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Magnetic fields in star cluster formation

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Motivation

- star formation regions observed to contain magnetic fields of significant strengths
- want to determine their role in the star formation process
Star formation is *clustered*!
Star formation is **clustered**!

Credit: J. Hatchell (Exeter)

Perseus molecular cloud
Dynamical models of star formation

e.g. Bate, Bonnell & Bromm (2003), Bonnell, Bate & Vine (2003), Bate & Bonnell (2005)
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- BUT star formation efficiency too high (all gas would eventually form stars).
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- discrepancy with *molecular cloud lifetimes?*
- observations indicate *magnetic fields* cannot be ignored!

---

e.g. Bate, Bonnell & Bromm (2003), Bonnell, Bate & Vine (2003), Bate & Bonnell (2005)
Smoothed Particle (Magneto)hydrodynamics

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\[ \rho(r) = \sum_{j=1}^{N} m_j W(|r - r_j|, h) \]
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solve the equations of MHD on moving, Lagrangian particles
The $\nabla \cdot B = 0$ constraint
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- use Euler potentials formulation for the magnetic field
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\mathbf{B} = \nabla \alpha \times \nabla \beta
\]

\[
\frac{d\alpha}{dt} = 0, \quad \frac{d\beta}{dt} = 0
\]

‘advection of magnetic field lines by Lagrangian particles’
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need accurate SPH derivatives (Price 2004)

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$d\alpha dt = 0$, $d\beta dt = 0$

‘Advection of magnetic field lines by Lagrangian particles’

Need accurate SPH derivatives (Price 2004)

$$\chi_{\mu \nu} \nabla^\mu \alpha_i = - \sum_j m_j (\alpha_i - \alpha_j) \nabla^\nu W_{ij}(h_i)$$

$$\chi_{\mu \nu} = \sum_j m_j (r_i^\mu - r_j^\mu) \nabla^\nu W_{ij}(h_i).$$

Add shock dissipation

$$\frac{d\alpha}{dt} = \sum_b m_b \frac{\alpha_B v_{sig}}{\bar{\rho}_{ab}} (\alpha_a - \alpha_b) \hat{r} \cdot \nabla_a W_{ab}$$

$$\frac{d\beta}{dt} = \sum_b m_b \frac{\alpha_B v_{sig}}{\bar{\rho}_{ab}} (\beta_a - \beta_b) \hat{r} \cdot \nabla_a W_{ab}$$
The $\nabla \cdot \mathbf{B} = 0$ constraint

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“advection of magnetic field lines by Lagrangian particles”

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\]

BUT: helicity constraints ($\mathbf{A} \cdot \mathbf{B} = \text{const}$): cannot represent certain fields. Field growth suppressed once clear mapping from initial to final particle distribution is lost.
Test problems

Mach 25 MHD shock (e.g. Balsara 1998)
(Price & Monaghan 2004a,b, Price 2004)

Orszag-Tang vortex (everyone)
(Price & Monaghan 2005, Rosswog & Price 2007)

Current loop advection (e.g. Gardiner & Stone 2007)
(Rosswog & Price 2007)
Effect of magnetic fields on single and binary star formation:

Price & Bate (2007), MNRAS 377, 77
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Magnetic fields in star cluster formation

Magnetic fields in star cluster formation


- 50 solar mass cloud
- diameter 0.375 pc, $n_{\text{H}_2} = 3.7 \times 10^4 \text{ cm}^{-3}$
- initial uniform B field
- $T \sim 10\text{K}$
- turbulent velocity field $P(k) \propto k^{-4}$
- RMS Mach number 6.7
- barytropic equation of state
- form sink particles at $10^{-11} \text{ g cm}^{-3}$
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as in Bate, Bonnell & Bromm (2003), but with magnetic fields...
Important parameters
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\[
\left( \frac{M}{\Phi} \right) / \left( \frac{M}{\Phi} \right)_{\text{crit}}
\]

magnetic field vs gravity
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\left( \frac{M}{\Phi} \right) / \left( \frac{M}{\Phi} \right)_{\text{crit}} \quad \text{magnetic field vs gravity}
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\[
\beta = \frac{c_s^2 \rho}{\frac{1}{2} B^2 / \mu_0} \quad \text{magnetic fields vs pressure}
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Important parameters

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magnetic fields vs pressure

\[ \frac{v_{\text{turb}}}{v_{\text{Alfven}}} \]

magnetic fields vs turbulence
Important parameters

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\]

magnetic field vs gravity
magnetic fields vs pressure
magnetic fields vs turbulence

these parameters are independent!
Important parameters

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\[ \frac{v_{\text{turb}}}{v_{\text{Alfven}}} \]

- magnetic fields vs turbulence

these parameters are independent!

Observations suggest molecular clouds are:

- mildly supercritical
  - have beta < 1
- marginally super-Alfvenic

Magnetic pressure-supported voids
Magnetic pressure-supported voids
Star formation rate
Effect on IMF
Effect on IMF
## Effect on IMF

<table>
<thead>
<tr>
<th></th>
<th>$N_{\text{BDs}}$</th>
<th>$N_{\text{stars}}$</th>
<th>ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hydro</td>
<td>44</td>
<td>14</td>
<td>3.14</td>
</tr>
<tr>
<td>$M/\Phi = 20$</td>
<td>51</td>
<td>18</td>
<td>2.83</td>
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<td>$M/\Phi = 10$</td>
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<td>$M/\Phi = 5$</td>
<td>15</td>
<td>14</td>
<td>1.07</td>
</tr>
<tr>
<td>$M/\Phi = 3$</td>
<td>8</td>
<td>7</td>
<td>1.14</td>
</tr>
</tbody>
</table>
even stronger field...
even stronger field...

t=0 yr
Mass/flux ratio = 3

log column density [g/cm²]
Which MHD regime is most realistic?
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$\beta > 1$
Which MHD regime is most realistic?

- $t_{ff} = 0.6$
- $t_{ff} = 0.8$
- $t_{ff} = 1$
- $t_{ff} = 1.2$

- $\beta > 1$
- $\beta < 1$

“Stripiness”
Which MHD regime is most realistic?

\[ \text{beta} > 1 \quad \text{beta} < 1 \]
Column density striations along field lines due to streaming motions in the gas
Column density striations along field lines due to streaming motions in the gas.
Column density striations along field lines due to streaming motions in the gas
Goldsmith, Heyer, Brunt et al. (2007)
“A hole...[where] it appears that some agent has been responsible for dispersing the molecular gas”

Goldsmith, Heyer, Brunt et al. (2007)
Summary
• (even supercritical) magnetic fields delay and suppress star formation.
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• (even supercritical) magnetic fields delay and suppress star formation.

• strong magnetic fields (beta < 1) lead to large scale voids, anisotropic turbulent motions and column density striations in collapsing molecular clouds which we should expect to observe.

• strongly inhibited accretion, resulting in a lower star formation rate and longer molecular cloud lifetimes.

• trend towards fewer brown dwarfs with increasing field strength.
Where we’re headed:
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- new vector potential formulation for the magnetic field evolution without restrictions associated with Euler potentials
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• new *vector potential* formulation for the magnetic field evolution without restrictions associated with Euler potentials

• using *physical resistivity* to solve the magnetic flux problem
Where we’re headed:

- **new vector potential** formulation for the magnetic field evolution without restrictions associated with Euler potentials

- using **physical resistivity** to solve the magnetic flux problem

- **MHD + radiation transport** (flux-limited diffusion)