Bias Correction of Persistence Measures in Fractionally Integrated Models: Supplementary Appendix

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Abstract

In this supplementary appendix we provide additional simulation results that complement Section 4.2 of Grose, Martin and Poskitt (2014).

Keywords: Long memory, ARFIMA, sieve bootstrap, bootstrap-based bias correction, sample autocorrelation function, impulse response function.

JEL Classification: C18, C22, C52

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Simulation Results for $T = 100$

We present here selected "small-sample" simulation results for bias correction of conventional estimates of two measures of persistence in the long memory context, namely the impulse response function $\psi(k)$, and the autocorrelation function $\rho(k)$. These results are for sample size $T = 100$, supplementing the $T = 500$ results presented in the main text Grose et al. (2014). The data generating process is ARFIMA$(1, d, 0)$ where d is the fractional integration parameter. Results are presented for $d = \{0.2, 0.4\}$, autoregression parameter $\phi = \{0.6, 0.9\}$, and selected lag lengths k between 1 and 99. The bias-adjustment is performed via the sieve boostrap, with consideration being given to two alternative approaches to choosing h, the order of the autoregression used in the sieve. For further details see Grose et al. (2014).

Bias correction of the sample IRF

Figures A1 to A3 display distributional results for the estimated impulse response function (both conventional, and bias-adjusted) for $T = 100$, $d = 0.4$, $\phi = 0.9$, and $h = [(\ln T)^2]$. Panels (i) to (v) in each figure plot:

- the Monte Carlo distribution of the unadjusted statistic $\widehat{\psi}(k)$;
- the Monte Carlo distribution of the bootstrap bias-adjusted statistic $\widehat{\psi}^{(BA)}(k)$;
- and the average bootstrap estimate of the distribution of $\widehat{\psi}(k)$, for $k = 1, 3, 6, 9, 12$.

The three distributions are indicated by the legend entries "MC", "MC-BA" and "BS-av" respectively. The vertical dotted line in each panel indicates the true value of $\psi(k)$ for each k.

Panel (vi) plots, for lags $k = 1, 2, ..., 99$:

- the true IRF $\psi(k)$ based on the parameters of the data generating process;
- the mean of the Monte Carlo distribution of $\widehat{\psi}(k)$;
- and the mean of the Monte Carlo distribution of $\widehat{\psi}^{(BA)}(k)$

(designated "True ψ", "$\overline{MC}$", and "$\overline{MC\text{-}BA}$" respectively).

Bias and RMSE results for both choices of h, and for all combinations of $d = 0.2, 0.4$ and $\phi = 0.6, 0.9$, are presented in Table A1.

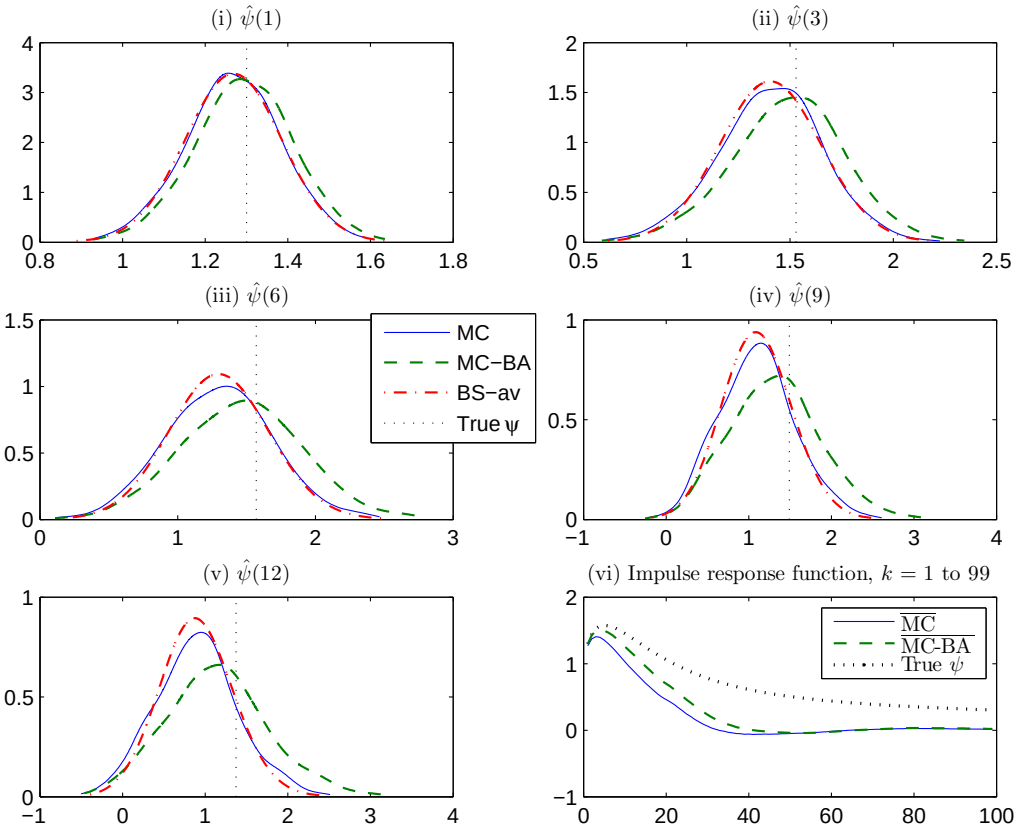

Figure A1. Bias correction of the sample IRF using the *raw sieve* bootstrap.

True process: ARFIMA$(1, d, 0)$; $T = 100$; $d = 0.4$; $\phi = 0.9$.

Key for Panels (1) to (v): MC": Monte Carlo distribution of the unadjusted statistic $\widehat{\psi}(k)$; "MC-BA": Monte Carlo distribution of the bootstrap bias-adjusted statistic $\widehat{\psi}^{(BA)}(k)$; "BS-av": the averaged bootstrap estimate of the distribution of $\widehat{\psi}(k)$. *Key for Panel (vi):* "$\overline{MC}$": mean of the Monte Carlo distribution of $\widehat{\psi}(k)$; "$\overline{MC\text{-}BA}$": mean of the Monte Carlo distribution of $\widehat{\psi}^{(BA)}(k)$. The true value of $\psi(k)$ is indicated by the use of small dots in all panels.

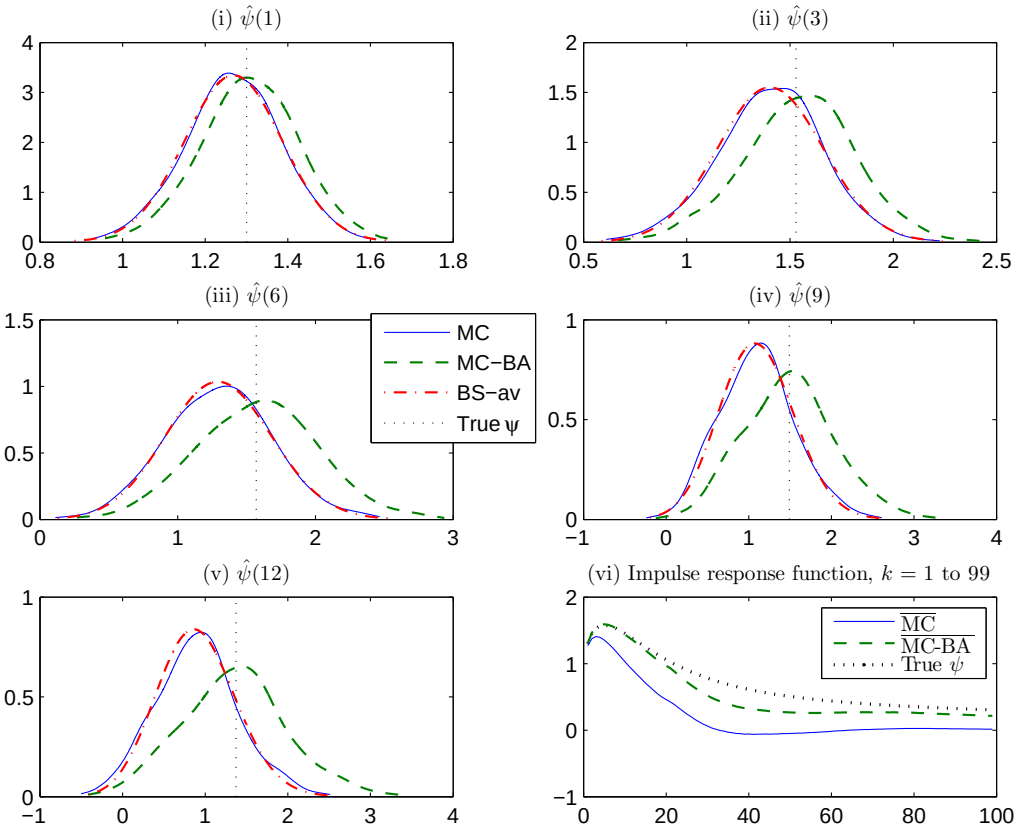

Figure A2. Bias correction of the sample IRF using the *pre-filtered sieve* bootstrap, based on the *true value of d as the pre-filter*.

True process: ARFIMA$(1, d, 0)$; $T = 100$; $d = 0.4$; $\phi = 0.9$.

Key for Panels (1) to (v): MC": Monte Carlo distribution of the unadjusted statistic $\widehat{\psi}(k)$; "MC-BA": Monte Carlo distribution of the bootstrap bias-adjusted statistic $\widehat{\psi}^{(BA)}(k)$; "BS-av": the averaged bootstrap estimate of the distribution of $\widehat{\psi}(k)$. *Key for Panel (vi):* "$\overline{MC}$": mean of the Monte Carlo distribution of $\widehat{\psi}(k)$; "$\overline{MC\text{-}BA}$": mean of the Monte Carlo distribution of $\widehat{\psi}^{(BA)}(k)$. The true value of $\psi(k)$ is indicated by the use of small dots in all panels.

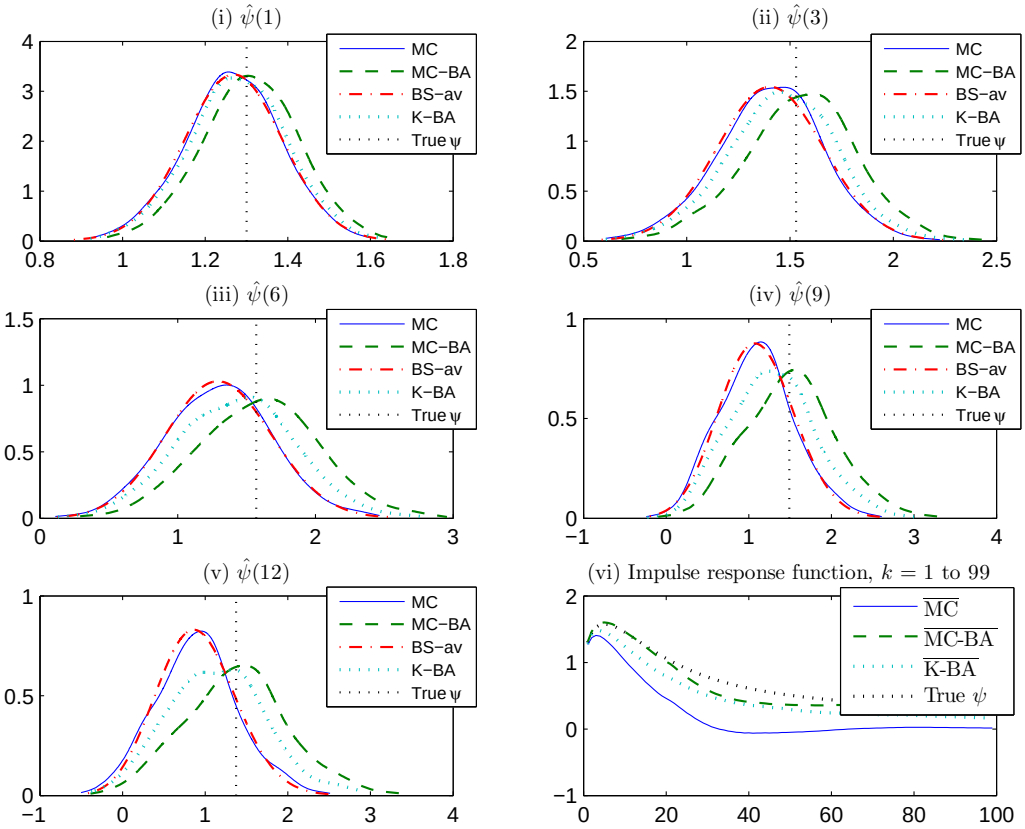

Figure A3. Bias correction of the sample IRF using the *pre-filtered sieve bootstrap*, based on the *SPLW estimate of d as the pre-filter.*

True process: ARFIMA$(1, d, 0)$; $T = 100$; $d = 0.4$; $\phi = 0.9$.

Key for Panels (1) to (v): "MC": Monte Carlo distribution of the unadjusted statistic $\widehat{\psi}(k)$; "MC-BA": Monte Carlo distribution of the bootstrap bias-adjusted statistic $\widehat{\psi}^{(BA)}(k)$; "BS-av": average bootstrap estimate of the distribution of $\widehat{\psi}(k)$, and "K-BA": Monte Carlo distribution of the bias-adjusted statistic $\widehat{\psi}^{(K)}(k)$ produced using Kilian's approach. *Key for Panel (vi):* "$\overline{MC}$": mean of the Monte Carlo distribution of $\widehat{\psi}(k)$; "$\overline{MC\text{-}BA}$": mean of the Monte Carlo distribution of $\widehat{\psi}^{(BA)}(k)$; "$\overline{K\text{-}BA}$": mean of the Monte Carlo distribution of $\widehat{\psi}^{(K)}(k)$. The true value of $\psi(k)$ is indicated by the use of small dots in all panels.

Table A1

Bias and root mean squared error (RMSE) of estimators of selected impulse response coefficients, for $T = 100$. Results for the unadjusted and both forms of bootstrap-based bias-adjusted estimators are documented.

		$\widehat{\psi}(k)$				$\widehat{\psi}^{(BA)}(k)$ (raw sieve)				$\widehat{\psi}^{(BA)}(k)$ (pre-filtered sieve)			
		$k=1$	$k=6$	$k=12$	av.	$k=1$	$k=6$	$k=12$	av.	$k=1$	$k=6$	$k=12$	av.
Panel A: $T = 100$; h based on AIC selection													
d	ϕ					Bias							
0.2	0.6	-0.0378	-0.0467	-0.0503	-0.0463	-0.0094	-0.0182	-0.0437	-0.0223	0.0049	0.0522	0.0973	0.0496
	0.9	-0.0406	-0.1974	-0.235	-0.1643	-0.0119	-0.0564	-0.0836	-0.0514	0.0260	0.0363	0.0343	0.0344
0.4	0.6	-0.0589	-0.1047	-0.1242	-0.0991	-0.0294	-0.0275	-0.0781	-0.0427	0.0179	0.0196	0.0423	0.0246
	0.9	-0.0143	-0.2984	-0.5071	-0.2734	0.0090	-0.0979	-0.2122	-0.0972	0.0092	0.0405	-0.0153	0.0175
						RMSE							
0.2	0.6	0.1075	0.1201	0.0862	0.1107	0.1006	0.1292	0.0972	0.1157	0.1133	0.139	0.1328	0.1338
	0.9	0.1293	0.2998	0.3325	0.2635	0.1170	0.2604	0.3207	0.2391	0.1051	0.2826	0.3173	0.2448
0.4	0.6	0.1327	0.1982	0.1868	0.1792	0.1181	0.1981	0.1974	0.1772	0.1076	0.1926	0.1648	0.1657
	0.9	0.1153	0.4577	0.6471	0.4134	0.1138	0.3844	0.5536	0.3558	0.1065	0.3843	0.5374	0.3497
Panel B: $T = 100$; $h = (\ln T)^2$													
d	ϕ					Bias							
0.2	0.6	-0.0216	-0.0673	-0.0627	-0.0548	-0.0005	-0.0238	-0.029	-0.0191	0.0127	0.0245	0.0335	0.0236
	0.9	-0.0264	-0.1667	-0.2416	-0.1482	-0.0024	-0.0538	-0.102	-0.0522	0.0132	0.0449	0.0627	0.0410
0.4	0.6	-0.0281	-0.1239	-0.1472	-0.1046	-0.0046	-0.0515	-0.0774	-0.0455	0.0095	0.0171	0.0191	0.0156
	0.9	-0.0318	-0.2682	-0.4737	-0.2571	-0.0061	-0.1028	-0.2288	-0.109	0.0096	0.0244	0.0063	0.0157
						RMSE							
0.2	0.6	0.1165	0.1759	0.1709	0.1608	0.1172	0.1814	0.1871	0.1677	0.1181	0.186	0.1926	0.1711
	0.9	0.1173	0.3264	0.4039	0.2905	0.1173	0.3216	0.4072	0.2889	0.1174	0.322	0.4143	0.2907
0.4	0.6	0.1180	0.2481	0.2644	0.219	0.1175	0.2464	0.2729	0.2202	0.1176	0.2453	0.2711	0.2189
	0.9	0.1191	0.4675	0.6863	0.4298	0.1182	0.4472	0.6551	0.4118	0.1174	0.4329	0.6233	0.3963

Bias correction of the sample ACF

Figures A4 and A5 display distributional results for the estimated autocorrelation function (both conventional, and bias-adjusted) for $T = 100$, $d = 0.4$ and $\phi = 0.9$. Panels (i) to (v) in each figure plot respectively:

- the Monte Carlo distribution of the unadjusted statistic $\widehat{\rho}(k)$;
- the Monte Carlo distribution of the bootstrap bias-adjusted statistic, $\widehat{\rho}^{(BA)}(k)$;
- the average bootstrap estimate of the distribution of $\widehat{\rho}(k)$;
- and the Monte Carlo distribution of the estimator adjusted using the (infeasible) asymptotic bias formula of Hosking (1996), $\widehat{\rho}^{(ASY)}(k)$, $k = 1, 3, 6, 9, 12$

The four plots are indicated by the legend entries "MC", "MC-BA", "BS-av" and "BA-asy" respectively. The vertical dotted line indicates the true value of $\rho(k)$ for each k.

In Panel (i) we also plot the sampling distribution of the feasible bias-adjusted estimator based on the bias expression of Lee and Ko (2009) (referred to hereafter as $\widehat{\rho}^{(LK)}(1)$, and designated "BA-LK" on the figure).

Panel (vi) plots, for lags $k = 1, 2, ..., 99$:

- the true ACF $\rho(k)$ (based on the parameters of the true data generating process);
- the mean of the Monte Carlo distribution of $\widehat{\rho}(k)$;
- and the mean of the Monte Carlo distribution of $\widehat{\rho}^{(BA)}(k)$ (designated "True ρ", "$\overline{MC}$", and "$\overline{MC\text{-}BA}$" respectively).

Bias and RMSE results for both choices of h, and for all combinations of $d = 0.2, 0.4$ and $\phi = 0.6, 0.9$, are presented in Table A2.

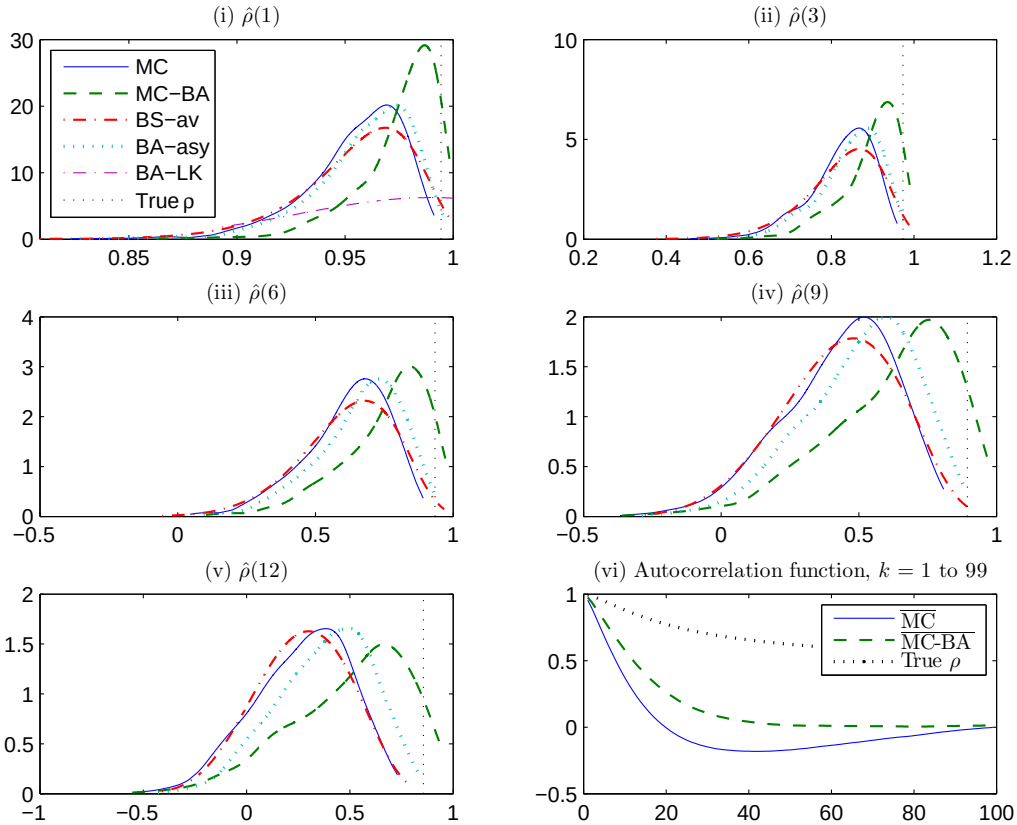

Figure A4. Bias correction of the sample ACF using the *raw sieve* bootstrap.

True process: ARFIMA$(1, d, 0)$; $T = 100$; $d = 0.4$; $\phi = 0.9$.

Key for Panels (1) to (v): "MC": Monte Carlo distribution of the unadjusted statistic $\widehat{\rho}(k)$; "MC-BA": Monte Carlo distribution of the bootstrap bias-adjusted statistic $\widehat{\rho}^{(BA)}(k)$; "BS-av": the average bootstrap estimate of the distribution of $\widehat{\rho}(k)$; "BA-asy": the Monte Carlo distribution of $\widehat{\rho}^{(ASY)}(k)$; "BA-LK": the Monte Carlo distribution of $\widehat{\rho}^{(LK)}(1)$. *Key for Panel (vi):* "$\overline{MC}$": mean of the Monte Carlo distribution of $\widehat{\rho}(k)$; "$\overline{MC\text{-}BA}$": mean of the Monte Carlo distribution of $\widehat{\rho}^{(BA)}(k)$. The true value of $\rho(k)$ is indicated by the use of small dots in all panels.

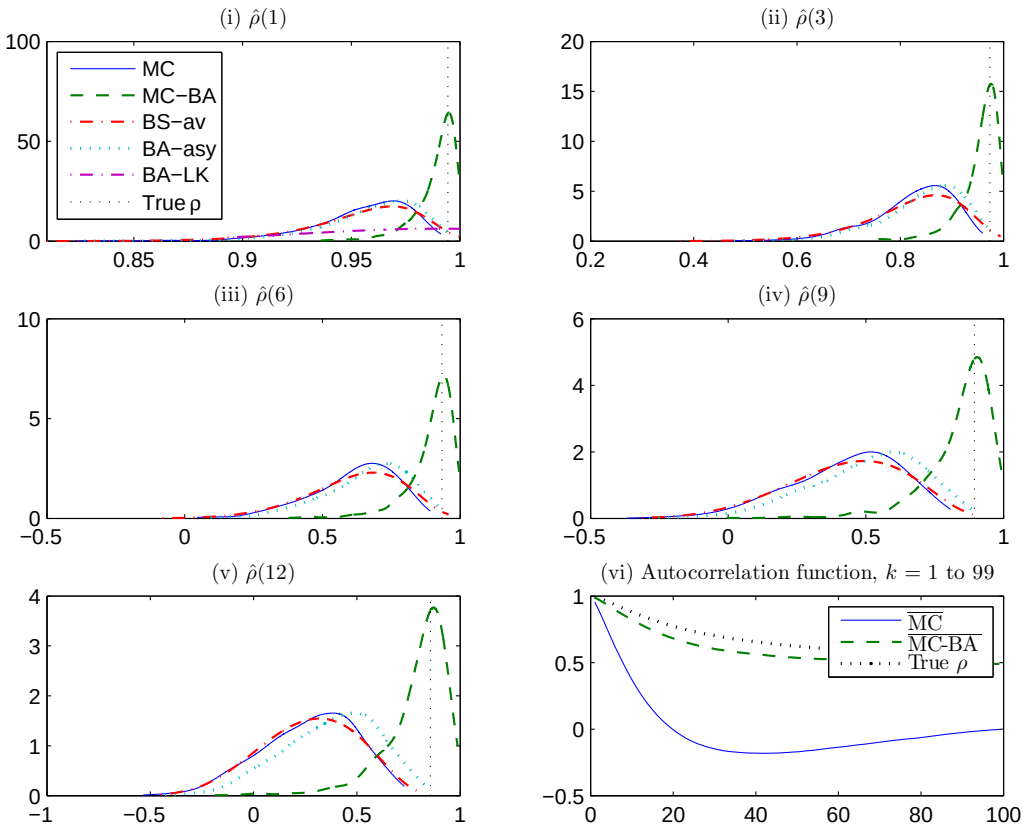

Figure A5. Bias correction of the sample ACF using the *pre-filtered sieve* bootstrap, based on the *true value of d as the pre-filter*

True process: ARFIMA$(1, d, 0)$; $T = 100$; $d = 0.4$; $\phi = 0.9$.

Key for Panels (1) to (v): "MC": Monte Carlo distribution of the unadjusted statistic $\widehat{\rho}(k)$; "MC-BA": Monte Carlo distribution of the bootstrap bias-adjusted statistic $\widehat{\rho}^{(BA)}(k)$; "BS-av": the average bootstrap estimate of the distribution of $\widehat{\rho}(k)$; "BA-asy": the Monte Carlo distribution of $\widehat{\rho}^{(ASY)}(k)$; "BA-LK": the Monte Carlo distribution of $\widehat{\rho}^{(LK)}(1)$. *Key for Panel (vi):* "$\overline{MC}$": mean of the Monte Carlo distribution of $\widehat{\rho}(k)$; "$\overline{MC\text{-}BA}$": mean of the Monte Carlo distribution of $\widehat{\rho}^{(BA)}(k)$. The true value of $\rho(k)$ is indicated by the use of small dots in all panels.

Table A2

Bias and root mean squared error (RMSE) of estimators of selected autocorrelation coefficients, for $T = 100$. Results for the unadjusted and both forms of bootstrap-based bias-adjusted estimators are documented.

		$\widehat{\rho}(k)$				$\widehat{\rho}^{(BA)}(k)$ (raw sieve)				$\widehat{\rho}^{(BA)}(k)$ (pre-filtered sieve)			
		$k=1$	$k=6$	$k=12$	av.	$k=1$	$k=6$	$k=12$	av.	$k=1$	$k=6$	$k=12$	av.
Panel A: $T = 100$; h based on AIC selection													
d	ϕ					Bias							
0.2	0.6	-0.0591	-0.1570	-0.1518	-0.1309	-0.0299	-0.0862	-0.0885	-0.0724	0.1727	0.6146	0.7291	0.5269
	0.9	-0.0453	-0.2570	-0.3835	-0.2324	-0.0198	-0.1144	-0.1774	-0.1051	0.0322	0.2403	0.4408	0.2356
0.4	0.6	-0.0959	-0.4249	-0.5256	-0.3629	-0.0673	-0.3150	-0.4072	-0.2724	0.0566	0.2899	0.3932	0.2537
	0.9	-0.0392	-0.3186	-0.5851	-0.3115	-0.0191	-0.1803	-0.3546	-0.1814	0.0055	0.0650	0.1424	0.0688
						RMSE							
0.2	0.6	0.0919	0.2241	0.2143	0.1890	0.0780	0.1997	0.1891	0.1665	0.1948	0.6583	0.7672	0.5649
	0.9	0.0575	0.3079	0.4409	0.2747	0.0383	0.2181	0.3288	0.1983	0.0322	0.2403	0.4408	0.2356
0.4	0.6	0.1099	0.4638	0.5587	0.3940	0.0864	0.3809	0.4676	0.3246	0.0602	0.2972	0.3981	0.2596
	0.9	0.0454	0.3537	0.6285	0.3408	0.0270	0.2406	0.4540	0.2375	0.0055	0.0650	0.1424	0.0688
Panel B: $T = 100$; $h = (\ln T)^2$													
d	ϕ					Bias							
0.2	0.6	-0.0591	-0.1570	-0.1518	-0.1309	-0.0314	-0.0969	-0.1062	-0.0824	0.1583	0.5749	0.6839	0.4920
0.2	0.9	-0.0453	-0.2570	-0.3835	-0.2324	-0.0232	-0.1461	-0.2521	-0.1403	0.0319	0.2384	0.4373	0.2338
0.4	0.6	-0.0959	-0.4249	-0.5256	-0.3629	-0.0690	-0.3286	-0.4364	-0.2869	0.0551	0.2837	0.3855	0.2484
0.4	0.9	-0.0392	-0.3186	-0.5851	-0.3115	-0.0209	-0.2056	-0.4238	-0.2116	0.0054	0.0637	0.1398	0.0675
						RMSE							
0.2	0.6	0.0919	0.2241	0.2143	0.189	0.0796	0.2164	0.2270	0.1845	0.1928	0.6452	0.7491	0.5536
0.2	0.9	0.0575	0.3079	0.4409	0.2747	0.0404	0.2358	0.3806	0.2206	0.0319	0.2384	0.4374	0.2338
0.4	0.6	0.1099	0.4638	0.5587	0.3940	0.0880	0.3963	0.5074	0.3426	0.0608	0.2955	0.3933	0.2578
0.4	0.9	0.0454	0.3537	0.6285	0.3408	0.0286	0.2614	0.5145	0.2633	0.0054	0.0638	0.1400	0.0675

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