Capturing the Business Cycle with Autoregressive Leading Indicator Models

George Athanasopoulos
Heather M. Anderson
Farshid Vahid

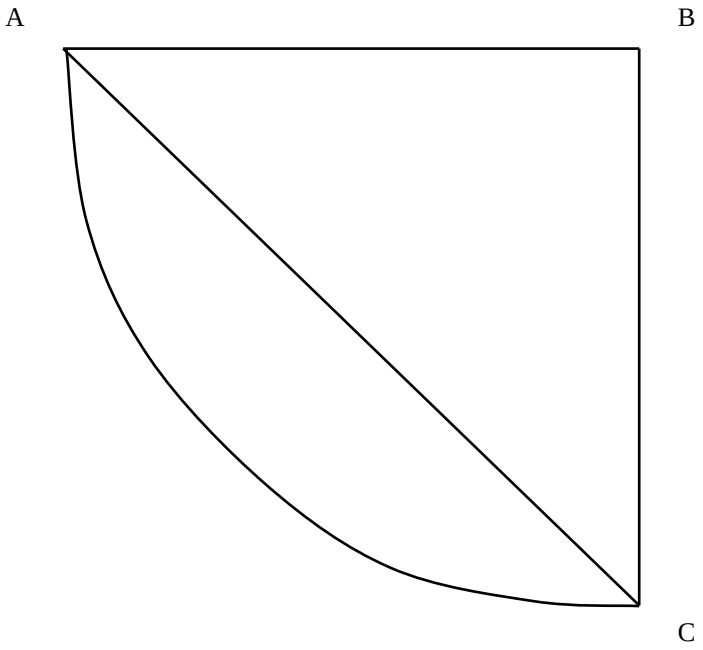
- Harding and Pagan (1999)
  - "Dissecting the Cycle"
- ISSUES
  - describing the business cycle
    - graphical V parametric
  - use of ARLI models
    - linear and non-linear
  - use of variables
    - real V financial
- Graphical V Parametric
  - graphical criticised
  - parametric
    - "detrended" series
    - ABS - Henderson Filters
    - NBER - "phase averaging"
    - Academics:
      - Hodrick Prescott
      - Band Pass

- Graphical Approach
  - turning points
  - algorithm should:
    - identify turning points
    - alternating
    - censoring rules
- BBQ by H&P
    - Bry & Boschan for quarterly data
  - turning points

$$y_t > (<) y_{t\pm k} \quad \text{for} \quad \mathrm{k} = 1, ..., \mathrm{K} \rightarrow \mathrm{K} = 2 \text{ qtrs}$$

  - Censoring criteria
    - minimum phase of cycle: 2qrts
    - minimum cycle duration: 5qrts
- Observing the Business Cycle
  - duration
  - amplitude
  - cumulative movements
  - H&P excess statistic

- Typical Recession Phase



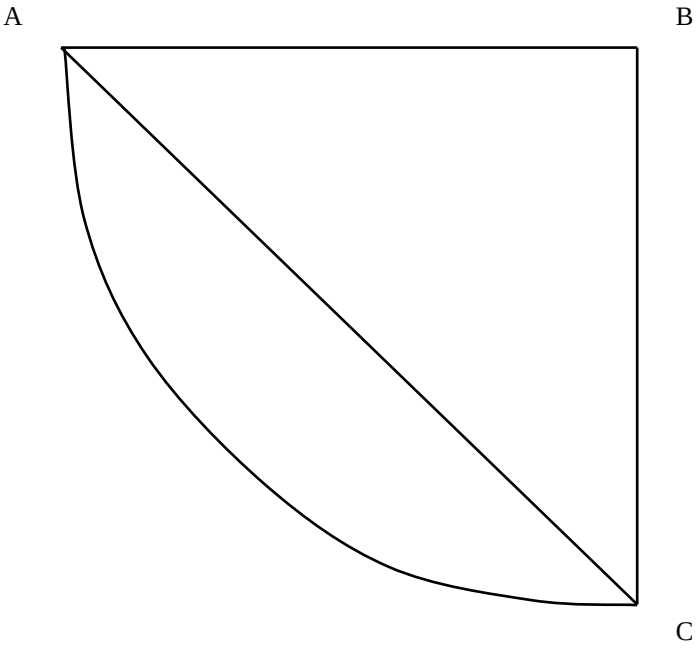
Cumulated loss : C_i (area)

- Approximation to C_i:

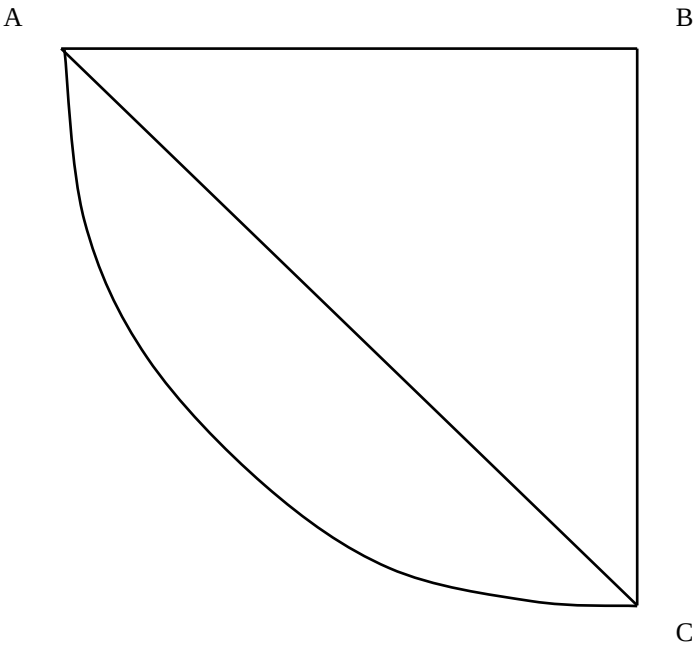
$$ABC: \quad C_{Ti} = \frac{1}{2} D_i A_i$$

  - Most models capture:
    - Duration
    - Amplitude
  - ie. Cumulative movement approximated by the triangle

- A better approximation

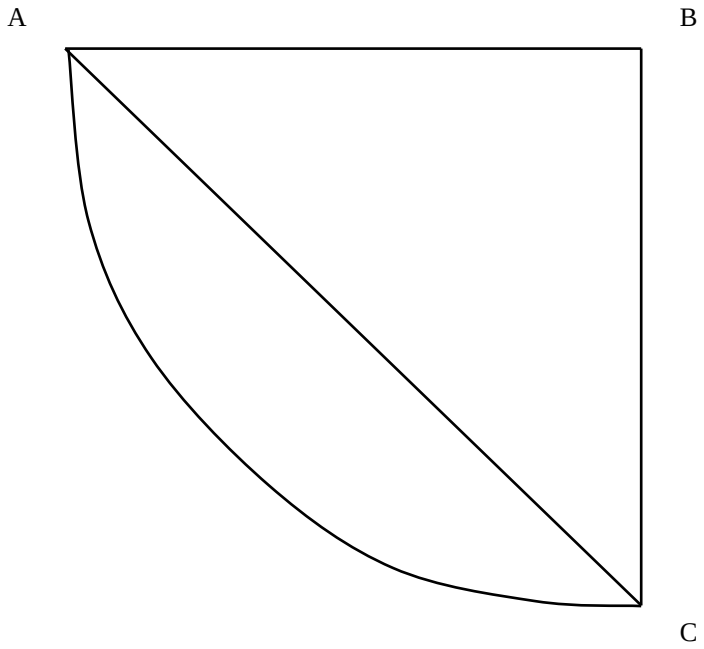


$$C_i = \sum_{t=1}^{N} r_t$$



$$\frac{\omega_t}{2} = s_t$$

$$\text{Total Area}: S_i = \frac{1}{2}\sum_{t=1}^{K} \alpha_t = \frac{1}{2} A_i$$

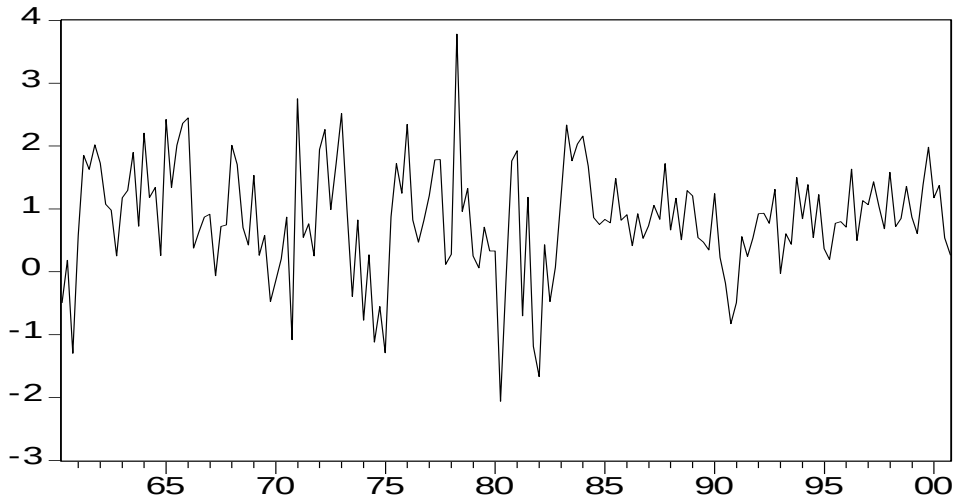


- H& P Excess Index

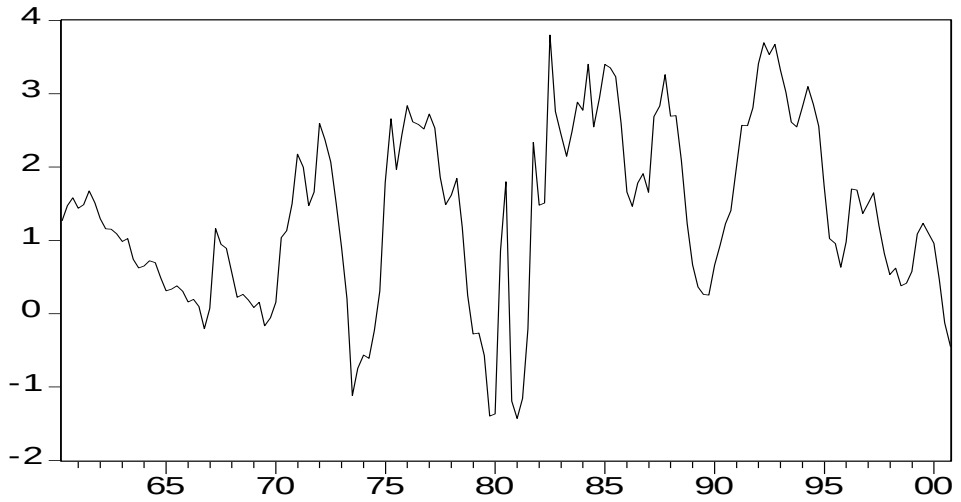
$$E_i = \frac{C_i - \frac{1}{2}A_i - \frac{1}{2}D_i A_i}{D_i}$$

- Tests for parametric models
  - can models capture the “SHAPE” of the business cycle

- **US data** (1960:1- 2000:1)
  - y_t - growth rate of real GDP



  - s_t - spread between
    - long term rates (10-yr T-bonds)
    - short term rates (3 mth T-bills)



- Model development for the **US**
  - basis - linear bivariate VAR(3)
  - non-linearity tests (by L/S/T)
    - H0: linear model
    - H1: non-linear STAR model
  - rejected H0 many times
- Models:
    - RW+drift / ARLI
    - Bi-NARLI / Com-NARLI
  - Bi-NARLI

$$y_t = \alpha_{10} + \alpha_{11} y_{t-1} + \alpha_{12} s_{t-1} + f_{yt}(\beta_{10} + \beta_{11} y_{t-1} + \beta_{12} s_{t-1})$$

$$f_{yt} = (1 + \exp\{-\gamma_1 (tr_{t-p} - c_1)\})$$

$$S_t = \alpha_{20} + \alpha_{21} y_{t-1} + \alpha_{22} s_{t-1} + f_{yt}(\beta_{20} + \beta_{21} y_{t-1} + \beta_{22} s_{t-1})$$

$$f_{st} = (1 + \exp\{-\gamma_2 (tr_{t-q} - c_2)\})$$

- Simulation procedure
  - 10,000 series
    - 163 observations
    - same initial conditions

HARDING & PAGAN *US* sample (1947:1-1997:1)

	US Data	RW + drift	Hamilton	Dur Dep	VAR(2)
Mean Duration (qrts)					
PT	3	2.3	4.4	4.8*	3.2
TP	17.8	16.4	20	16.9	23.2
Mean Amplitude (%)					
PT	-2.5	-1.5*	-2.8	-3.3*	-1.8
TP	20.2	16.6	27.3	25.0	23.3
Cumulation (%)					
PT	-4.1	-1.8*	-8.2	-8.5*	-3.8
TP	256	254	496	293	549
Excess (%)					
PT	-0.1	0.0	0.0	0.0	0.0
TP	1.1	-0.0*	-0.0*	0.0*	-0.0*

	US Data	RW + DRIFT	ARLI	Bi-NARLI	Com-NARLI
Mean Duration (qrts)					
PT	3.8	2.51* (2,3.55)	3.28 (2,5)	2.8765 (2,4)	3.08 (2,4.4)
TP	20.4	35.12 (16.5,70.5)	28.51 (14.37,54.5)	36.99 (17.2,72.5)	32.61 (15.8,66)
Mean Amplitude (%)					
PT	-2.14	-1.1057* (-1.79,-0.54)	-1.52 (-2.58,-0.74)	-1.7119 (-3.11,-0.63)	-1.84 (-3.22,-0.79)
TP	22.86	32.44 (15.1,64.41)	27.99 (13.4,54.19)	35.51 (16.43,69.2)	34.38 (16.2,69.43)
Cumulation (%)					
PT	-4.23	-1.5* (-3.12,-0.53)	-3.15 (-7.57,-0.85)	-2.8081 (-6.39,-0.67)	-3.35 (-7.48,-0.91)
TP	342	977 (173,2839)	704 (138,2084)	1101 (194,3301)	954 (181,2851)
Excess (%)					
PT	-0.0979	-0.0015 (-0.15,0.15)	0 (-0.16,0.16)	0.01 (-0.22,0.26)	0.017 (-0.19,0.23)
TP	1.3636	0.023* (-1.19,1.25)	0.02* (-1.21,1.31)	0.0788 (-1.2,1.399)	0.0632 (-1.3,1.398)

The Australian case

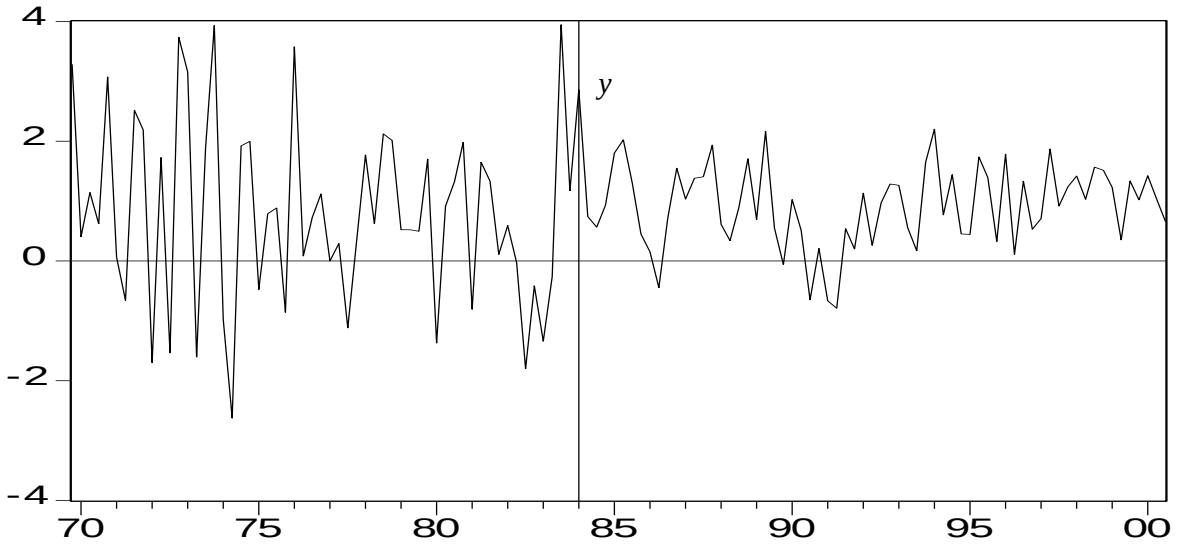
- (sample 1969:3-2000:3)

– no significant dynamics in y_t

– no evidence of $s_t \xrightarrow{GC} y_t$

– ie. random walk with drift

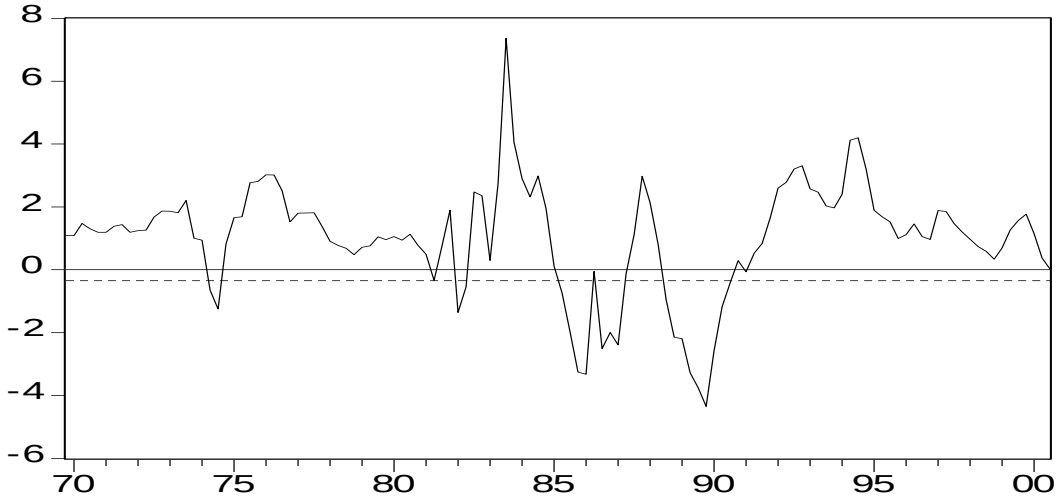
- y_t - growth rate of real GDP



- AR variance depends on variance of innovations
- break in 1984:1

$$\hat{y}_t = \underset{(0.08)}{0.83} + \underset{(0.04)}{0.08}\, s_{t-2}$$

- s_t - spread between
  - 15 yr T-bonds
  - 3 mth T-notes



- non-linearity tests supported TAR

$$\hat{s}_t = \underset{(0.13)}{0.18} + \underset{(0.07)}{0.81} s_{t-1} + (s_{t-1} < -0.35) * (\underset{(0.14)}{-0.28} s_{t-2} - \underset{(0.21)}{0.69} y_{t-2})$$

- Simulation results

	Aus Data	RW + drift	RW + drift + hetero	TAR
Mean Duration (qrts)				
PT	3.67	2.81 (2,4)	3.86 (2.25,6.3)	3.025 (2,4.75)
TP	29.5	21.02 (10,41)	10.71* (4.8,21.67)	20.26 (6.5,45.5)
Mean Amplitude (%)				
PT	-3.13	-1.74** (-2.64,-0.97)	-5.34 (-8.62,-2.78)	-2.37 (-4.1,-1.03)
TP	28.27	22.11 (10.41,42.42)	17.8 (8.27,34.2)	22.48 (7.9,48.03)
Cumulation (%)				
PT	-5.2	-2.75 (-5.57,-1.05)	-12.76 (-30.17,-3.52)	-4.3 (-10.34,-1.12)
TP	457.8	391 (68,1143)	151.19 (23.41,478.4)	406.49 (30.1,1338)
Excess (%)				
PT	0.19	-0.0009 (-0.2,0.2)	-0.0039 (-0.62,0.62)	-0.0062 (-0.32,0.31)
TP	1.53	0.0206 ** (-1.14,1.18)	0.0597 (-1.46,1.65)	0.2726 (-1.22,2.02)

- Conclusions
  - H&P algorithm
  - Non-Linearity
  - Bivariate
    - (financial/monetary variables)

- Further Research
  - Forecasting
  - Other countries
    - (UK / Canada / Netherlands)
  - Other leading indicators