HRD). In fact, evolutionary tracks are constructed for a given (initial) mass, but show the path in the CMD as a function of time. To determine ages we need the opposite – a curve that shows how a population of co-evolutionary stars will appear at a given time. Hence these tracks are called *isochrones* and they show the CMD path at a given time, with the mass varying along the isochrone.

The first step is to determine the abundance for the cluster, usually [Fe/H] which is assumed to scale with Z. We know that this is not strictly true but it is a good start. However, for globular clusters we know that the alpha elements are enhanced, typically with [α/Fe] = +0.3-0.4. So here the nexus between Fe and Z is clearly broken!

It is tedious running lots of models that are the same except for mass. So we have done that for you. You will see a list of files all with names like HRDMx.dat; these are the resultant evolutionary tracks for the approximate composition of M4, where x is the (initial) mass of the star in that file. Within each file the data given is age (in Gyr), $\log(T_e)$ and $\log(L/L_\odot)$.

The composition of M4 is thought to be [Fe/H] = -1.15 and [α/Fe] = +0.38; and I have assumed Y=0.25. To get models for a relatively old cluster, we need low masses. I have constructed models for masses between $0.65M_\odot$ and $0.90M_\odot$. The lower masses will have hardly moved from the ZAMS so I have not evolved them very far. The table below gives you the maximum age in each file.

The data for M4 is taken from Mochejska et al. (200), *Astrophys J*, 124, 1486.

THE ASTRONOMICAL JOURNAL, 124:1486–1496, 2002 September

CLUSTERS AGES EXPERIMENT: HOT SUBDWARFS AND LUMINOUS WHITE DWARF CANDIDATES IN THE FIELD OF THE GLOBULAR CLUSTER M4[1]

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The data is in the file M4data.dat and it contains $\text{Log}(T_e)$ and $\text{Log}(L/L_\odot)$. Note that quite some analysis has been done to obtain these values! Not least of all is the conversion from observed apparent magnitude and colour to luminosity and effective temperature. One needs the distance to convert to absolute magnitude and then one needs the bolometric correction (BC) to convert from the magnitude in a given filter (usually M_V) to the total (bolometric) magnitude, which is what the theorists calculate when they determine the luminosity. The two are of course related through:

$$M_{bol} = -2.5 \log (L/L_\odot) + 4.74$$

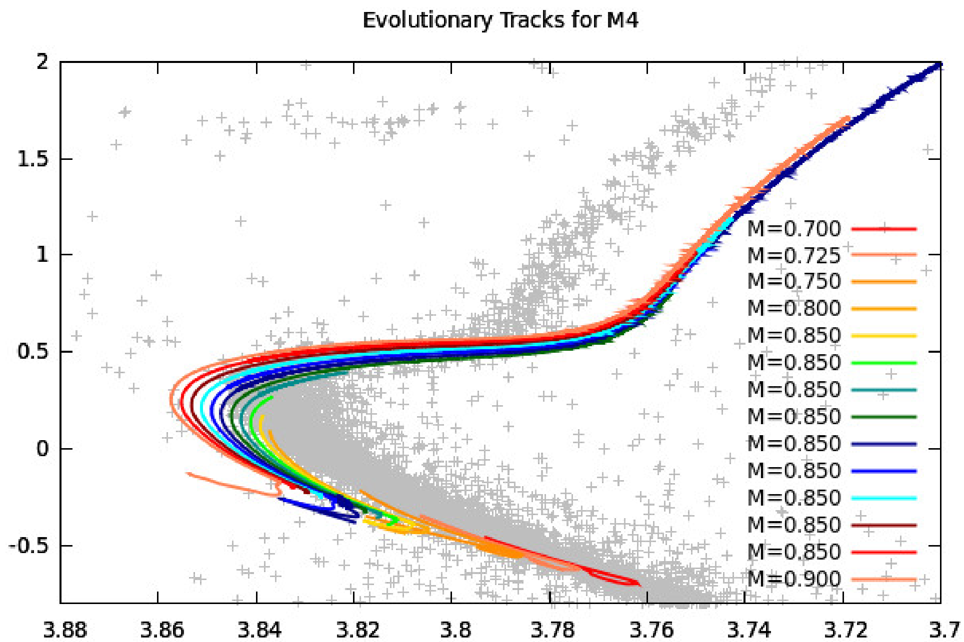
where 4.74 is the bolometric magnitude for the Sun. We also need the reddening, to convert the observed colour to the real colour and then to effective temperature.

Table 1. Maximum ages for stars provided

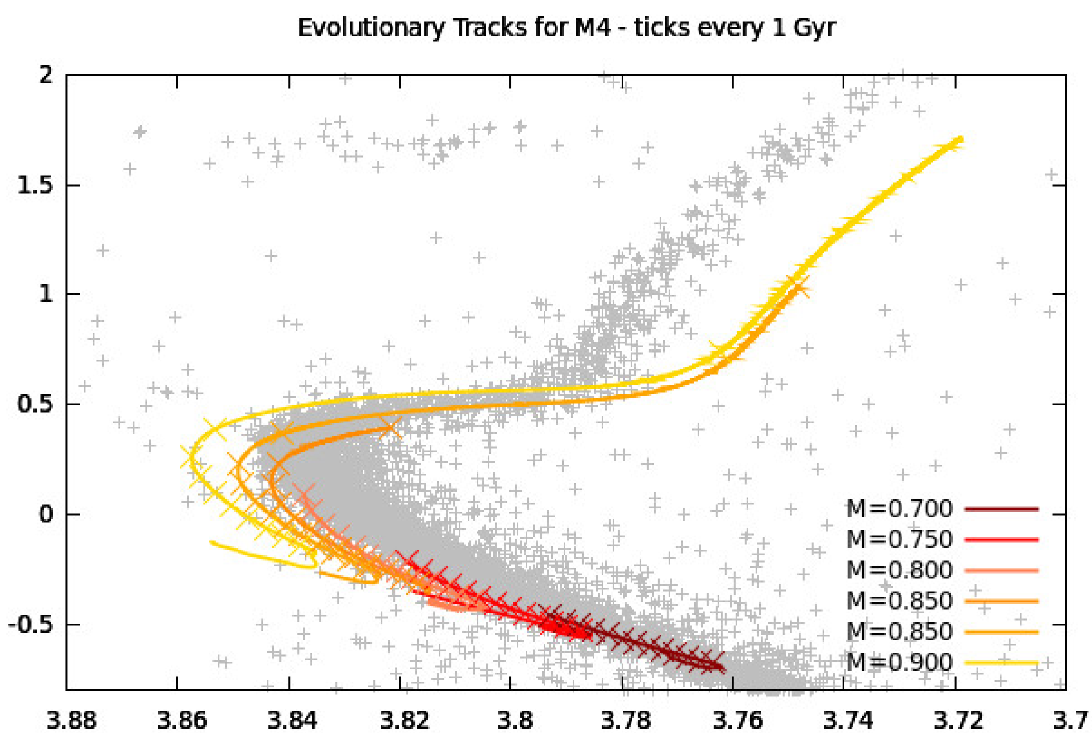
$M/M_\odot$	Maximum age (Gyr)
0.65	ZAMS only
0.70	13
0.725	13
0.75	13
0.80	13
0.81	13
0.82	13
0.83	13
0.84	13
0.85	12.75
0.86	12
0.87	11.5
0.88	11
0.89	10.5
0.90	10.3

a) Plot the evolutionary tracks for some of the masses in the HR diagram.
   i. Are they a good fit?
   ii. Where are they good and where are they bad?
   iii. What could we do to improve the fit?
   iv. Are we learning physics or playing with fitting things?
   v. You may like to plot some with tick marks every Δt for some chosen Δt. You may need a different Δt on the main sequence as opposed to the giant branch! Why?

Fit is not perfect! Tracks seem too blue and the RGB is way to cool.

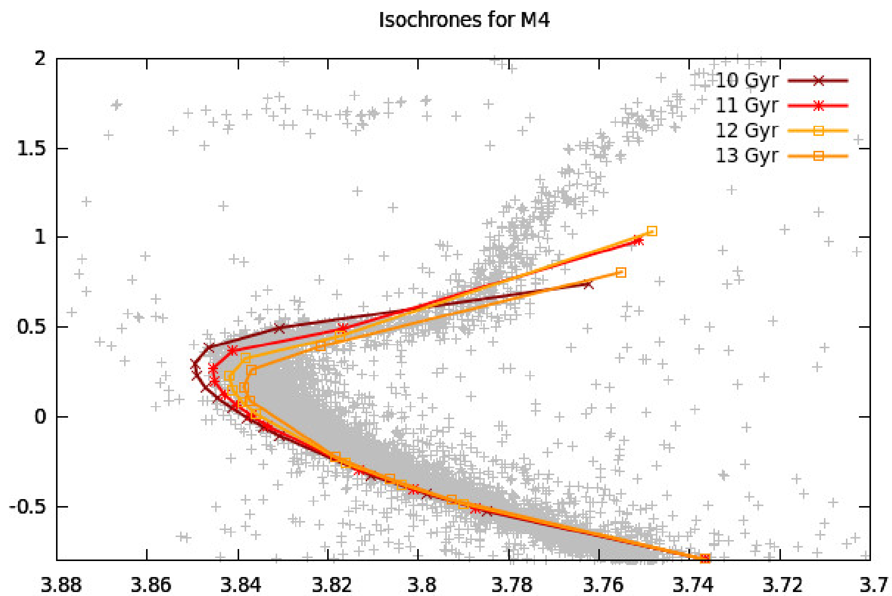


The RGB can be moved by altering the mixing-length.
The turnoff can be moved by including CNO elements properly, via the opacity.
We are indeed learning physics – such as what affects the position and can our models get it right with realistic choices?
Of course, reddening, distance and BC are also up for negotiation.



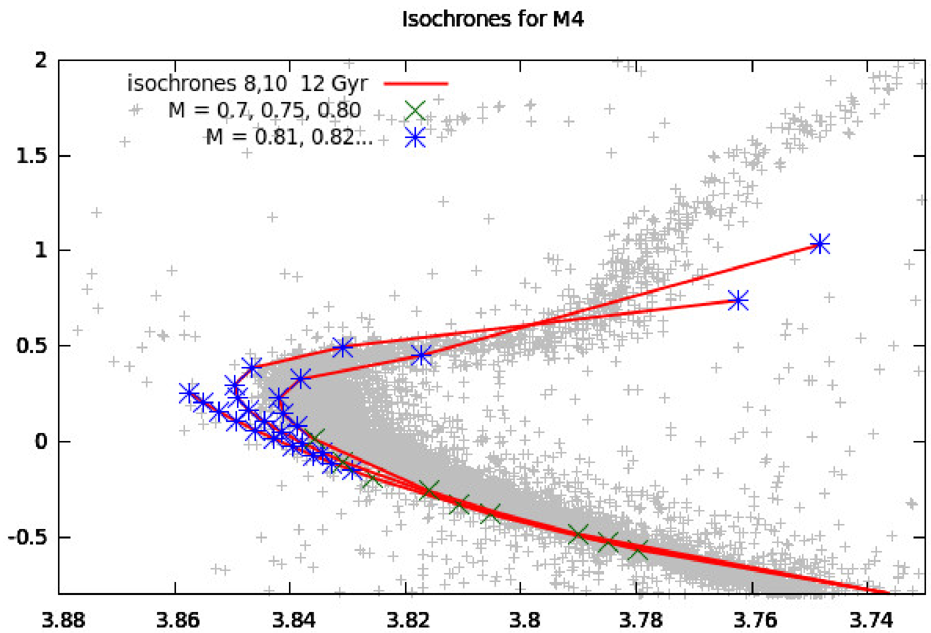
b) Now interpolate within the files to construct an isochrone for a chosen age. Linear interpolation is fine because the time-steps are small. You will need ages around the 10 to 13 Gyr range.

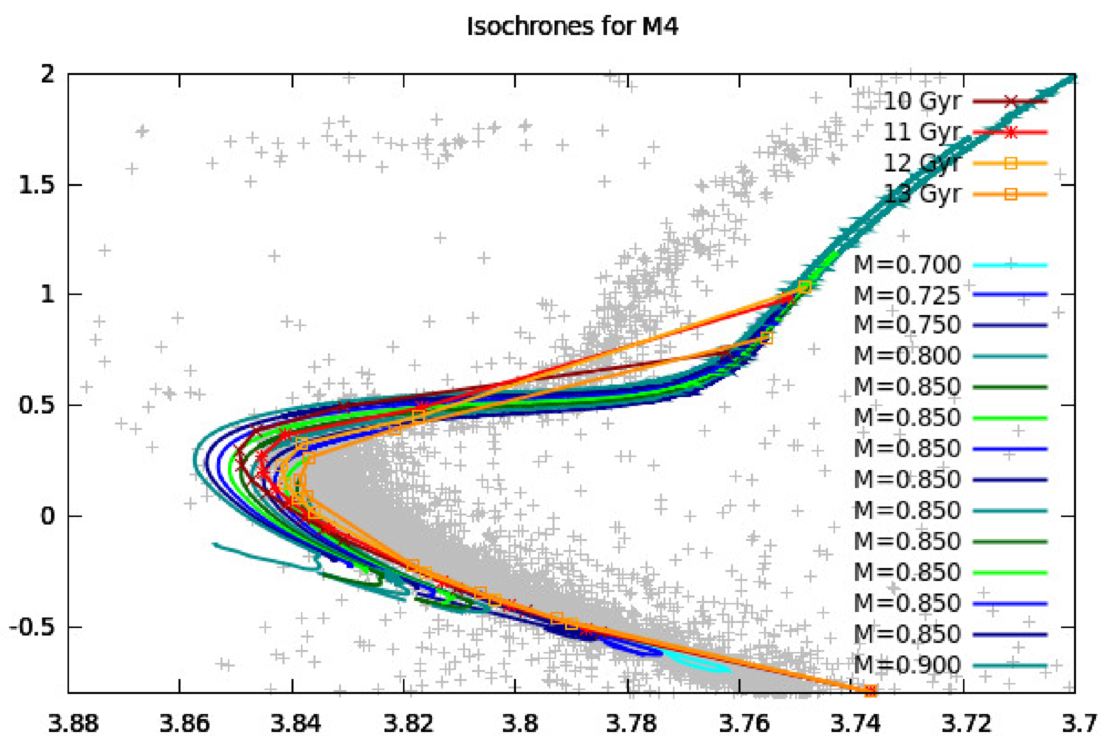
   i. Are they a good fit?
   ii. How can we resolve the giant branch better?
   iii. It might be worth plotting one isochrone with symbols every 0.01 in $M_{\odot}$.



They suffer form same problems as the tracks, obviously.

Note that there are not enough tracks for a smooth subgiant branch and RGB. We need more models with a smaller step in mass to get this more smooth.

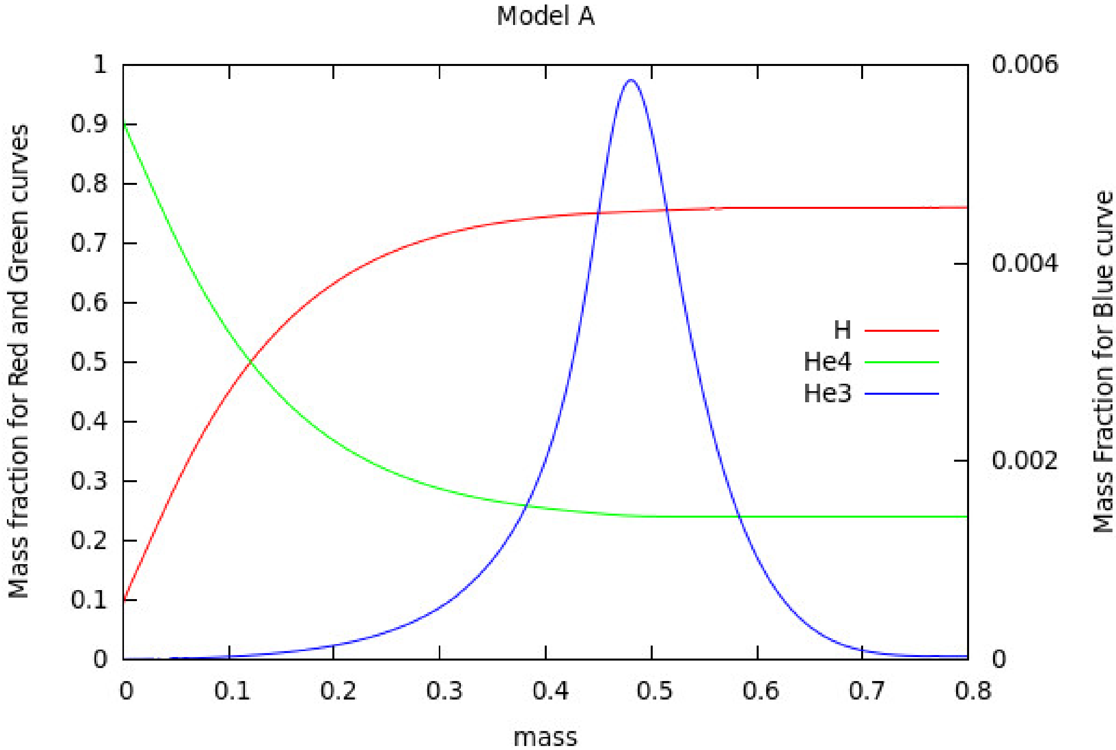

Isochrones for M4
isochrones 8,10 12 Gyr
M = 0.7, 0.75, 0.80
M = 0.81, 0.82...
2
1.5
1
0.5
0
-0.5
3.88
3.86
3.84
3.82
3.8
3.78
3.76
3.74

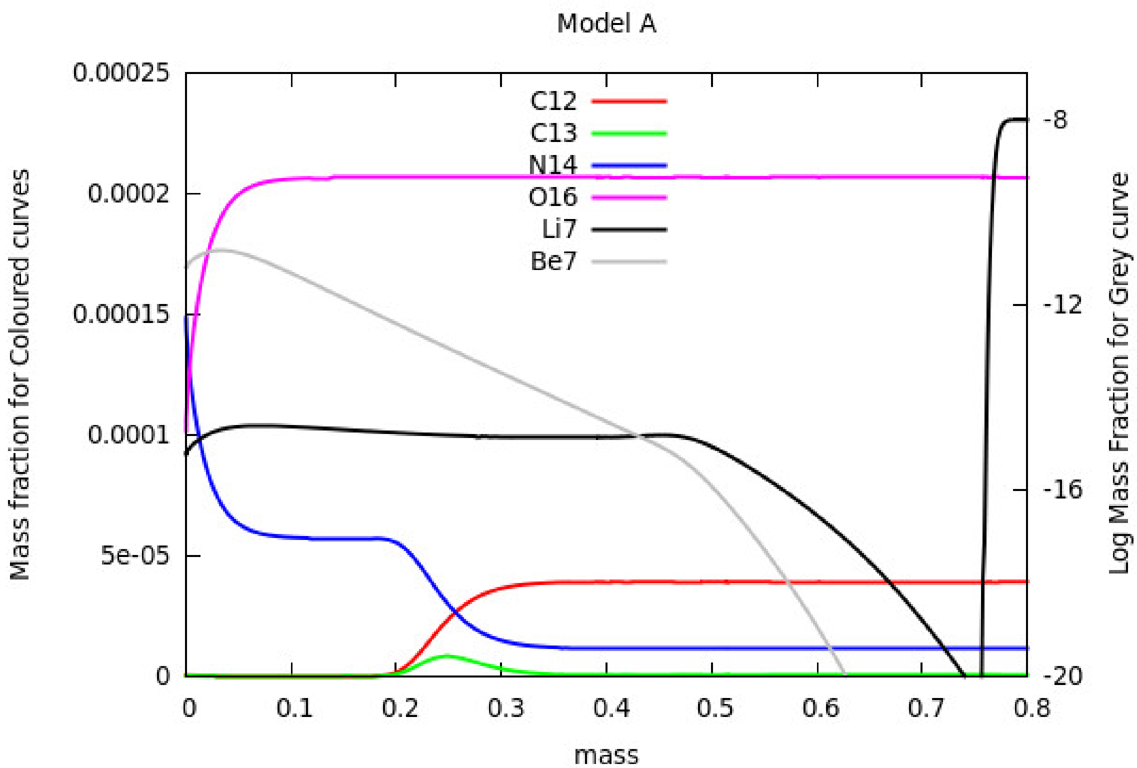

Isochrones for M4
10 Gyr
11 Gyr
12 Gyr
13 Gyr
M=0.700
M=0.725
M=0.750
M=0.800
M=0.850
M=0.850
M=0.850
M=0.850
M=0.850
M=0.850
M=0.850
M=0.850
M=0.850
M=0.900
2
1.5
1
0.5
0
-0.5
3.88
3.86
3.84
3.82
3.8
3.78
3.76
3.74
3.72
3.7

2. Abundance Profiles: The Key to Understanding a Star's Structure

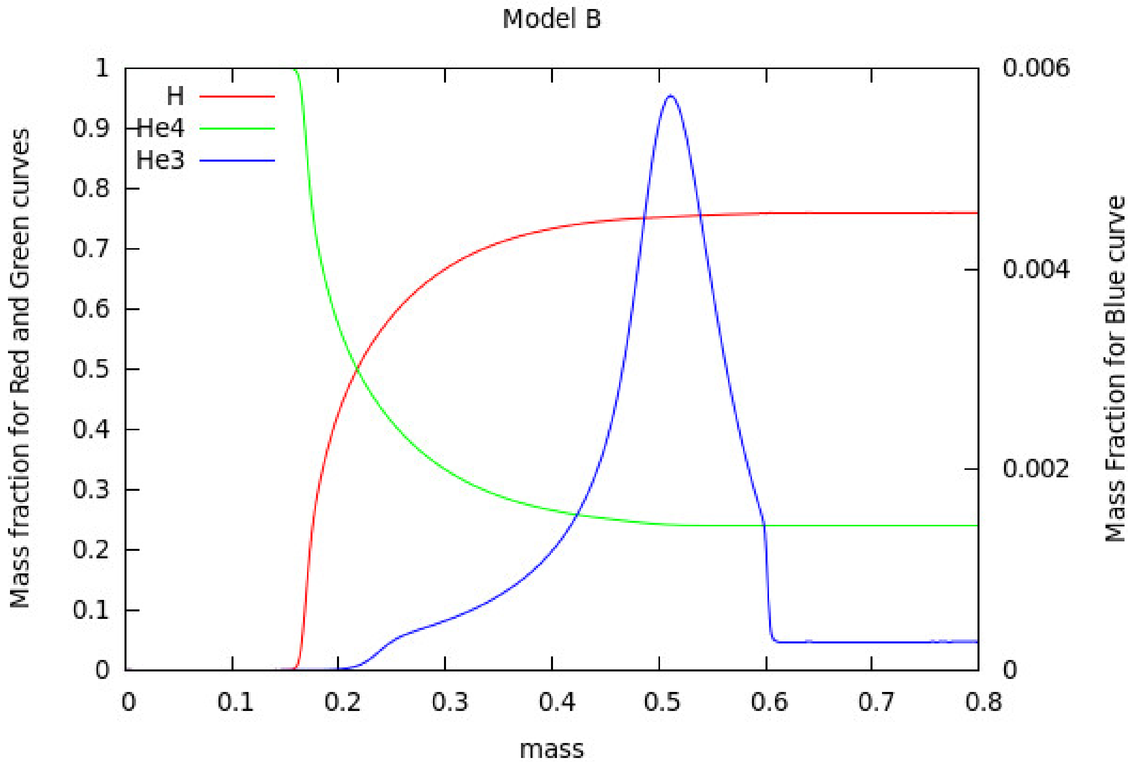
M=0.8 Z=0.00039 Y=0.24

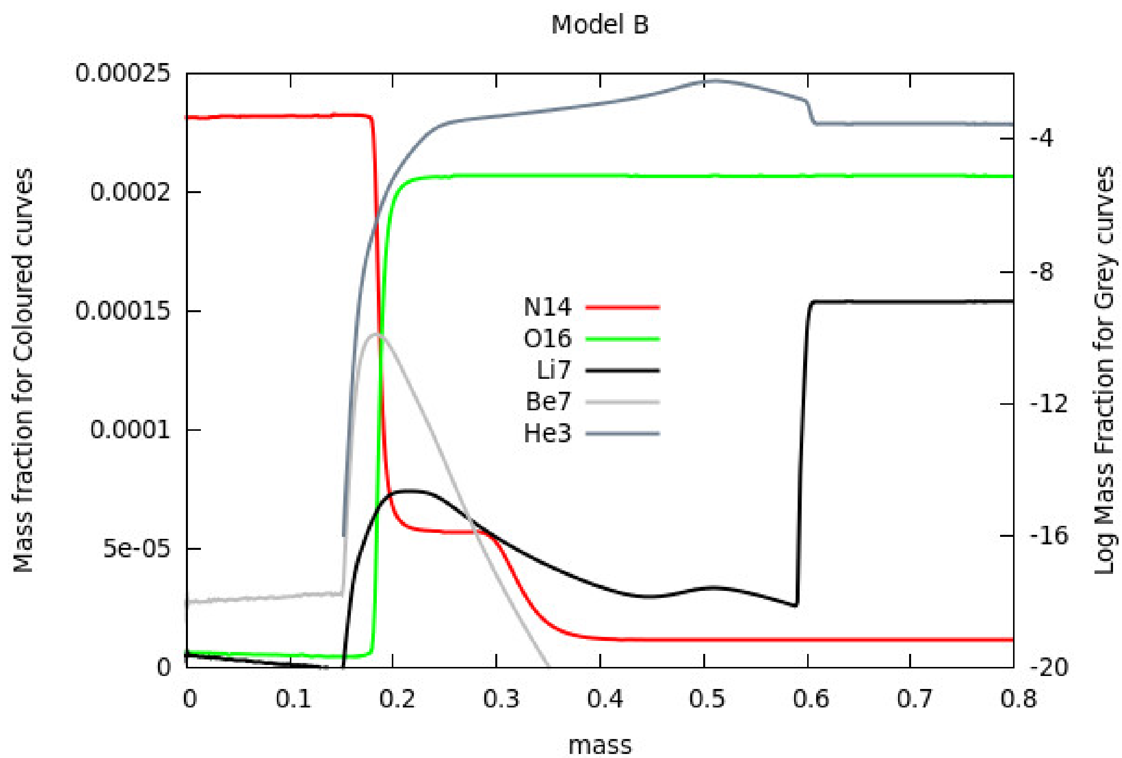
Model A: Core H Burning



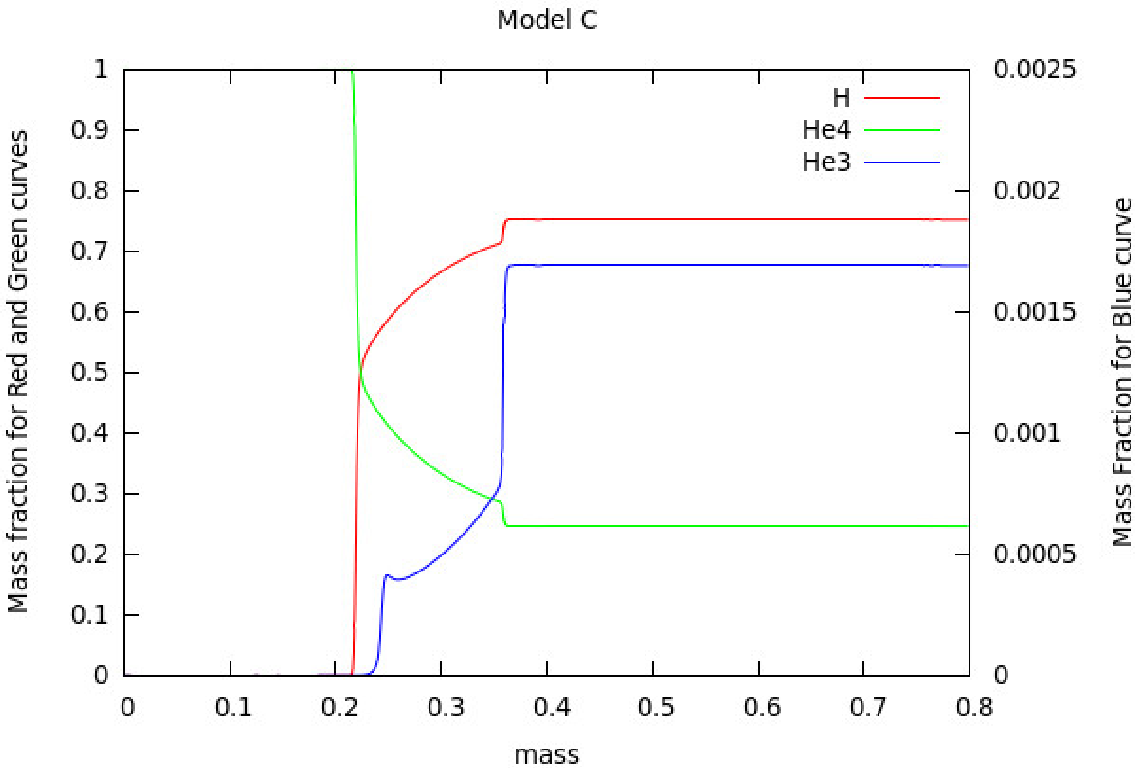


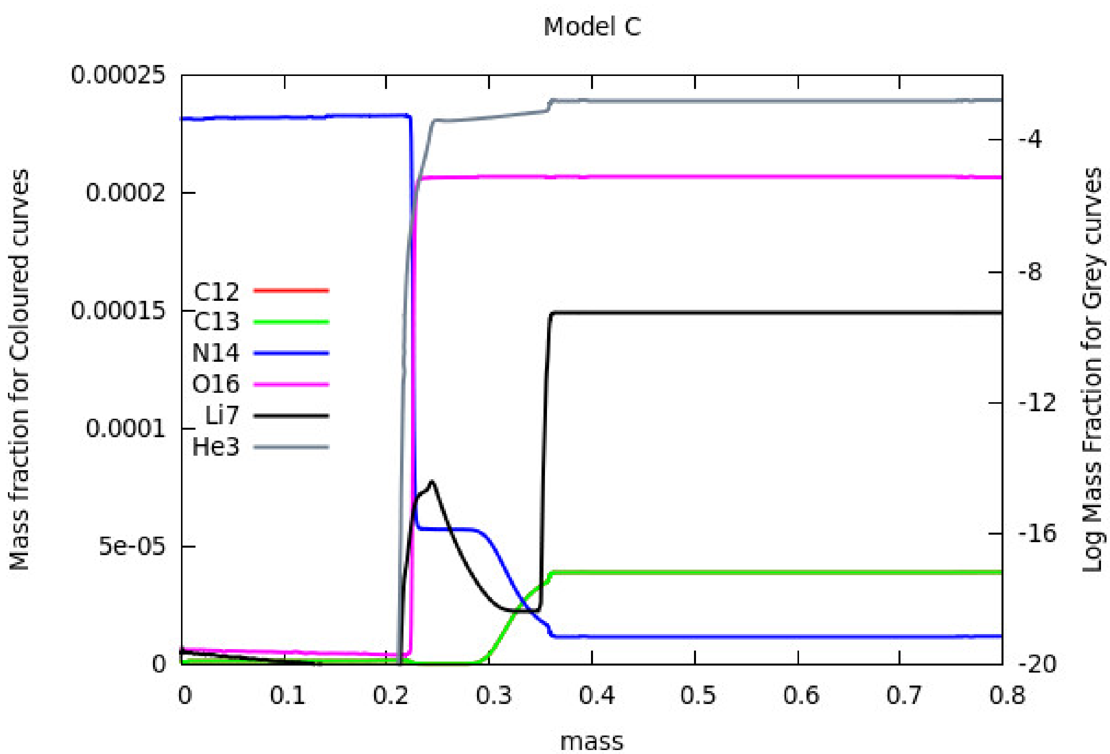
Model B: SubGiant Branch



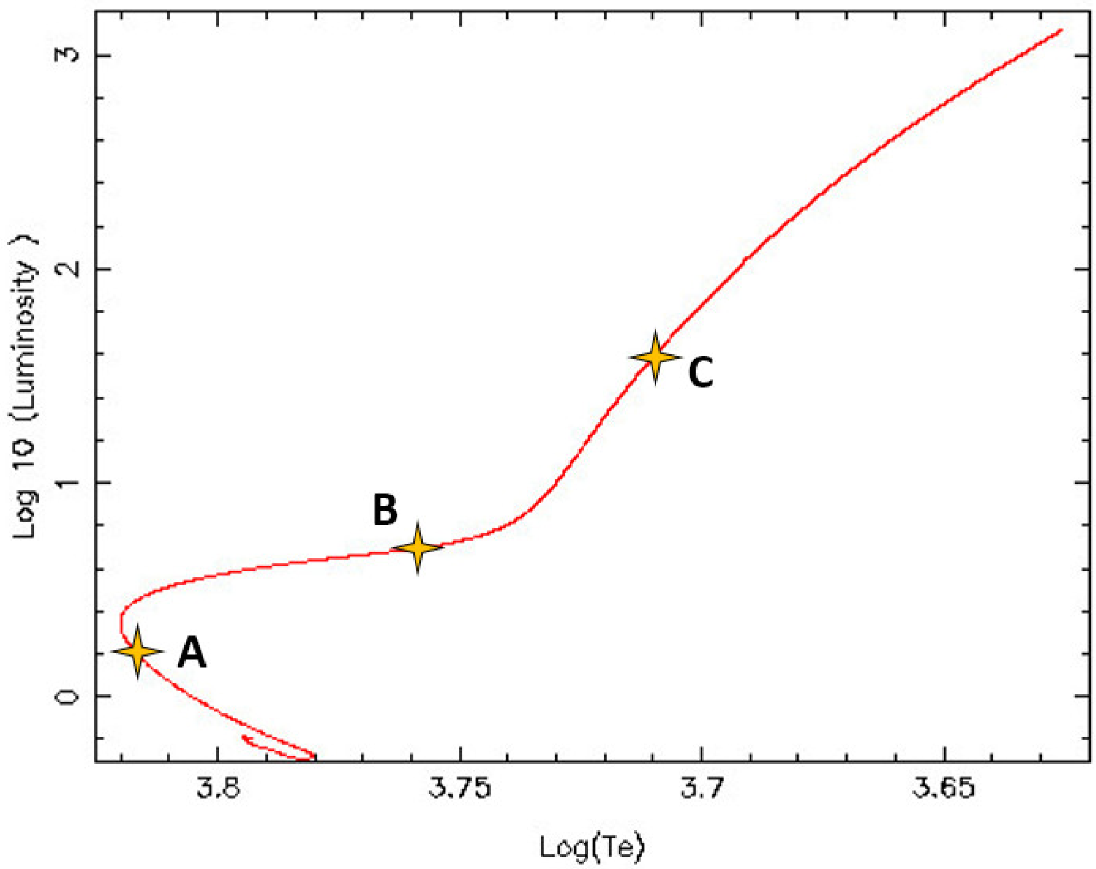


Model C


Model C
H
He4
He3
Mass fraction for Red and Green curves
Mass Fraction for Blue curve
mass
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.0025
0.002
0.0015
0.001
0.0005
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8

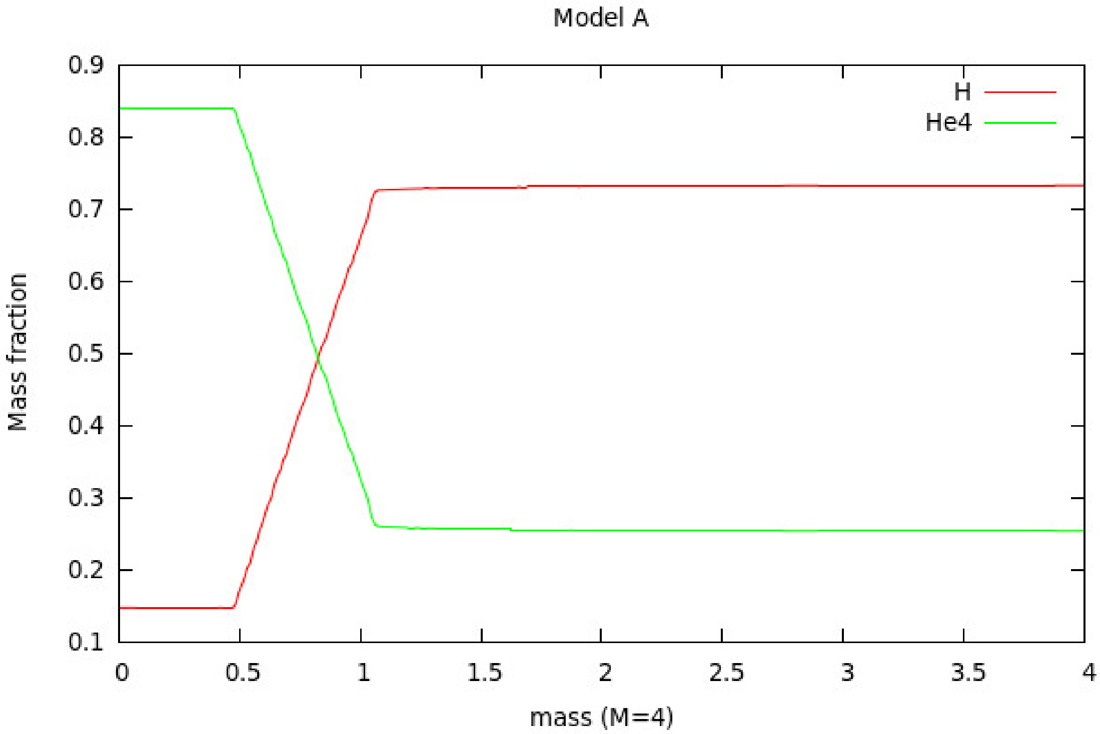

Model C
C12
C13
N14
O16
Li7
He3
Mass fraction for Coloured curves
Log Mass Fraction for Grey curves
mass
0.00025
0.0002
0.00015
0.0001
5e-05
0
-4
-8
-12
-16
-20
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8

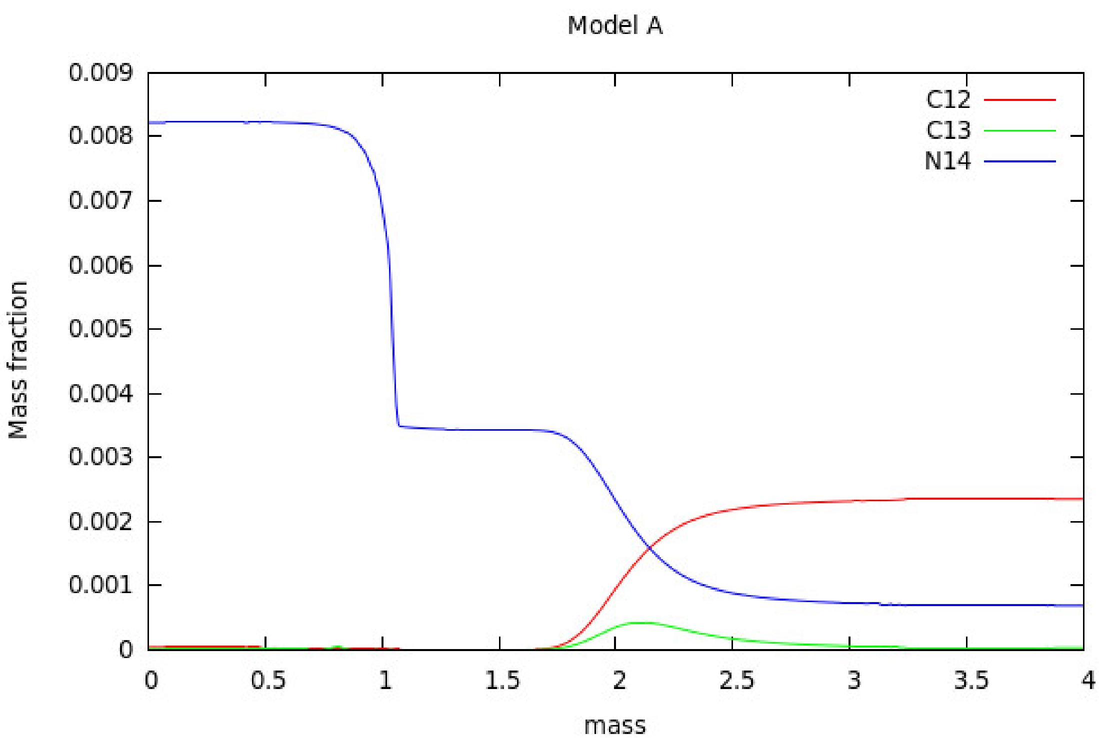
Resulting HR Diagram



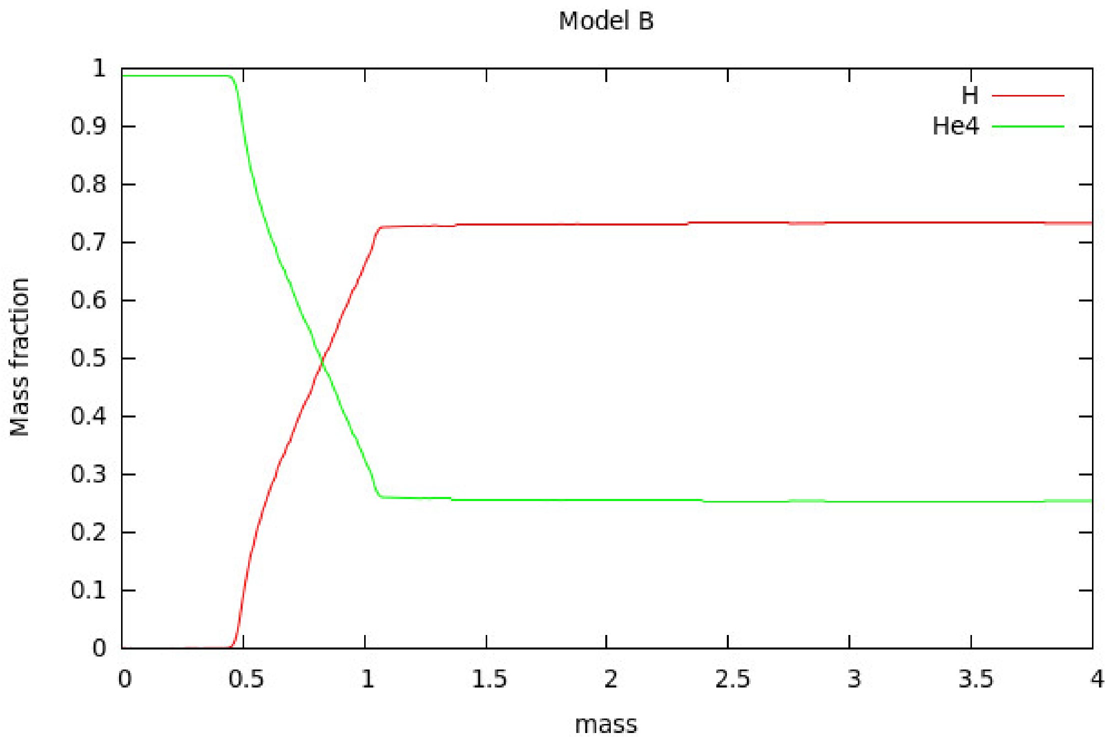
M=4 Z=0.0134 Y=0.254

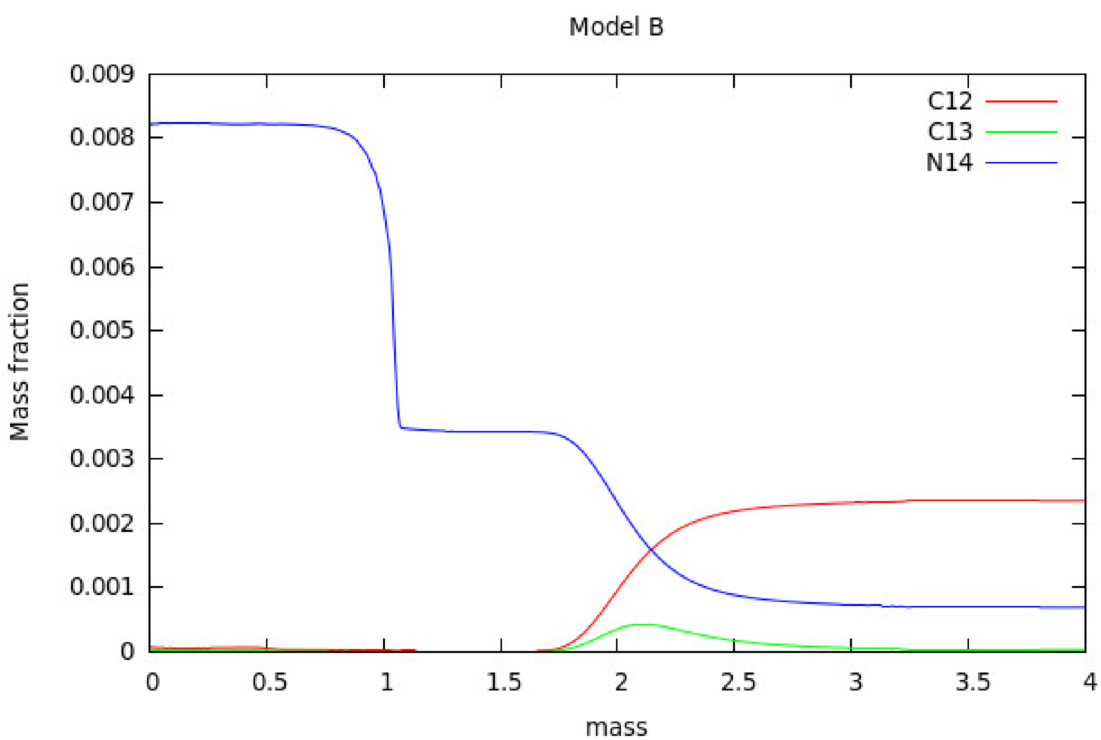
Model A: Core H Burning



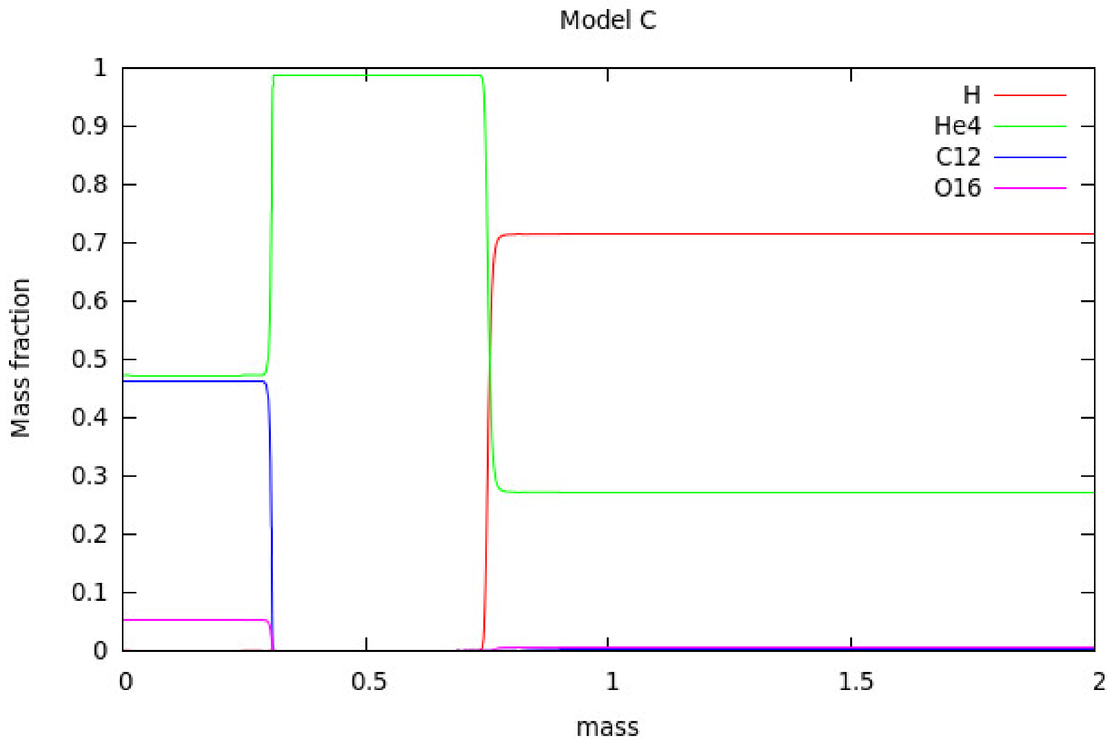


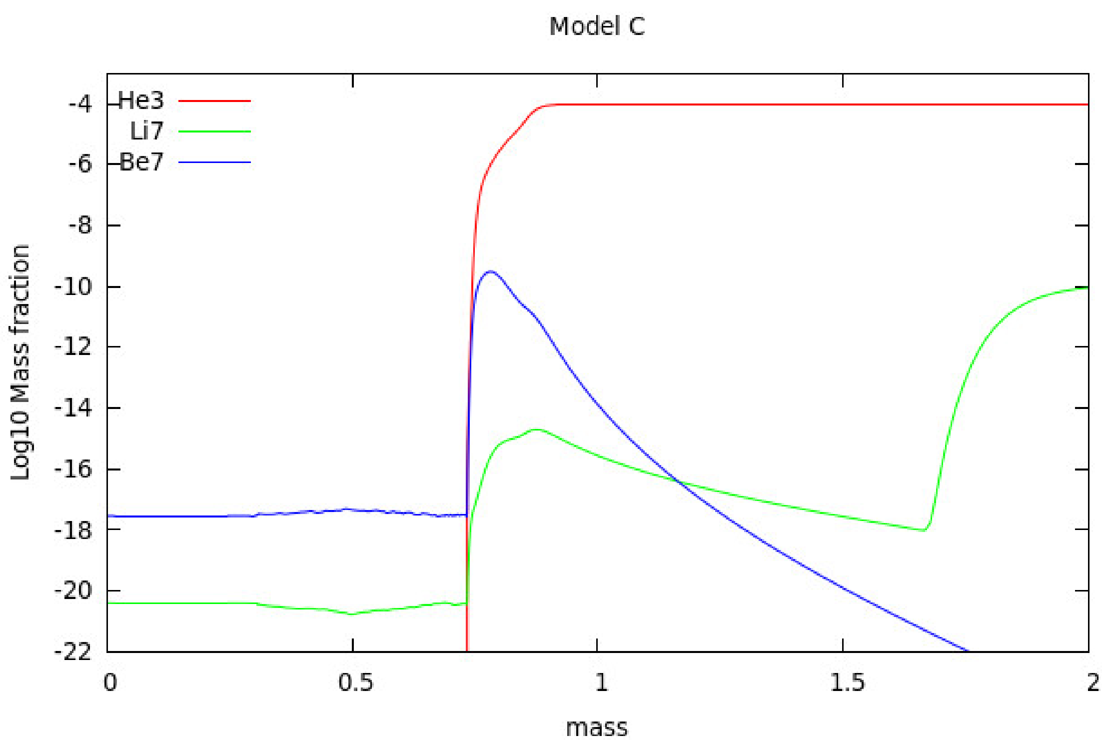
Model B: Sub-giant branch



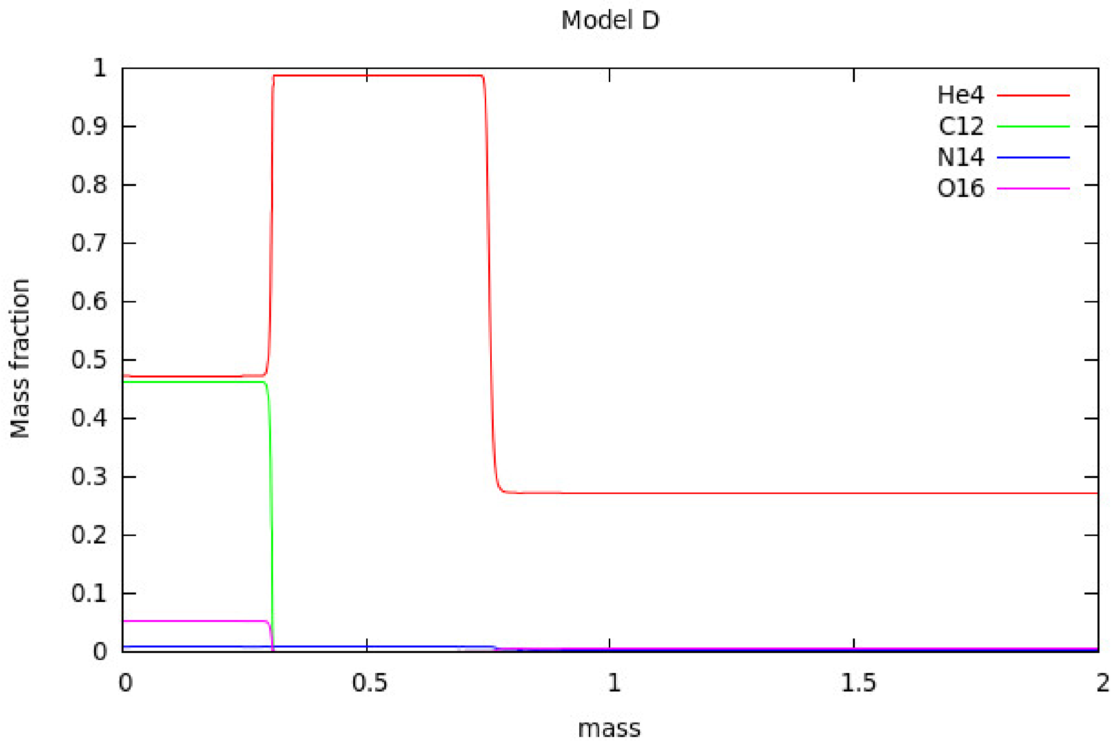


Model C: Core He Burning

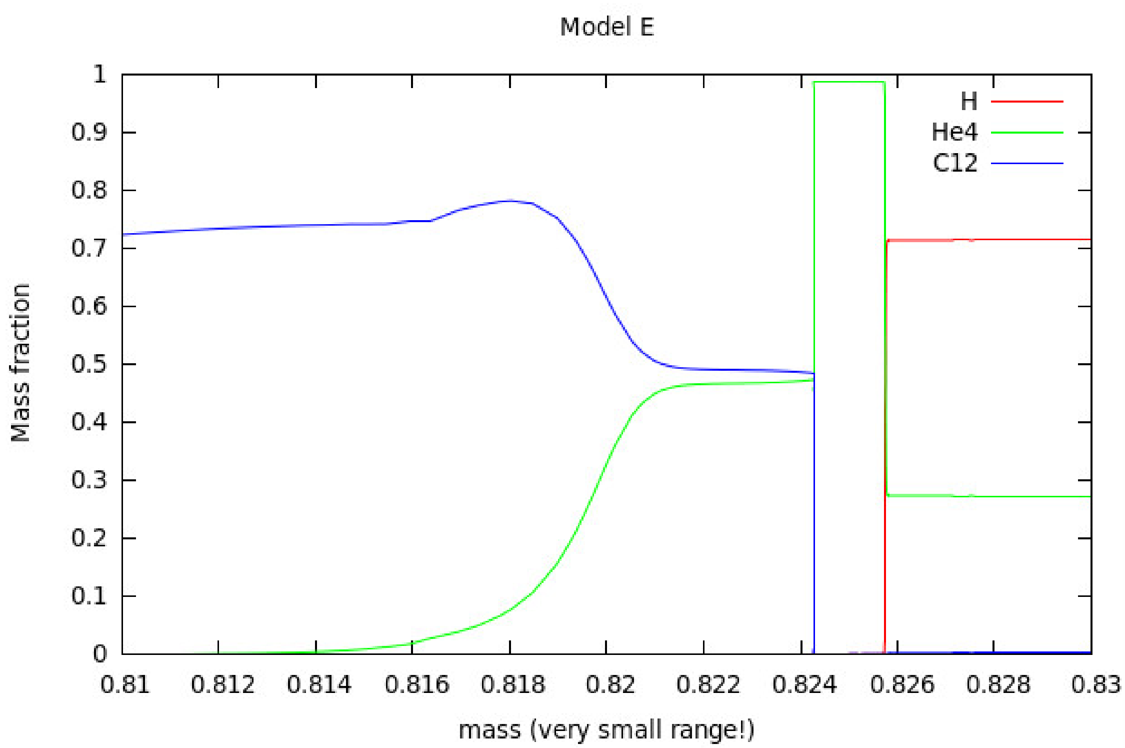




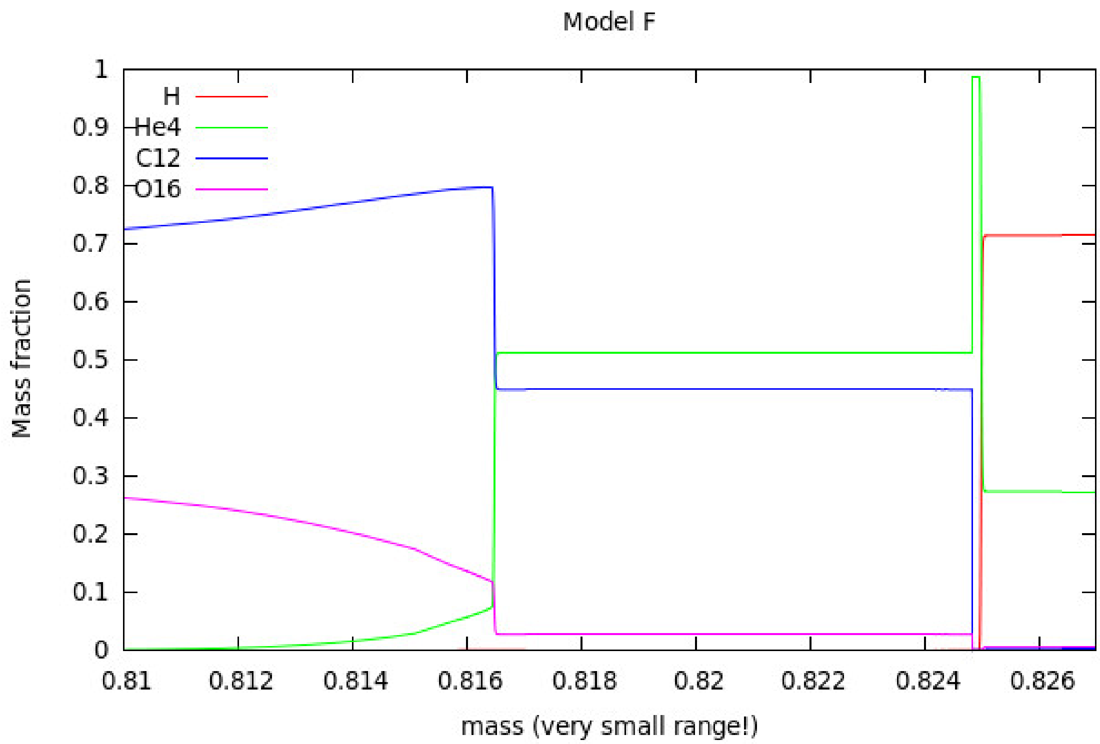
Model D: Core He Exhaustion



Model E: Interpulse Phase on the AGB

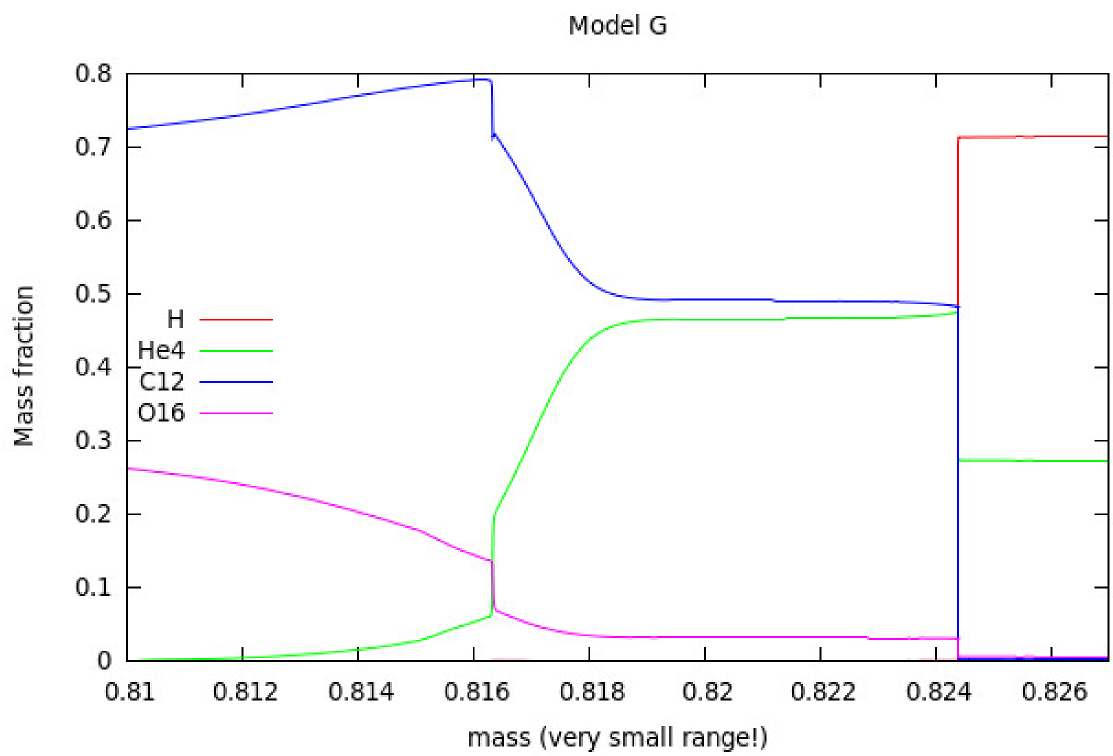


Model F: During a Pulse on the AGB

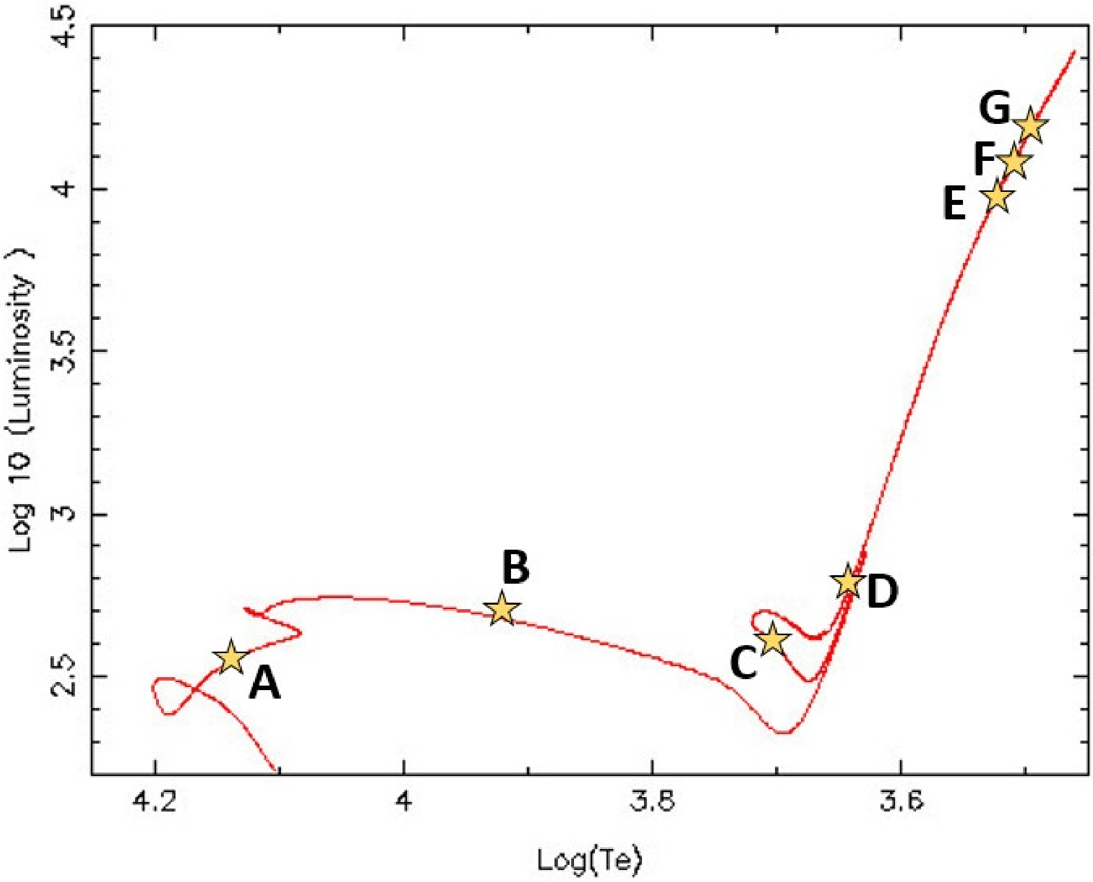


Model G: Third Dredge-Up on the AGB

Note that the x-scale is the same on Models F and G, to help identify the dredge-up.



Resulting HR Diagram



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