School of Physics and Astronomy
February 2016

Monash Centre for Astrophysics

Lab Exercises for Low Mass Stars

1. Isochrones for the globular cluster M4

One common use of stellar models is to determine ages for stellar populations. This is usually done through fitting evolutionary tracks to an observed colour-magnitude diagram (CMD, or HR diagram, HRD). In fact, evolutionary tracks are constructed for a given (initial) mass, but show the path in the CMD as a function of time. To determine ages we need the opposite – a curve that shows how a population of co-evolutionary stars will appear at a given time. Hence these tracks are called *isochrones* and they show the CMD path at a given time, with the mass varying along the isochrone.

The first step is to determine the abundance for the cluster, usually [Fe/H] which is assumed to scale with Z. We know that this is not strictly true but it is a good start. However, for globular clusters we know that the alpha elements are enhanced, typically with [α/Fe] = +0.3-0.4. So here the nexus between Fe and Z is clearly broken!

It is tedious running lots of models that are the same except for mass. So we have done that for you. You will see a list of files all with names like HRDMx.dat; these are the resultant evolutionary tracks for the approximate composition of M4, where x is the (initial) mass of the star in that file. Within each file the data given is age (in Gyr), $\log(T_e)$ and $\log(L/L_\odot)$.

The composition of M4 is thought to be [Fe/H] = -1.15 and [α/Fe] = +0.38; and I have assumed Y=0.25. To get models for a relatively old cluster, we need low masses. I have constructed models for masses between $0.65M_\odot$ and $0.90M_\odot$. The lower masses will have hardly moved from the ZAMS so I have not evolved them very far. The table below gives you the maximum age in each file.

The data for M4 is taken from Mochejska et al. (200), *Astrophys J*, 124, 1486.

THE ASTRONOMICAL JOURNAL, 124:1486–1496, 2002 September

CLUSTERS AGES EXPERIMENT: HOT SUBDWARFS AND LUMINOUS WHITE DWARF CANDIDATES IN THE FIELD OF THE GLOBULAR CLUSTER M4[1]

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Received 2002 February 28; accepted 2002 May 23

The data is in the file M4data.dat and it contains Log(T_e) and Log ($L/L_\odot$). Note that quite some analysis has been done to obtain these values! Not least of all is the conversion from observed apparent magnitude and colour to luminosity and effective temperature. One needs the distance to convert to absolute magnitude and then one needs the bolometric correction (BC) to convert from the magnitude in a given filter (usually M_V) to the total (bolometric) magnitude, which is what the theorists calculate when they determine the luminosity. The two are of course related through:

$$M_{bol} = -2.5 \log (L/L_\odot) + 4.74$$

where 4.74 is the bolometric magnitude for the Sun. We also need the reddening, to convert the observed colour to the real colour and then to effective temperature.

Table 1. Maximum ages for stars provided

$M/M_\odot$	Maximum age (Gyr)
0.65	ZAMS only
0.70	13
0.725	13
0.75	13
0.80	13
0.81	13
0.82	13
0.83	13
0.84	13
0.85	12.75
0.86	12
0.87	11.5
0.88	11
0.89	10.5
0.90	10.3

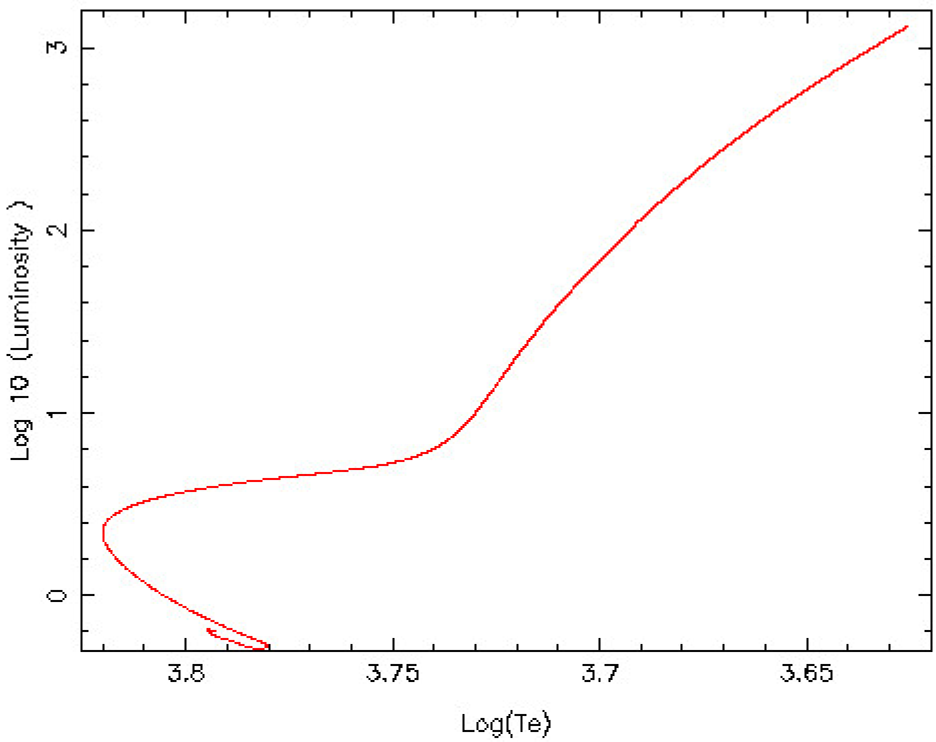
a) Plot the evolutionary tracks for some of the masses in the HR diagram.
   i. Are they a good fit?
   ii. Where are they good and where are they bad?
   iii. What could we do to improve the fit?
   iv. Are we learning physics or playing with fitting things?
   v. You may like to plot some with tick marks every Δt for some chosen Δt. You may need a different Δt on the main sequence as opposed to the giant branch! Why?

b) Now interpolate within the files to construct an isochrone for a chosen age. Linear interpolation is fine because the time-steps are small. You will need ages around the 10 to 13 Gyr range.
   i. Are they a good fit?
   ii. How can we resolve the giant branch better?
   iii. It might be worth plotting one isochrone with symbols every 0.01 in $M_\odot$.

2. Abundance Profiles: The Key to Understanding a Star's Structure

A good way to understand stellar physics is to consider the composition as a probe, because it reflects the physics that has occurred in the star. In this exercise you will follow the evolution of a low mass star and an intermediate mass star. We will give you plots of composition profiles, but without labels on the curves. Your job is to determine which species (in fact, isotope) is plotted on each figure. Then you can deduce the phase of evolution and place the given model in the correct place in the HR diagram.

Let's start with the low mass star. Here is the HR diagram for the evolution from just before the ZAMS through to nearly the tip of the red giant branch.



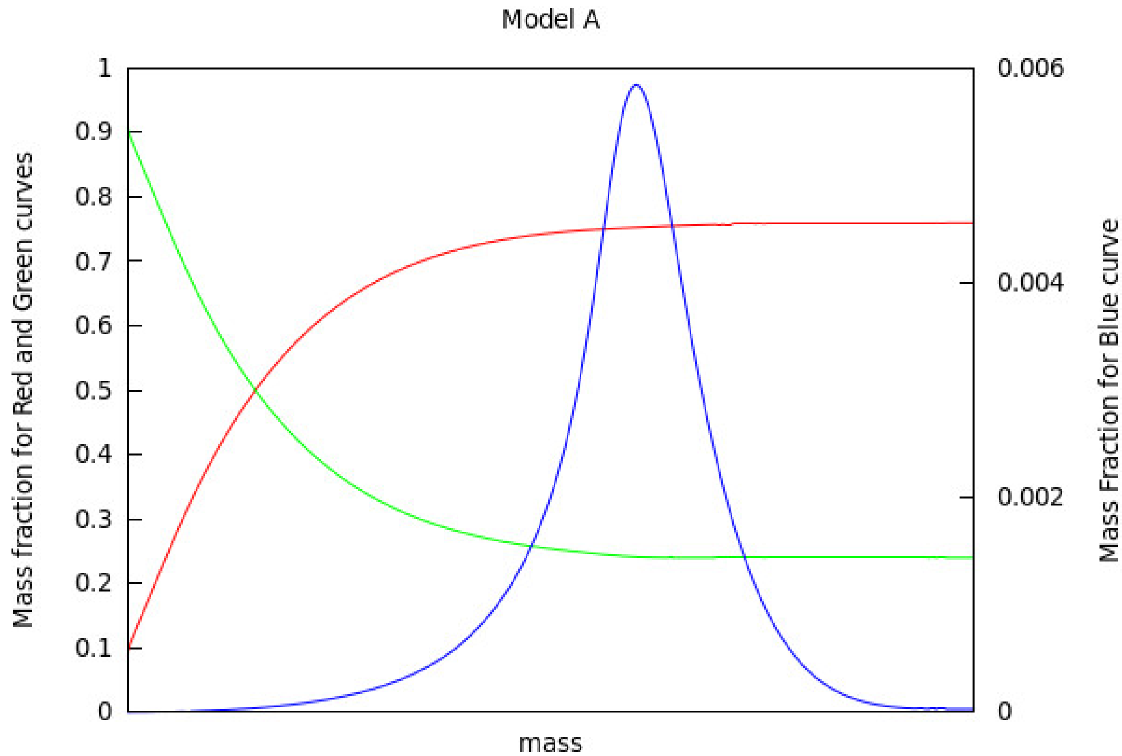
When you have worked out the phase of each of the three models presented below, return to this figure and mark where each model (A, B and C) fits on the star's evolutionary track.

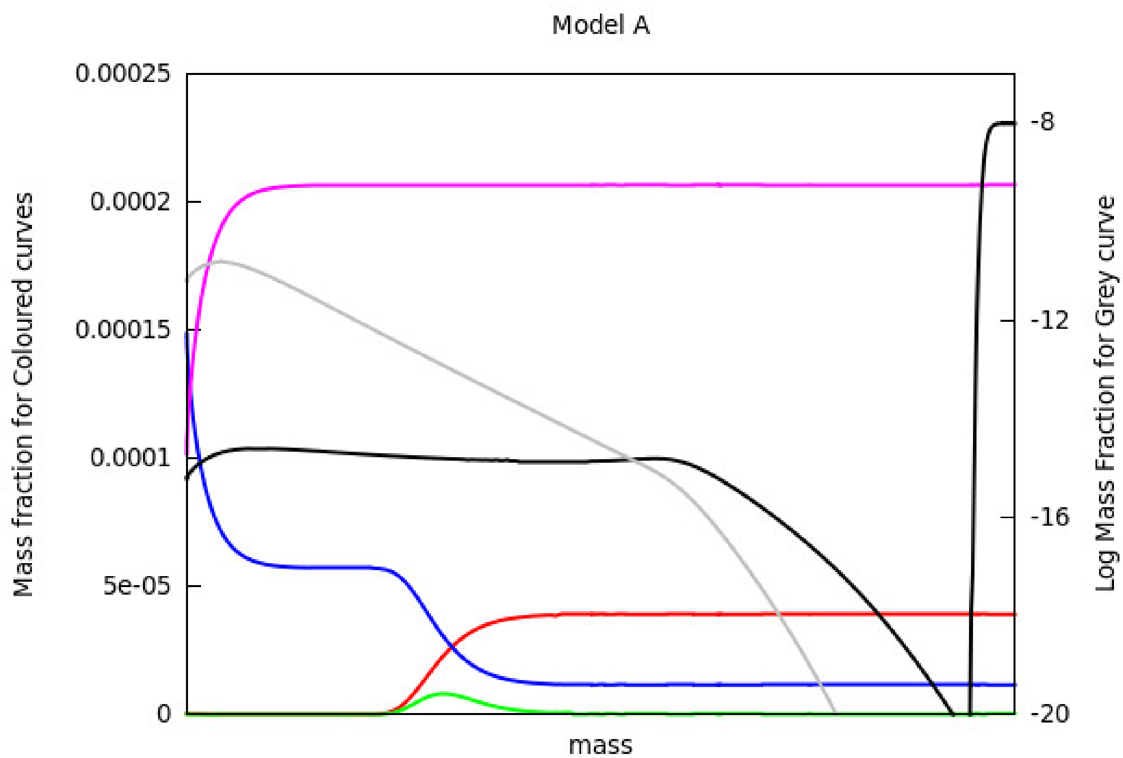
The figures below plot various species which are taken from the following list:

H, ^{3}He, ^{4}He, ^{7}Li, ^{7}Be, ^{12}C, ^{13}C, ^{14}N, ^{16}O

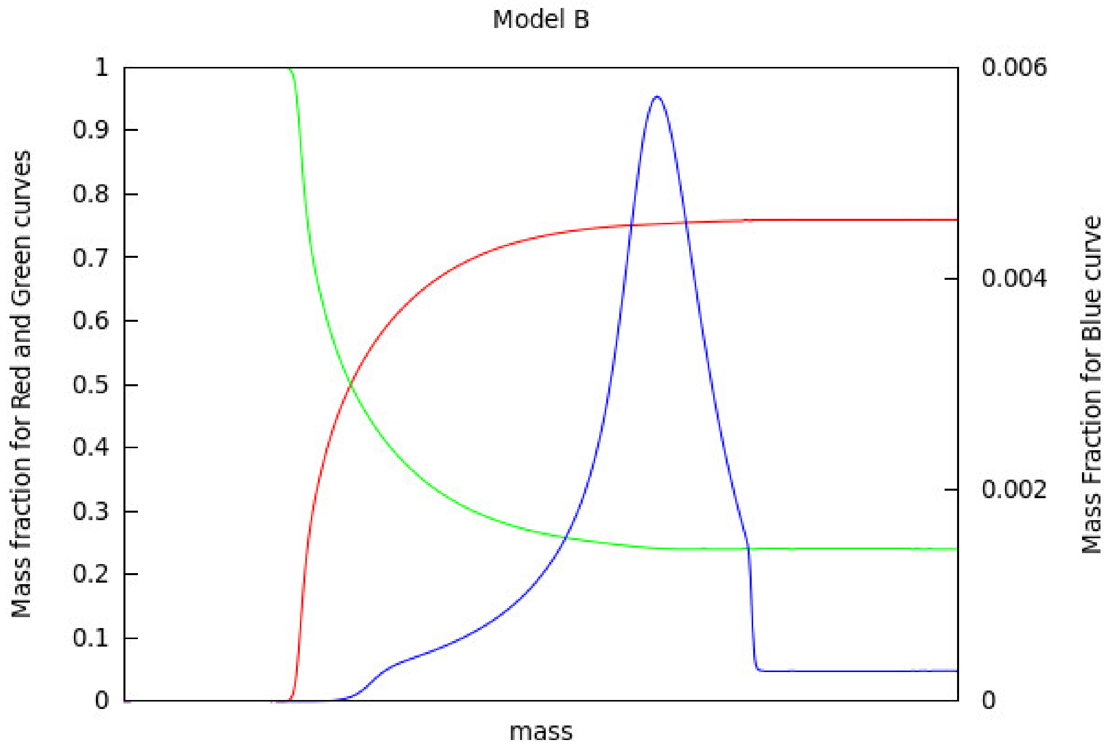
Write your estimates for each Model on the figures below. Sometimes there are two plots per model. In these case, both plots have the same x-axis scale (although it is not shown). Hence you can line them up and see the behaviour of the different elements (on quite different y-axis scales). You may want to estimate the values of the x-axis also.

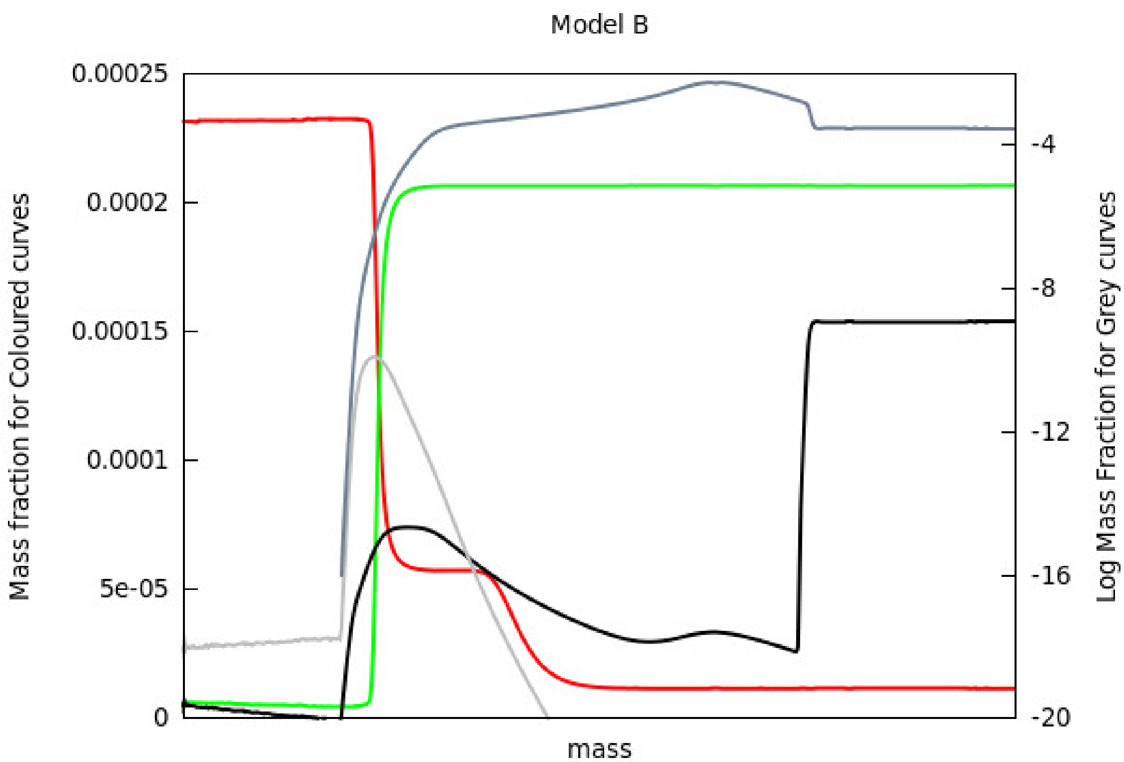
Model A


Model A
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.006
0.004
0.002
0
Mass fraction for Red and Green curves
Mass Fraction for Blue curve
mass

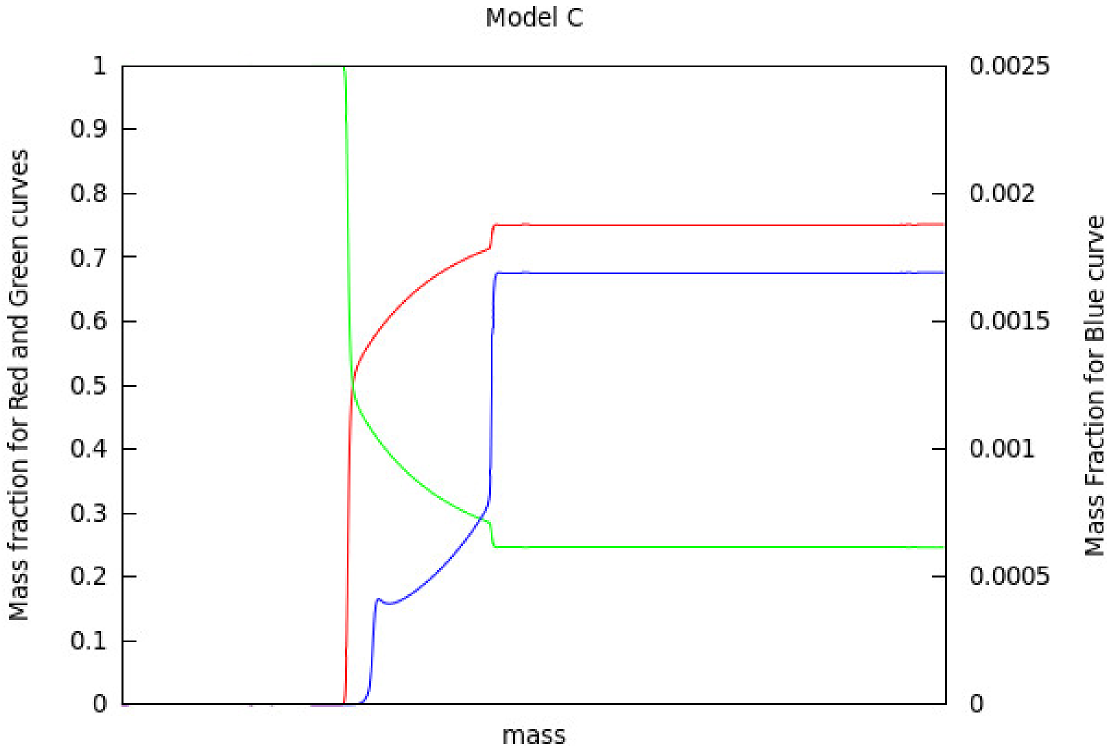

Model A
0.00025
0.0002
0.00015
0.0001
5e-05
0
-8
-12
-16
-20
Mass fraction for Coloured curves
Log Mass Fraction for Grey curve
mass

Model B

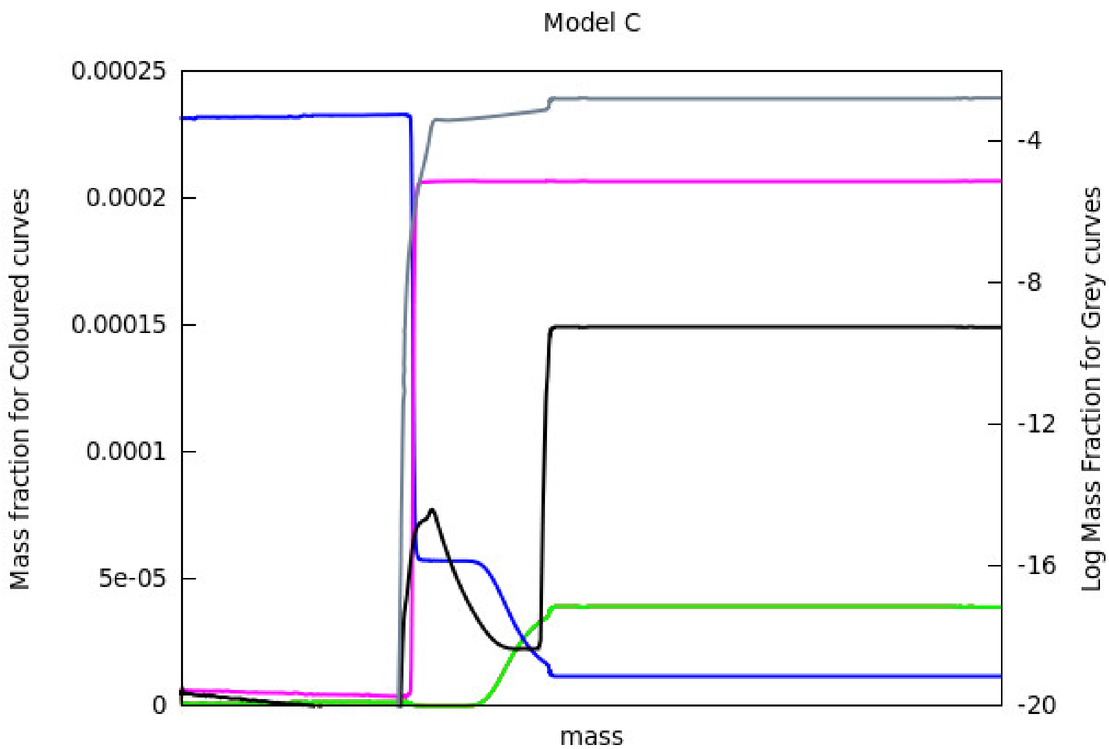

Model B
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Mass fraction for Red and Green curves
0.006
0.004
0.002
0
Mass Fraction for Blue curve
mass


Model B
0.00025
0.0002
0.00015
0.0001
5e-05
0
Mass fraction for Coloured curves
-4
-8
-12
-16
-20
Log Mass Fraction for Grey curves
mass

Model C

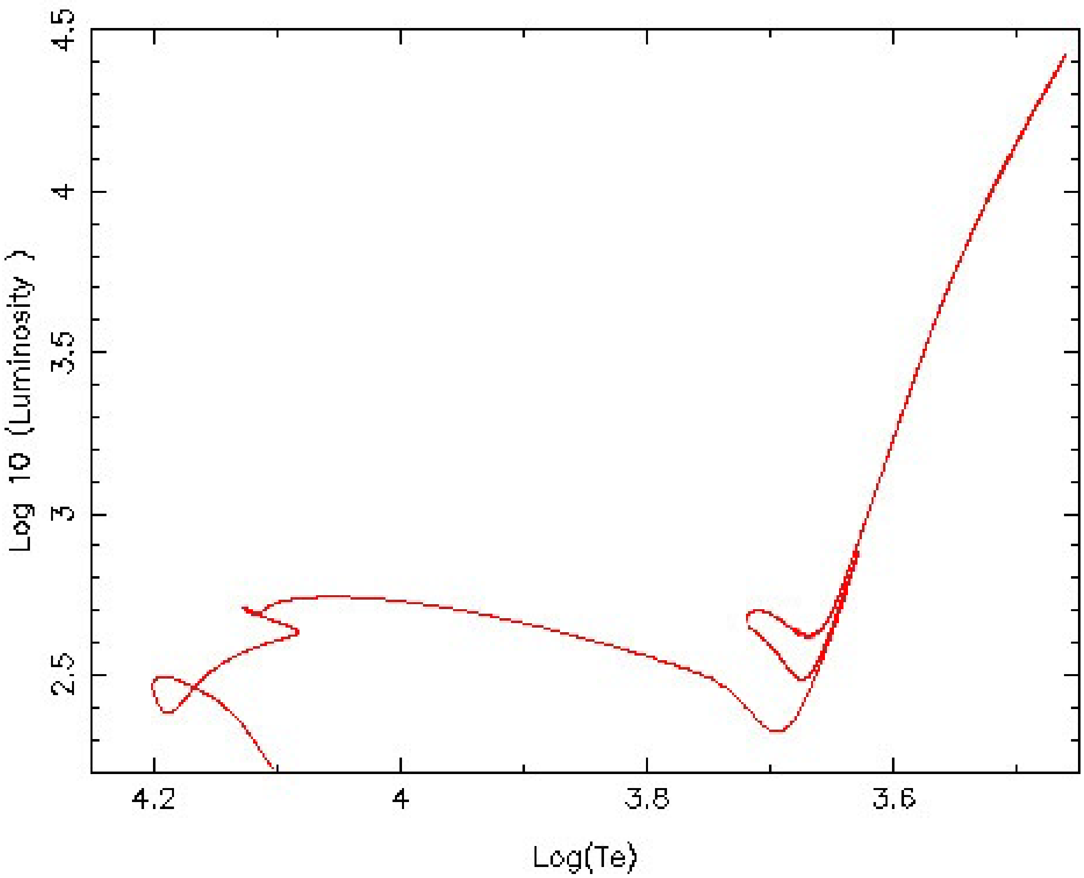


Model C
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Mass fraction for Red and Green curves
0.0025
0.002
0.0015
0.001
0.0005
0
Mass Fraction for Blue curve
mass



Model C
0.00025
0.0002
0.00015
0.0001
5e-05
0
Mass fraction for Coloured curves
-4
-8
-12
-16
-20
Log Mass Fraction for Grey curves
mass

Now Let's move to the intermediate mass star. Here is the HR diagram for the evolution from just before the ZAMS through to the thermally pulsing AGB stage.



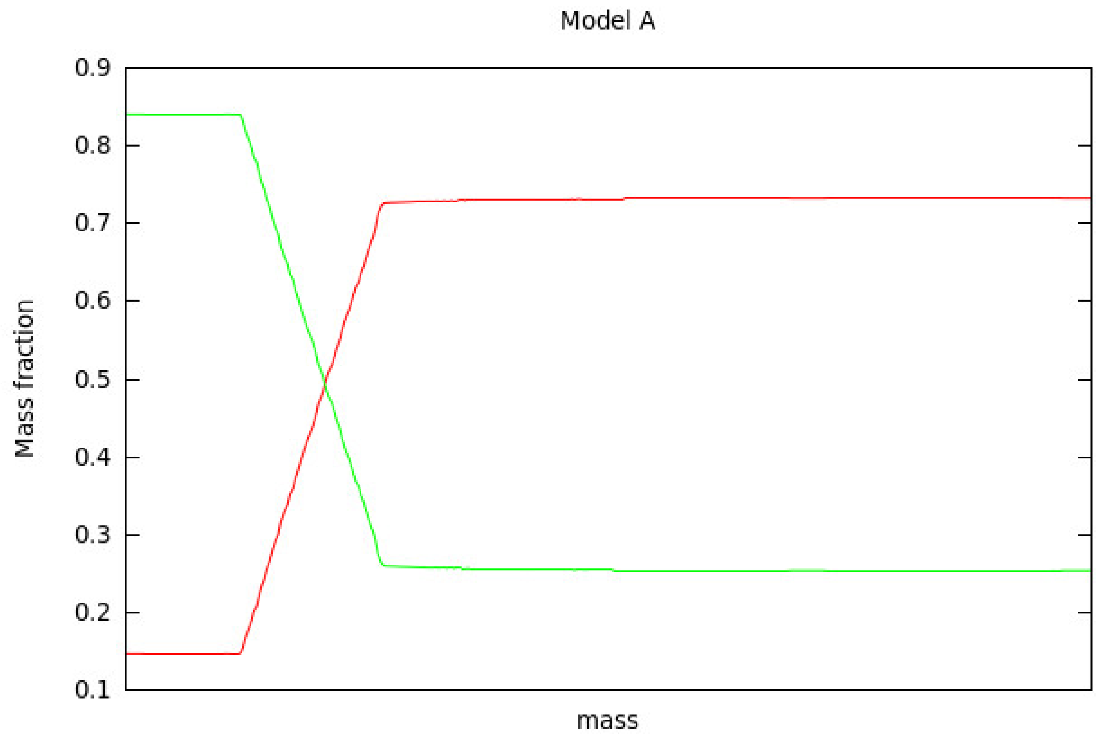
When you have worked out the phase of each of the models presented below, return to this figure and mark where each model (A-G) fits on the star's evolutionary track.

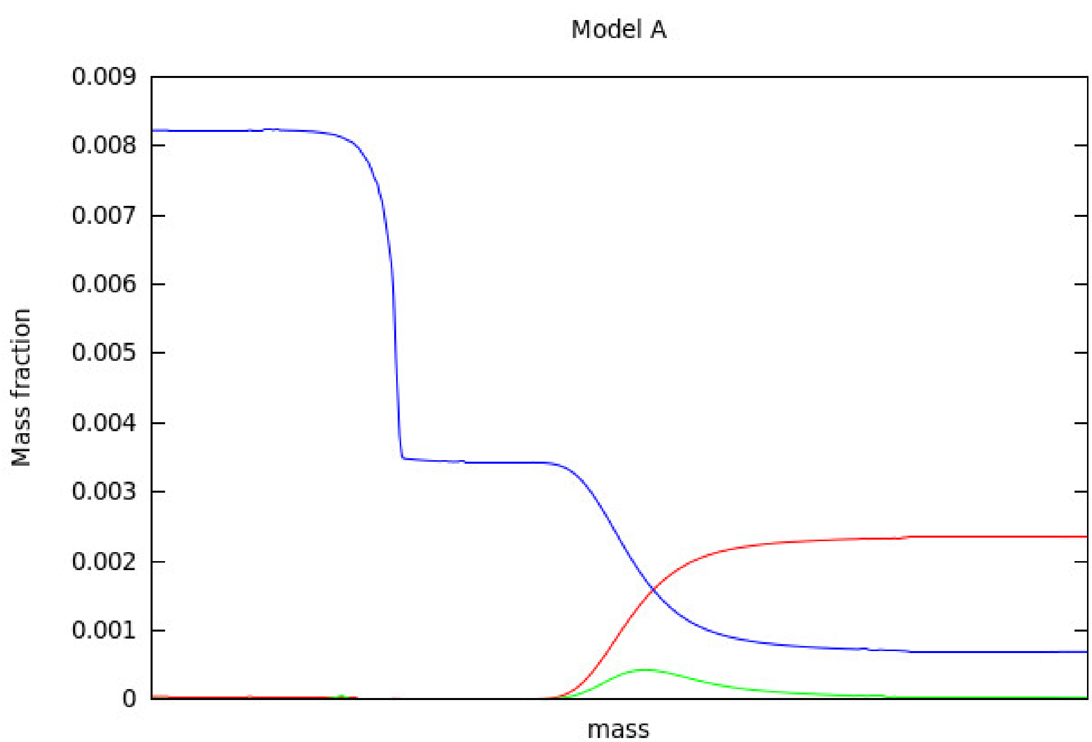
The figures below plot various species which are taken from the following list:

H, ^{3}He, ^{4}He, ^{7}Li, ^{7}Be, ^{12}C, ^{13}C, ^{14}N, ^{16}O

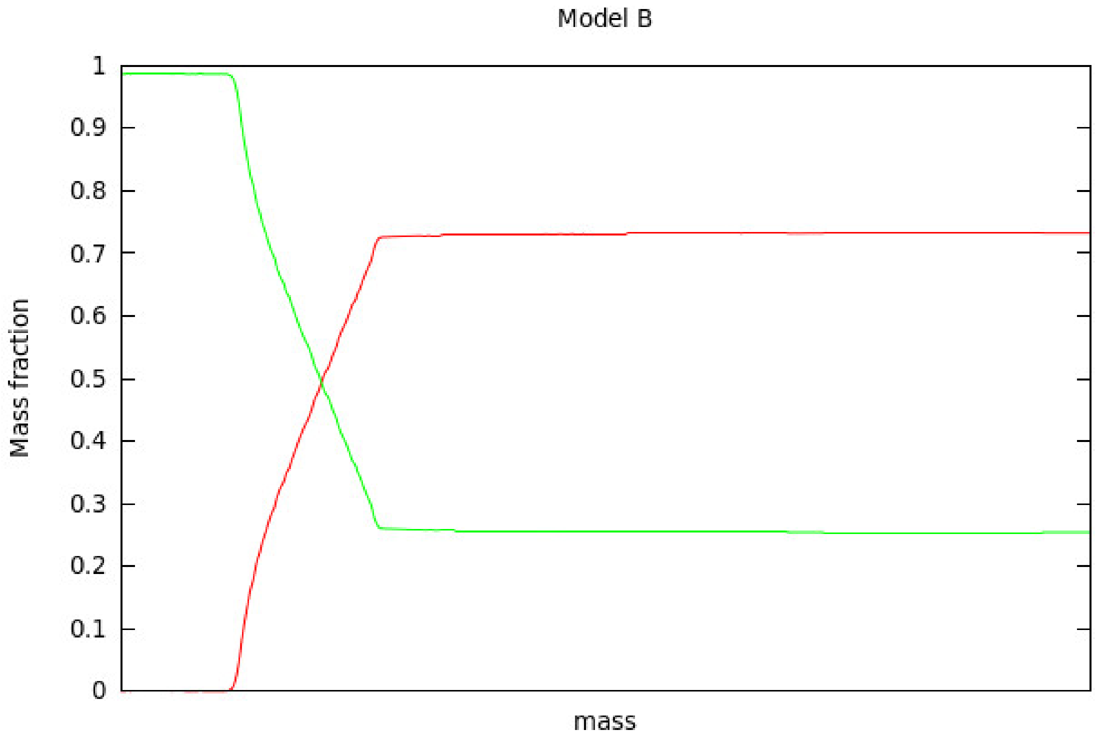
Write your estimates for each Model on the figures below. Sometimes there are two plots per model. In these case, both plots have the same x-axis scale (although it is not shown). Hence you can line them up and see the behaviour of the different elements (on quite different y-axis scales). You may want to estimate the values of the x-axis also.

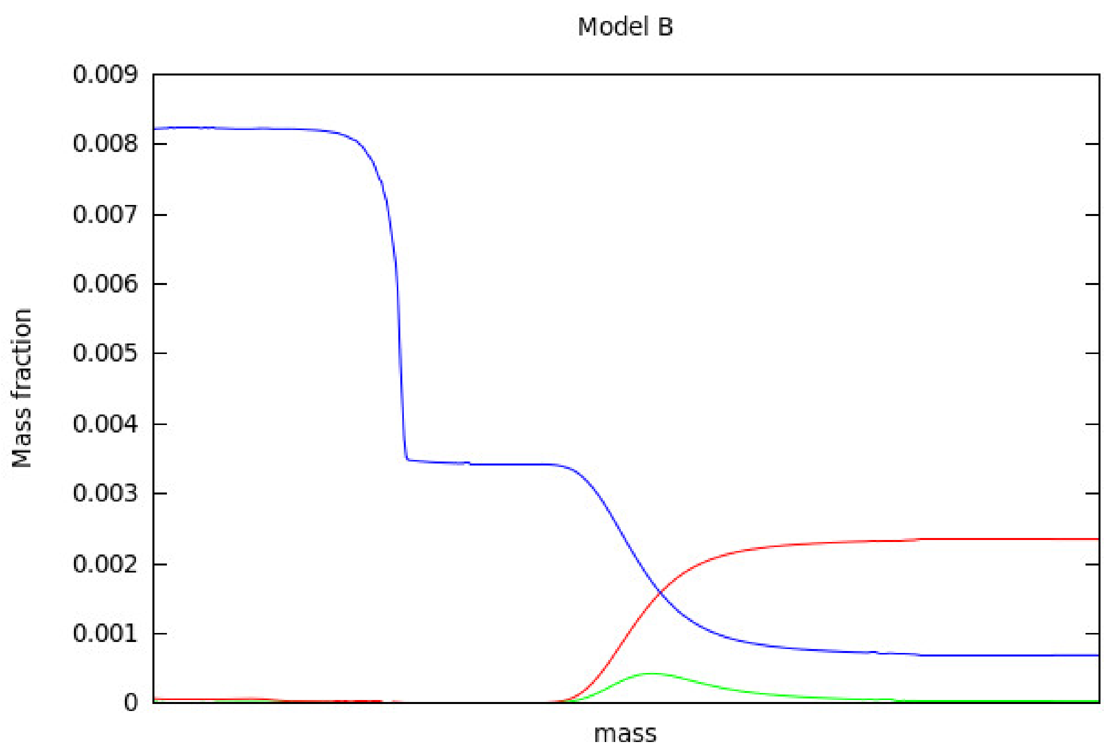
Model A


Model A
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
Mass fraction
mass

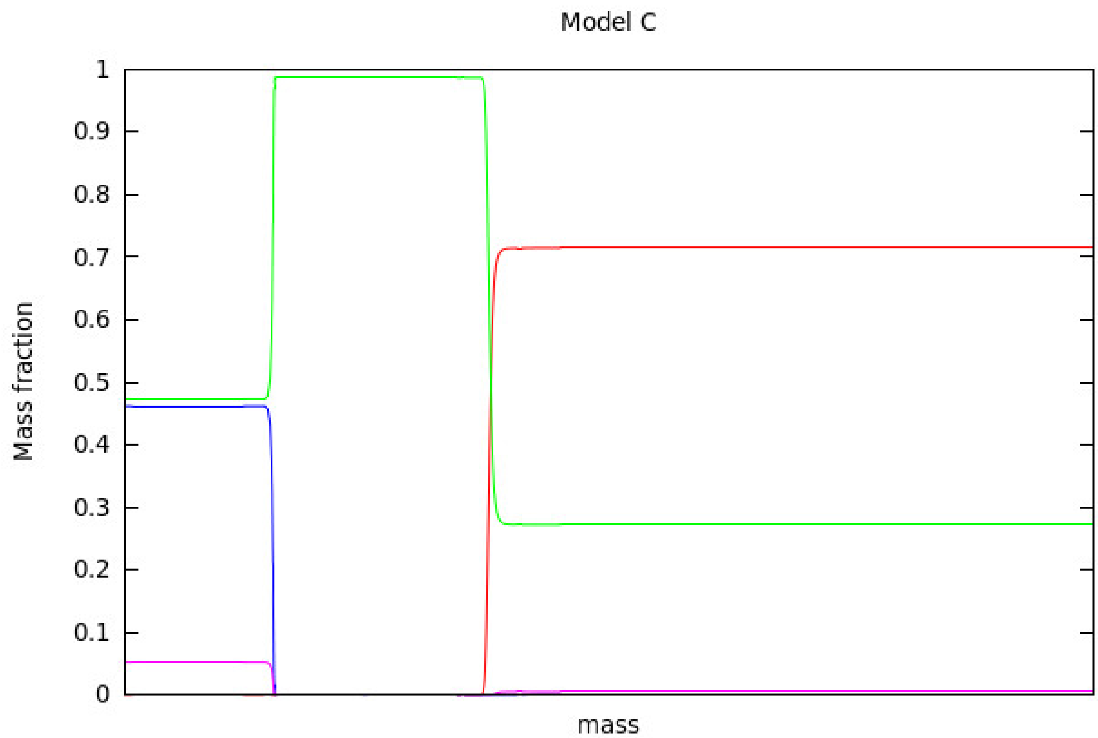

Model A
0.009
0.008
0.007
0.006
0.005
0.004
0.003
0.002
0.001
0
Mass fraction
mass

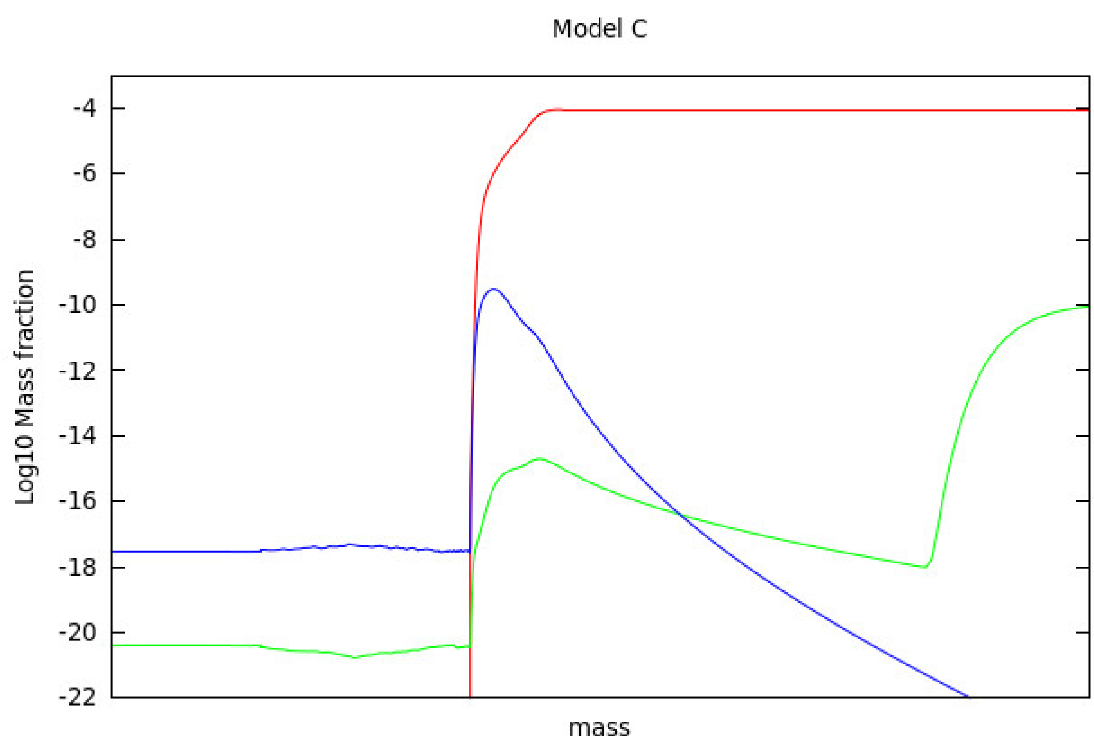
Model B


Model B
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Mass fraction
mass

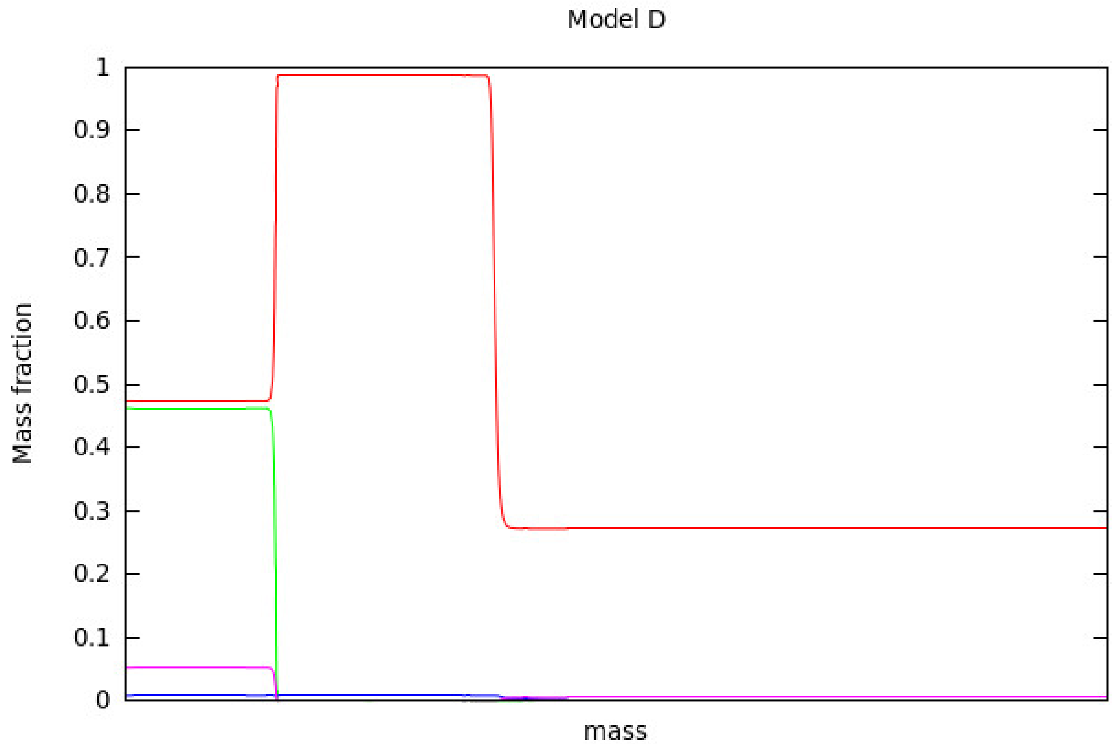

Model B
0.009
0.008
0.007
0.006
0.005
0.004
0.003
0.002
0.001
0
Mass fraction
mass

Model C

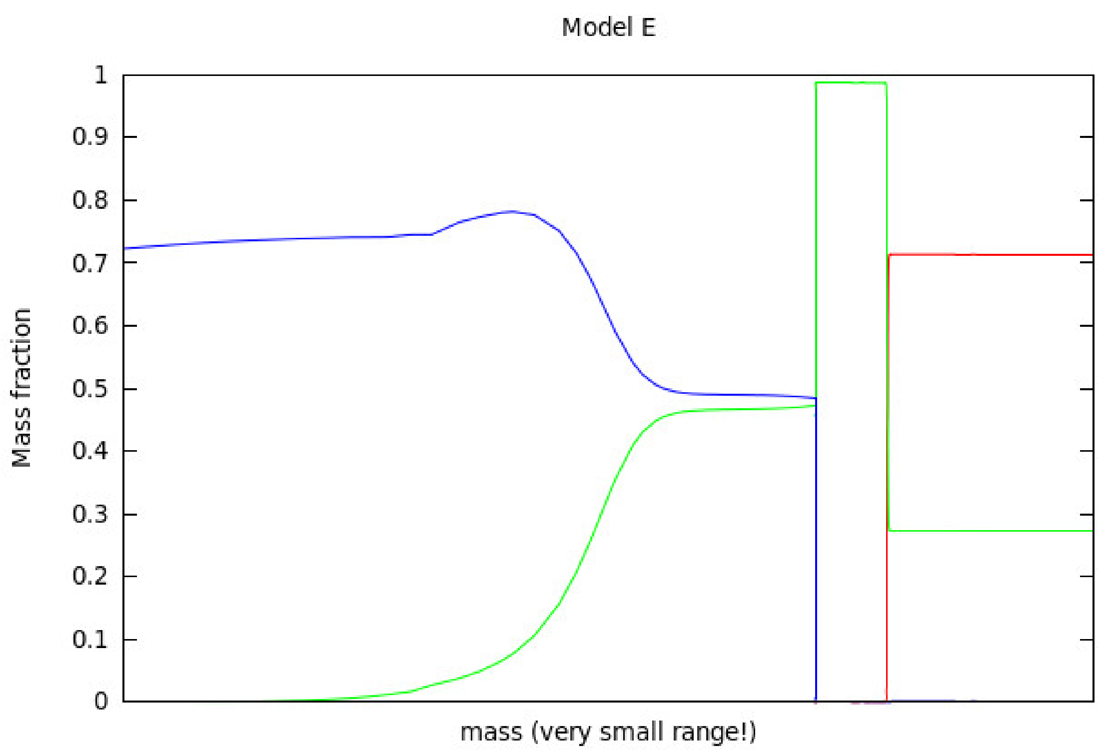

Model C
Mass fraction
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
mass


Model C
Log10 Mass fraction
-4
-6
-8
-10
-12
-14
-16
-18
-20
-22
mass

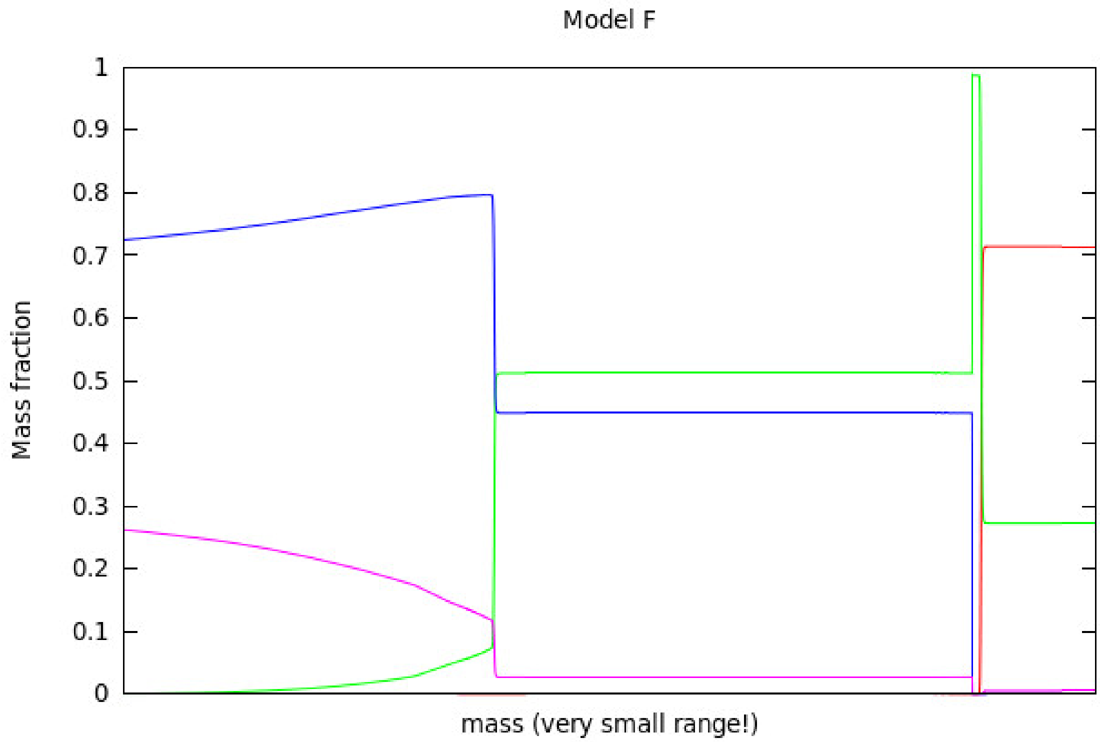
Model D



Model E



Model F



Model G
Note that the x-axis for the plots of Models F and Is the same so you can compare them, That is a good hint to what is going on!