

Can Structural Small Open-Economy Models Account for the Influence of Foreign Disturbances?*

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Abstract

This paper demonstrates that an estimated, structural, small open-economy model of the Canadian economy cannot account for the substantial influence of foreign-sourced disturbances identified in numerous reduced-form studies. The benchmark model assumes uncorrelated shocks across countries and implies that U.S. shocks account for less than 3 percent of the variability observed in several Canadian series, at all forecast horizons. Accordingly, model-implied cross-correlation functions between Canada and U.S. are essentially zero. Both findings are at odds with the data. A specification that assumes correlated cross-country shocks partially resolves this discrepancy, but still falls well short of matching reduced-form evidence. One central difficulty resides in the model's inability to account for comovement without generating counterfactual implications for the real exchange rate, the terms of trade and Canadian inflation.

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1 Introduction

This paper investigates whether an estimated microfounded semi-small open-economy model can reproduce the observed comovement in international business cycles. Focusing on Canada as the semi-small open economy, the starting point for the analysis is the large body of empirical work that identifies a significant influence of U.S. shocks on Canadian economic fluctuations.

There has been ample theoretical work seeking to replicate the observed comovement in economic activity across countries. Until recently, the empirical validation of these models largely relied on calibrations aimed at matching selected moments in the data — see the contributions of Backus et al. (1992, 1995), Stockman and Tesar (1995) and Baxter (1995) for a review. The new open-economy macroeconomics (NOEM) has since produced significant theoretical advancements in international macroeconomic modeling. Given the empirical success of closed-economy models built on similar foundations, it is not surprising that there is a growing literature estimating NOEM models. These include amongst others: Ambler et al. (2003), Bergin (2003, 2004), Del Negro (2003), Ghironi (2000), Justiniano and Preston (2008b), Lubik and Schorfheide (2003, 2005), Lubik and Teo (2005) and Rabanal and Tuesta (2005).

To our knowledge, the ability of these NOEM models to explain the observed comovement in economic fluctuations has not been previously systematically analyzed in this empirical literature. This paper fills this gap by evaluating a workhorse semi-small NOEM model in this particular dimension. The focal point is the model’s ability to replicate the fraction of the variance in Canadian macroeconomic series attributed to U.S. shocks. We also contrast the cross-country correlation functions in the model and data, particularly for output.

The analysis is pursued using generalizations of the semi-small open-economy framework proposed by Gali and Monacelli (2005).¹ Following Monacelli (2005), we allow for deviations from the law of one price. In addition, we consider incomplete asset markets, a large set of disturbances, and incorporate other real and nominal rigidities (e.g., wage stickiness, indexation and habits) which have been found crucial in fitting closed-economy models as documented by Christiano et al. (2005) and Smets and Wouters (2007).

The model is estimated using Bayesian methods with data for Canada and the United

¹The model is technically a semi-small open-economy model, where domestic goods producers have some market power, but we shall nonetheless refer to it as a small open economy. Note also that our analysis appeals to an earlier interpretation in Gali and Monacelli (2005) of a small-large country pair, rather than as an analysis of a continuum of small open economies.

States. Our baseline specification assumes that shocks across these two countries are independent. This contrasts with much of the international real business cycle literature which often assumes correlated cross-country technology shocks, but is consistent with all of the empirical NOEM studies cited above.² Under independent shocks, the channels of transmission embedded in the model (e.g. risk sharing and expenditure switching effects) must account for the cross-country comovement in aggregate fluctuations.

The main contribution of this paper is to document that the baseline specification fails to account for the influence of foreign shocks. A structural variance decomposition reveals that all U.S. shocks combined cannot explain more than 3 percent of the variability in Canadian output, interest rates or inflation. Furthermore, model-implied cross-correlation functions between these two countries are estimated to be essentially zero. Both findings are in stark contrast with reduced-form empirical evidence in the same data. These results are shown to be robust across alternative specifications, priors and detrending methods.

Model parameters chosen based on previous calibrated studies can deliver both large shares of domestic variance being attributed to U.S. shocks and substantial cross-country correlation in some series. Therefore, our findings indicate that the inability to reproduce some international correlations — known as the quantity anomaly in the case of output (see Baxter and Crucini (1995)) — is exacerbated in estimated models. The results also suggest caution in extrapolating to the international dimension the empirical success of related closed-economy models.

A second contribution of this paper is to document that the international comovement problem can only be partially resolved by introducing disturbances that are correlated across countries. To do this, each Canadian structural shock is written as the sum of two orthogonal components: a disturbance common to the same type of shock in the U.S. block, and a country-specific disturbance. This decomposition can be viewed as a rough approximation to reduced-form dynamic factor models that have been used for business cycle analysis.³

When all U.S. shocks are common to the domestic block the DSGE model gets closer to matching the reduced-form variance decomposition. However, there are at least three reasons for not viewing this specification as a panacea for the model's inability to replicate the observed influence of foreign disturbances. First, at medium to long horizons the fraction of output

²For example, Gali and Monacelli (2005) consider the role of technology spillovers in their calibration study. But likelihood-based empirical studies have typically excluded this possibility.

³In a closed-economy setting, Boivin and Giannoni (2006) establish a formal link between DSGE and dynamic factor models.

variation explained by U.S. disturbances is still below the reduced-form evidence. Second, this specification engenders an extreme version of the exchange rate disconnect puzzle — see Devereux and Engel (2002). Third, some of the induced correlations are difficult to rationalize on structural grounds.

A third contribution of our analysis is to elucidate reasons for the model’s failure in this crucial dimension. The inability to match the comovement in the data gets reflected in cross-correlation amongst supposedly orthogonal innovations in our baseline model. These correlations point to a complex pattern of covariation, beyond pairing the same type of disturbance across countries, explaining the limited success of the common shocks models. More promising guidance for future research is given by the observation that while U.S. shocks can a priori match some bivariate cross-country correlations, they also have strong counterfactual predictions, particularly involving the real exchange rate, the terms of trade and domestic inflation. This tension helps understand, at least in part, why the estimated model shuts down international linkages and indicates ample scope to improve the transmission mechanism of foreign disturbances in this class of models.

This paper broadly relates to the international business cycle literature and recent empirical work with NOEM models. More closely related is Adolfson et al. (2007) who present a state-of-the-art model, more richly specified than the one considered here. While their model performs very well in several dimensions, an earlier version, Adolfson et al. (2005), reported variance decompositions revealing little transmission of foreign-sourced disturbances from the European Union to Sweden — a property that is not remarked upon. Similar observations apply to an extension of this framework by Christiano et al. (2009), and de Walque et al. (2005) in a two-country model for the U.S. and the Euro Area. We also build on Schmitt-Grohe (1998) who evaluates whether a calibrated small open-economy real business cycle model can replicate impulse responses to a single foreign output shock, extracted from a bivariate U.S.-Canada vector autoregression.⁴ Our results suggest that in estimation the failure to capture international linkages may be worse than when the model is calibrated.

⁴Schmitt-Grohe (1998) concludes that financial and trade linkages are not capable of reproducing the strong response of Canadian hours, output and investment to innovations in U.S. GNP. She suggests that these difficulties might be alleviated by the introduction of sticky prices. Our analysis reveals that the inability to capture the influence of foreign shocks persists in an estimated model even when various nominal rigidities are considered.

2 Evidence on International Linkages

A central empirical regularity that international business cycle models seek to explain is the observed cross-country comovement amongst economic variables. This section documents a number of statistics suggesting comovement is a salient feature of U.S. and Canadian business cycles, understanding that earlier literature testifies to the generality of these insights in other economies. This close link is not surprising considering the U.S. accounts for 75 percent of Canada's average trade share.⁵

2.1 Data

We use data for twelve series that in section 4 constitute the observable states in the estimated DSGE model. These are: real per-capita output, inflation, nominal interest rates, real wages and hours in both the U.S. and Canada, as well as the bilateral terms of trade and the real exchange rate. Details of the data are in appendix A. Consistent with the model presented later, output and real wages are expressed in log-deviations from a common linear trend. The real exchange rate and the terms of trade are given in log-differences. Section 6 evidences the robustness of our results to alternative detrending of these series. Inflation and interest rates are expressed as percentages and, like hours, are not transformed, except that all series are demeaned. The sample runs from 1982q1 to 2007q1, although the first 8 quarters are used to initialize the Kalman filter.

2.2 Reduced-Form Evidence

The solid black lines in figure 1 give the sample cross-correlations between Canadian and lagged U.S. series, at lags zero through four. The remaining lines correspond to the estimated DSGE model and are discussed in section 4. For presentation purposes only we exclude these statistics for the terms of trade and the real exchange rate but discuss them later on.

For many series these cross-correlations are large at various lags and rarely equal to zero. For example, the contemporaneous correlations between Canadian and U.S. output, inflation, nominal interest rates and wages are: 0.69, 0.45, 0.83 and 0.72, respectively. This is consistent with earlier studies on international comovement, such as Backus et al. (1992), Stockman and Tesar (1995) and Ambler et al. (2004).

⁵In our sample, 1/2 the share of U.S. imports in total Canadian imports plus 1/2 the share of total Canadian exports oriented to the U.S. equals 75.1 percent.

We rely on two statistical models to compute the variance share of these Canadian series that is attributable to U.S. shocks. The first model is a VAR subject to the exclusion restriction of no feedback from Canada to the U.S. that is embedded in the DSGE model. It is formally a seemingly unrelated regression (SUR). Variance decompositions are obtained with a Cholesky decomposition of the SUR innovations with no attempt to identify any particular shock. We only wish to infer the variance shares explained by disturbances also affecting the U.S. block.⁶ The SUR is estimated with the efficient block-recursive Gibbs algorithm proposed by Zha (1999). Details are in appendix B.

Table 1 reports variance shares attributable to all foreign shocks in a SUR with 4 lags at 1, 4, and 8 quarter horizons, and the stationary, or long-horizon, variance. We report medians and 90 percent posterior probability bands. In the short, medium and long run, U.S. disturbances account for a substantial fraction of variation in Canadian series. For example, at a 4 quarter horizon, shares vary from 25 percent for hours to 44 percent for output. At long horizons, contributions vary from 65 percent for inflation to 76 percent for output. The latter is almost identical to the 74 percent share for U.S. shocks in the smaller, but overidentified, structural SUR model of Cushman and Zha (1997).

The SUR analysis is limited by sample considerations to a dozen series. An alternative is to estimate a dynamic moving average factor model, which can encompass richer sources of shocks and channels of transmission by accommodating a larger number of series. Hence, we also mention variance decomposition estimates from such a model, estimated for the U.S. and Canada on a similar sample. The reader is referred to an earlier version of this paper, Justiniano and Preston (2006), which builds on Justiniano (2007), for further details.

To explain a panel of 32 series (16 for each country) formal model comparisons dictate including four factors, two of which are common to both countries (foreign factors), with the remaining two exclusive to the Canadian economy (domestic). The factors and idiosyncratic, series-specific, components follow independent autoregressive processes of order three. Measures of fit also suggest the presence of moving average dynamics in the loadings, indicating that spillover effects may be important for some variables.

Justiniano and Preston (2006) show that the median share of the long-horizon variance of Canadian output, inflation, interest rates, the terms of trade and the real exchange rate explained by the two foreign factors is 0.71, 0.15, 0.31, 0.22 and 0.11. While differences in sample and data preclude direct comparisons with the SUR results, this distinct methodology

⁶The results obtained with this identification procedure are invariant to re-ordering of the series.

clearly indicates an important role for U.S. shocks in explaining Canadian business cycles, particularly for output. Similar findings are reported by Kose et al. (2003, 2008), Lumsdaine and Prasad (2003) and Bowden and Martin (1995) with related methodologies.

Taken together, these various statistics suggest strong comovement between Canadian and U.S. business cycles. The remainder of the paper explores whether a structural model can similarly capture these international linkages.

3 The Model

Building on Gali and Monacelli (2005), Monacelli (2005) and Justiniano and Preston (2008b), the following section details a small open-economy model, allowing for habit formation, indexation of prices, labor market imperfections and incomplete markets. These papers extend the microfoundations described by Clarida et al. (1999) and Woodford (2003) for analyzing monetary policy in a closed-economy setting to an open-economy context.

3.1 Households

Each household maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t \tilde{\varepsilon}_{g,t} \left[\frac{(C_t - H_t)^{1-1/\sigma}}{1-1/\sigma} - \frac{\tilde{\varepsilon}_{l,t} N_t^{1+\varphi}}{1+\varphi} \right]$$

where N_t is the labor input; $H_t \equiv hC_{t-1}$ is an external habit taken as exogenous by the household and $0 < h < 1$; $\sigma^{-1}, \varphi > 0$ are the inverse elasticities of intertemporal substitution and labor supply; and $\tilde{\varepsilon}_{g,t}$ and $\tilde{\varepsilon}_{l,t}$ denote preference and labor supply shocks respectively. C_t is a composite consumption index

$$C_t = \left[(1-\tau)^{\frac{1}{\eta}} (C_{H,t})^{\frac{\eta-1}{\eta}} + \tau^{\frac{1}{\eta}} (C_{F,t})^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$$

where $C_{H,t}$ and $C_{F,t}$ are Dixit-Stiglitz aggregates of the available domestic and foreign produced goods given by

$$C_{H,t} = \left[\int_0^1 C_{H,t}(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}} \quad \text{and} \quad C_{F,t} = \left[\int_0^1 C_{F,t}(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}$$

where $\eta > 0$ gives the elasticity of substitution between domestic and foreign goods; $\theta > 1$ is the elasticity of substitution between types of differentiated domestic or foreign goods; and τ the relative weight of these goods in the overall consumption bundle.

Assuming the only available assets are one-period domestic and foreign bonds, optimization occurs subject to the flow budget constraint

$$P_t C_t + D_t + S_t B_t = D_{t-1} R_{t-1} + S_t B_{t-1} R_{t-1}^* \phi_t(A_t) + \Pi_{H,t} + \Pi_{F,t} + W_t N_t + T_t \quad (1)$$

for all $t > 0$, where D_t and B_t denote holdings of one-period domestic and foreign bonds with gross interest rates R_t and R_t^* . S_t is the nominal exchange rate. The price indices P_t , $P_{H,t}$ and P_t^* correspond to the domestic CPI, domestic goods prices and foreign prices and are defined below. Households receive wages W_t for labor supplied and $\Pi_{H,t}$ and $\Pi_{F,t}$ denote profits from equity holdings in domestic and retail firms. T_t denotes taxes and transfers.

Following Benigno (2001), Kollmann (2002) and Schmitt-Grohe and Uribe (2003), the function $\phi_t(\cdot)$ is interpretable as a debt elastic interest rate premium given by

$$\phi_t = \exp \left[-\chi \left(A_t + \tilde{\phi}_t \right) \right] \quad \text{where} \quad A_t \equiv \frac{S_{t-1} B_{t-1}}{\bar{C}_F P_{t-1}}$$

is the real quantity of outstanding foreign debt expressed in terms of domestic currency as a fraction of steady-state consumption of the imported good, \bar{C}_F , and $\tilde{\phi}_t$ a risk premium shock. This ensures stationarity of the foreign debt level in a log-linear approximation to the model.

Implicitly underwriting this expression for the budget constraint is the assumption that all households in the domestic economy receive an equal fraction of both domestic and retail firm profits and that labor income risk is pooled across agents. Absent this assumption, which imposes complete markets within the domestic economy, the analysis would require modeling the distribution of wealth across agents. This assumption also ensures that households face identical decision problems and choose identical state-contingent plans for consumption.

The household's optimization problem requires allocation of expenditures across all types of domestic and foreign goods both intratemporally and intertemporally. This yields the following set of optimality conditions. The demand for each category of consumption good is

$$C_{H,t}(i) = (P_{H,t}(i) / P_{H,t})^{-\theta} C_{H,t} \quad \text{and} \quad C_{F,t}(i) = (P_{F,t}(i) / P_{F,t})^{-\theta} C_{F,t}$$

for all i with associated aggregate price indexes for the domestic and foreign consumption bundles given by $P_{H,t}$ and $P_{F,t}$. The optimal allocation of expenditure across domestic and foreign goods implies the demand functions

$$C_{H,t} = (1 - \tau) (P_{H,t} / P_t)^{-\eta} C_t \quad \text{and} \quad C_{F,t} = \tau (P_{F,t} / P_t)^{-\eta} C_t \quad (2)$$

where $P_t = \left[(1 - \tau) P_{H,t}^{1-\eta} + \tau P_{F,t}^{1-\eta} \right]^{\frac{1}{1-\eta}}$ is the consumer price index. Allocation of expenditures on the aggregate consumption bundle satisfies

$$\Xi_t = \tilde{\varepsilon}_{g,t} (C_t - H_t)^{-1/\sigma} \quad (3)$$

and portfolio allocation is determined by the optimality conditions

$$\Xi_t S_t P_t = \beta E_t [R_t^* \phi_{t+1} \Xi_{t+1} S_{t+1} P_{t+1}] \quad (4)$$

$$\Xi_t P_t = \beta E_t [R_t \Xi_{t+1} P_{t+1}] \quad (5)$$

for Lagrange multiplier Ξ_t attached to the constraint (1). The latter when combined with (3) gives the Euler equation.

The household problem in the foreign economy is similarly described with the exceptions now noted. Because the foreign economy is approximately closed (the influence of the domestic economy is negligible), the available consumption bundle comprises the continuum of foreign produced goods $C_{F,t}^*(j)$ for $j \in [0, 1]$. Foreign households need only decide how to allocate expenditures across these goods in any time period t and also over time. Foreign debt in the foreign economy is in zero net supply, using the property that the domestic economy engages in negligible financial asset trade. There is no access to domestic debt markets for foreign agents. Conditions (3) and (5) continue to hold with all variables taking superscript “*”.

3.2 Optimal Labor Supply

Following Erceg et al. (2000) and Woodford (2003), assume a single economy-wide labor market and that producers of the domestic good hire the same bundle of labor inputs at common wage rates. Firm j produces good j with technology $Y_t(j) = \tilde{\varepsilon}_{a,t} f(N_t(j))$ where $\tilde{\varepsilon}_{a,t}$ is a neutral technology shock and $f(\cdot)$ satisfies the usual Inada conditions. The labor input used in the production of each good j and associated aggregate wage index are given by the CES aggregators

$$N_t(j) \equiv \left[\int_0^1 N_t(k)^{\frac{\theta_w - 1}{\theta_w}} dk \right]^{\frac{\theta_w}{\theta_w - 1}} \quad \text{and} \quad W_t = \left[\int_0^1 W_t(k)^{1 - \theta_w} dk \right]^{\frac{1}{1 - \theta_w}}$$

for $\theta_w > 1$. Firm j 's demand for each type of labor k is determined by maximizing the former index for a given level of wage payment. This gives the demand function

$$N_t(k) = N_t(j) \left(\frac{W_t(k)}{W_t} \right)^{-\theta_w}. \quad (6)$$

Households supply their labor under monopolistic competition. They face a Calvo-style wage-setting problem, having the opportunity to re-optimize their wage with probability $1 - \alpha_w$ each period, where $0 < \alpha_w < 1$. As in Christiano et al. (2005) and Woodford (2003), households not re-optimizing adjust their wage according to the indexation rule

$$\log W_t(k) = \log W_{t-1}(k) + \gamma_w \pi_{t-1}$$

where $0 \leq \gamma_w \leq 1$ measures the degree of indexation to the previous-period's inflation rate and $\pi_t = \log(P_t/P_{t-1})$. Since all households having the opportunity to reset their wage face the same decision problem, they set a common wage, W_t .

The household's wage-setting problem in period t is to maximize

$$E_t \sum_{T=t}^{\infty} (\alpha_w \beta)^{T-t} \left[\Xi_T W_t(k) \left(\frac{P_{T-1}}{P_{t-1}} \right)^{\gamma_w} N_T(k) - \frac{\tilde{\varepsilon}_{l,t} N_T(k)^{1+\varphi}}{1+\varphi} \right]$$

by choice of $W_t(k)$ subject to the labor demand function (6). The first-order condition for this problem is

$$E_t \sum_{T=t}^{\infty} (\alpha_w \beta)^{T-t} \left[\Xi_T \left(\frac{P_{T-1}}{P_{t-1}} \right)^{\gamma_w} \left(N_T(k) + W_t(k) \frac{\partial N_T(k)}{\partial W_t(k)} \right) - \tilde{\varepsilon}_{l,t} N_T^{\varphi} \frac{\partial N_T(k)}{\partial W_t(k)} \right] = 0. \quad (7)$$

Households in the foreign block face an identical problem, with appropriate substitution of foreign variables and technology and preference parameters.

3.3 Domestic Producers

There is a continuum of monopolistically competitive domestic firms producing differentiated goods. Calvo-style price setting is assumed, allowing for indexation to past domestic goods-price inflation. In any period t , a fraction $1 - \alpha_H$ of firms set prices optimally, while a fraction $0 < \alpha_H < 1$ of goods prices are adjusted according to the indexation rule

$$\log P_{H,t}(i) = \log P_{H,t-1}(i) + \gamma_H \pi_{H,t-1}, \quad (8)$$

where $0 \leq \gamma_H \leq 1$ measures the degree of indexation to the previous-period's inflation rate and $\pi_{H,t} = \log(P_{H,t}/P_{H,t-1})$. Since all firms having the opportunity to reset their price in period t face the same decision problem, they set a common price $P'_{H,t}$. Firms setting prices in period t face a demand curve

$$y_{H,T}(i) = \left(\frac{P_{H,t}(i)}{P_{H,T}} \cdot \left(\frac{P_{H,T-1}}{P_{H,t-1}} \right)^{\gamma_H} \right)^{-\theta} (C_{H,T} + C_{H,T}^*) \quad (9)$$

for all $T \geq t$ and take aggregate prices and consumption bundles as parametric.

The firm's price-setting problem in period t is to maximize the expected present discounted value of profits

$$E_t \sum_{T=t}^{\infty} \alpha_H^{T-t} Q_{t,T} \left[P_{H,t}(i) \left(\frac{P_{H,T-1}}{P_{H,t-1}} \right)^{\gamma_H} y_{H,T}(i) - W_T f^{-1} \left(\frac{y_{H,T}(i)}{\tilde{\varepsilon}_{a,t}} \right) \right]$$

subject to the demand curve (9), where $Q_{t,T}$ is interpreted as a stochastic discount factor evaluated at aggregate income. This implies the first-order condition

$$E_t \sum_{T=t}^{\infty} \alpha_H^{T-t} Q_{t,T} y_{H,T}(i) \left[P_{H,t}(i) \left(\frac{P_{H,T-1}}{P_{H,t-1}} \right)^{\gamma_H} - \frac{\theta}{\theta-1} P_{H,T} MC_T \right] = 0 \quad (10)$$

where MC_t is the marginal cost function of firm i .

Foreign firms face an analogous problem. Thus the optimality condition takes an identical form, with all variables taking the superscript “*” and the subscript H being changed to F . Preferences and shocks are allowed to differ and the small open-economy assumption implies that P_t^* is equivalent to $P_{F,t}^*$.

3.4 Retail Firms

Retail firms in the small open economy import foreign differentiated goods for which the law of one price holds at the docks. In determining the domestic currency price of imported goods they are monopolistically competitive. Pricing power leads to a violation of the law of one price in the short run.

Like domestic firms, retail firms face a Calvo-style price-setting problem allowing for indexation to past inflation. A fraction $1 - \alpha_F$ of firms set prices optimally, while a fraction $0 < \alpha_F < 1$ of goods prices are adjusted according to an indexation rule analogous to (8) with indexation parameter $0 < \gamma_F < 1$. Firms setting prices in period t face a demand curve

$$C_{F,T}(i) = \left(\frac{P_{F,t}(i)}{P_{F,T}} \cdot \left(\frac{P_{F,T-1}}{P_{F,t-1}} \right)^{\gamma_F} \right)^{-\theta} C_{F,T} \quad (11)$$

for all $T \geq t$ and take aggregate prices and consumption bundles as parametric. The firm's price-setting problem in period t is to maximize the expected present discounted value of profits

$$E_t \sum_{T=t}^{\infty} \alpha_F^{T-t} Q_{t,T} C_{F,T}(i) \left[P_{F,t}(i) \left(\frac{P_{F,T-1}}{P_{F,t-1}} \right)^{\gamma_F} - S_T P_{F,T}^*(i) \right]$$

subject to the demand curve, (11), and implies the first-order condition

$$E_t \sum_{T=t}^{\infty} \alpha_F^{T-t} Q_{t,T} \left[P_{F,t}(i) \left(\frac{P_{F,T-1}}{P_{F,t-1}} \right)^{\gamma_F} - \frac{\theta}{\theta-1} \tilde{c}_T P_{F,T}^*(i) \right] = 0.$$

In the foreign economy there is no analogous optimal pricing problem. Because imports form a negligible part of the foreign consumption bundle, variations in the import price have a negligible effect on the evolution of the foreign price index, P_t^* , and need not be analyzed.

3.5 International Risk Sharing and Prices

Optimality conditions for domestic and foreign bond holdings imply the uncovered interest rate parity condition

$$E_t \Xi_{t+1} P_{t+1} [R_t - R_t^* (S_{t+1}/S_t) \phi_{t+1}] = 0, \quad (12)$$

placing a restriction on the relative movements of domestic and foreign interest rates, and changes in the nominal exchange rate.

The terms of trade is defined as $P_{F,t}/P_{H,t}$. The real exchange rate is given by $S_t P_t^*/P_t$. Since $P_t^* = P_{F,t}^*$, when the law of one price fails to hold $\tilde{\Psi}_{F,t} \equiv S_t P_t^*/P_{F,t} \neq 1$, which defines what Monacelli (2005) calls the law of one price gap. The models of Galí and Monacelli (2005) and Monacelli (2005) are respectively characterized by whether or not $\tilde{\Psi}_{F,t} = 1$.

3.6 Monetary and Fiscal Policy

Monetary policy is conducted according to an interest rate rule

$$\frac{R_t}{\bar{R}} = \left(\frac{R_{t-1}}{\bar{R}} \right)^{\rho_i} \left[\left(\frac{P_t}{P_{t-1}} \right)^{\phi_\pi} \left(\frac{Y_t}{\bar{Y}} \right)^{\phi_y} \left(\frac{Y_t}{Y_{t-1}} \right)^{\phi_{\Delta y}} \left(\frac{S_t}{S_{t-1}} \right)^{\phi_S} \right]^{(1-\rho_i)} \tilde{\varepsilon}_{m,t}$$

where \bar{R} and \bar{Y} are steady-state values of nominal interest rates and output and $\tilde{\varepsilon}_{m,t}$ is an exogenous disturbance. Policy responds to contemporaneous values of inflation, output, output growth and the growth rate in the nominal exchange rate. Evidence for rules that respond to exchange rates in various small open economies is found in Lubik and Schorfheide (2005b) and Justiniano and Preston (2008b). Fiscal policy is specified as a zero debt policy.

3.7 Exogenous Disturbances

All shocks have unit means. In log deviations from steady state the following assumptions are made. In the foreign block, the technology, preference and labor disutility shocks are first-order autoregressive processes. The monetary policy innovation and cost-push shock in the pricing of foreign goods are i.i.d. In the domestic block, technology, preference, labor disutility shocks and the cost-push shock in imported goods pricing are first-order autoregressive processes, as is the risk premium shock. The monetary policy shock and the cost-push shock to domestic price

setters are i.i.d. Justiniano and Preston (2006) discuss identification issues which motivate these specifications.

3.8 General Equilibrium

Equilibrium requires that all markets clear. Goods market clearing requires

$$Y_{H,t} = C_{H,t} + C_{H,t}^* \quad \text{and} \quad Y_t^* = C_t^* \quad (13)$$

in the domestic and foreign economies respectively. The model is closed assuming foreign demand for the domestically produced good is specified as

$$C_{H,t}^* = \left(\frac{P_{H,t}^*}{P_t^*} \right)^{-\lambda} Y_t^*$$

where $\lambda > 0$. This demand function is standard in small open-economy models (see Kollmann (2002) and McCallum and Nelson (2000)) and nests the specification in Monacelli (2005) by allowing λ to be different from η , the domestic elasticity of substitution across goods in the domestic economy, giving additional flexibility in the transmission mechanism of foreign disturbances to the domestic economy. Our results are unaffected by the parametrization of this demand function.⁷ The dynamics of Y_t^* and other foreign variables remain specified by the structural relations developed above. Domestic debt is in zero net supply so that $D_t = 0$ for all t .⁸

The analysis considers a symmetric equilibrium in which all domestic producers setting prices in period t set a common price $P_{H,t}$. Similarly, all domestic retailers and foreign firms each choose a common price $P_{F,t}$ and P_t^* . Analogous conditions hold for wage setters in the domestic and foreign economies. Finally, we assume households have identical initial wealth, so that each faces the same period budget constraint and make identical consumption and portfolio decisions.

4 Estimation Methodology and Data

4.1 Estimation and Priors

Model parameters are estimated using Bayesian methods now used extensively in the empirical macroeconomics literature — see Schorfheide (2000) for a seminal reference and Justiniano

⁷Constraining λ to equal η results in identical insights from the estimation.

⁸A similar condition holds for the foreign economy once it is noted that domestic holdings of foreign debt, B_t , is negligible relative to the size of the foreign economy.

and Preston (2006) for further details in the context of the model estimated here. We work with a log-linear approximation of the model in a neighborhood of a non-stochastic steady state. The observables used in estimation were described in section 2.

The first column of table 2 presents the priors for the coefficients, indicating the density, mean and standard deviation. They are motivated by earlier work reported in Justiniano and Preston (2008b), are fairly uncontroversial, and accord with other studies adopting Bayesian inference. Several parameters, not well identified, are calibrated. The discount factor is fixed at 0.99. The elasticities of demand across varieties of goods and labor inputs in both the domestic and foreign block are set equal to 8, as in Woodford (2003). Following Benigno (2001), the parameter governing the interest rate elasticity of debt is fixed at 0.01.⁹

Priors that are particularly germane to the transmission of foreign shocks deserve further comment. The densities for the degree of openness, τ , and the the elasticity of substitution between home and foreign goods, η , are chosen to generate a tight distribution for the steady-state share of imports to GDP, centered at 0.27 as in the data.¹⁰ For τ we specify a beta density with mean 0.29, matching the average trade share in our sample, and a tight standard deviation of 0.1. For η we choose a normal with mean 0.9 and also small dispersion of 0.1. Our results are even stronger with looser priors on τ , which produce implausibly low estimates.¹¹

For the exogenous shocks, priors are guided both by closed-economy estimates of similar disturbances for the U.S. and consistency of the implied degree of volatility and persistence with the corresponding observables in each country. Our baseline specification also includes a “tilt” towards foreign block disturbances, which are assumed twice as volatile and more persistent than their domestic counterparts.

4.2 Estimates and Model Fit

Table 2 reports parameter estimates for the baseline model.¹² The robustness of our results to alternative priors and specifications is addressed later. Parameter estimates for the baseline model are reasonable. The degree of price stickiness in home-produced goods, both in the domestic and foreign blocks of the model, is high. However, estimates for the foreign economy agree with Levin et al. (2005). Note that cost-push shocks to the domestic and foreign

⁹In Justiniano and Preston (2006) we evidence the robustness of our results to alternative calibrations with the elasticities of demand equal to 4 or when setting the interest elasticity of interest rate debt to 1e-4.

¹⁰We are grateful to one of the referees for this suggestion.

¹¹Our results are unchanged by calibrating τ at 0.29.

¹²We initialize multiple chains using random starting values after launching 50 optimization runs to ensure they all converge to the same mode. Convergence of the MCMC chains is diagnosed looking at trace plots and the potential scale reduction factors for variances and 90% posterior bands.

Phillips' curve are white noise and we do not rely on shocks to the inflation target to impart inflationary inertia. The Calvo adjustment parameters for wages in the domestic and foreign economies are similar to those reported in Del Negro et al. (2007) on a longer sample for the U.S. Imported goods prices are re-optimized most frequently, every 2 quarters.

The degree of habit persistence is close to 0.6 in both countries, tightly estimated and in line with values in Boldrin et al. (2001). The intertemporal elasticity of substitution and elasticity of labor supply accord with earlier macroeconomic studies of this kind. The estimated coefficients of the policy rule align with conventional wisdom. Technology and preference shocks are highly persistent in both countries. This is also true of risk premium and imported goods cost-push shocks in Canada. The median estimate for the elasticity of substitution across home and foreign goods is 0.86, below the value of 1.5 used in calibrations by Chari et al. (2002) and Schmitt-Grohe (1998), but consistent with estimates in Gust et al. (2008). Finally, the posterior density for the degree of openness lies well in the left tail of our very tight prior.

In Justiniano and Preston (2006) we show that the model matches the volatility and persistence of the data *within* blocks.¹³ The rest of the paper is devoted to the model's performance *across* blocks.

5 Accounting for the Influence of Foreign Shocks

This section documents the central result of the paper: the baseline model with independent shocks is unable to account for international comovement. Two pieces of evidence are adduced. First, variance decompositions reveal that U.S. disturbances explain a negligible fraction of variation in the domestic economy. Second, model-implied cross-country correlations are very close to zero. Both findings are clearly at odds with the reduced-form evidence discussed in section 2.

5.1 Variance Decompositions in the DSGE model

Using the draws from the posterior distribution of model parameters, table 3 reports the posterior variance shares in the domestic series — including the real exchange rate and terms of trade — that is attributable to all five foreign disturbances, at several forecast horizons.¹⁴

¹³This is also evident from the unreported cross-correlation functions within each block.

¹⁴According to the *prior* variance decomposition — see table 3 in Justiniano and Preston (2008a) — U.S. shocks combined account for roughly 40% of Canadian output and hours fluctuations, half of the variability in inflation, nominal interest, terms of trade and real exchange rate, and, about 30% of the variance in real wages,

We report medians and 90 percent posterior probability bands. Simulated moments, which also account for small-sample uncertainty, are discussed in the on-line appendix and yield very similar conclusions.

Regardless of forecast horizon, virtually none of the observed variation in domestic series is attributable to foreign disturbances. For output, interest rates, inflation, hours and wages, their maximum contribution at a horizon of 1 quarter is 3 percent. At longer horizons U.S. shocks explain at most 1 percent. Furthermore, the 95 percentiles for the variance shares of these series never exceeds 4 percent.¹⁵ For the real exchange rate and terms of trade, these statistics reveal a slightly larger contribution of foreign shocks, but still, below 7 percent. Compared with the reduced-form evidence in table 1, it is clear that this specification of the model cannot account for the influence of foreign shocks.

5.2 Cross-Country Correlations in the DSGE Model

Section 2 discussed the empirical cross-correlations between Canadian and U.S. series shown in figure 1 (solid). Here we revisit that figure focusing on the moments implied by the estimated model. These population statistics are computed using the posterior distribution of the DSGE parameters and the model’s state-space representation. We report median (dotted) and [5,95] percent posterior probability bands (dashed).

The median model-implied population cross-correlations are virtually zero at all horizons. The DSGE model cannot replicate the common fluctuations of domestic series with U.S. variables. Virtually all data cross-correlations lie outside the posterior probability bands of the corresponding model moments. This mismatch between model and data is also evident for the real exchange rate and the terms of trade (not shown for space considerations).¹⁶

Section 8 demonstrates that the lack of meaningful effects from foreign shocks in the domestic series is not an inherent feature of the DSGE model. Moreover, the inability to explain the influence of foreign disturbances is not unique to the estimated model of this

across different horizons.

¹⁵A simulation-based decomposition of the stationary variance (using the same posterior draws) constructed by feeding artificial sequences of domestic and foreign shocks one block at a time is presented in table 1A of the on-line appendix. There it is shown that the median shares are essentially identical to the those reported here, for all series, while the upper-ends of the posterior probability bands are roughly 0.02 points higher.

¹⁶Figures 1A and 6A in the on-line appendix present simulated cross-correlations which account for small-sample uncertainty. While the median estimates are virtually identical, the posterior probability bands are only very slightly wider, so long as care is taken to decompose the correlations into a “true” component and “spurious” component that arises in small samples, but vanishes in population. Failure to account for this “spurious” small-sample correlation produces posterior probability bands that may seem too wide and inconsistent with all other evidence on the model’s inability to generate comovement. We are very grateful to one of the referees for pointing out this apparent inconsistency in an earlier version of the paper.

paper. Adolfson et al. (2005) estimate a richer model which fits the data very well in several dimensions but also reveals, for Sweden, negligible variance shares for shocks originating in the rest of the world. While the authors do not comment on this issue, their estimated model includes features such as a stochastic trend, investment, variable capital utilization and a working capital channel, whose absence here could have been suspected as culprit for our results. This is also true of Christiano et. al. (2009) which advances that analysis by including financial frictions and unemployment. Similarly, de Walque et al. (2005) fail to identify significant cross-country linkages in an estimated two-country model for the U.S. and the Euro area, suggesting that the small open-economy assumption is not responsible for our findings either.

6 Robustness

The benchmark specification makes a range of assumptions, both on model structure and its match with data. Table 4 presents the estimated contribution of foreign disturbances to the variability of Canadian series for a number of alternative specifications. Further robustness checks are conducted in Justiniano and Preston (2006). To present a worst-case scenario against our findings, the numbers reported are for the horizon at which the share for output is greatest. A comparison with the first column, which replicates our baseline specification, makes clear that our central result remains intact.

Column 2 presents the decomposition when the prior standard deviations of all shocks are uniform between $1e-4$ and 10, while the prior for the persistence parameters is a fairly flat Beta density with mean 0.5 and dispersion 0.25. Compared with the benchmark results there is clearly little difference in the variance decompositions with this more agnostic prior. Column 3 estimates the model using maximum likelihood.¹⁷ Comovement again fails. The most notable difference in parameter estimates resides in the openness coefficient which is found to be 0.01 — essentially shutting down open-economy linkages. These two exercises suggest our priors are not responsible for the absence of comovement.

The next two columns evaluate the sensitivity of our conclusions to the choice of observables used to confront the model with data. Column 4 reports shares when output and wages are

¹⁷Since the inverse Frisch elasticities are poorly identified, the optimization routines sometimes got stuck at the upper bound of admissible values, set equal to 5. Regardless of whether this parameter is estimated or calibrated at 5, the variance shares of U.S. shocks for Canada are negligible.

in first differences rather than in level deviations from a common trend. The results are unchanged. Column 5 includes the observed terms of trade and the real exchange rate in levels rather than differences. This matters little for the contribution of U.S. shocks in Canada, except for a somewhat larger share for the terms of trade and real exchange rate. Column 6 specifies cost-push shocks in imports as i.i.d., as opposed to persistent disturbances. Once again, the variance shares are small, dropping to zero for the terms of trade.

Coordinated policy responses could perhaps explain part of the comovement in Canadian and U.S. business cycles. In the baseline specification monetary policies are assumed to be independently determined. However, interest rate decisions in Canada might be influenced by changes in U.S. interest rates beyond what can be accounted for with an explicit response to the exchange rate. Given the estimated degree of price stickiness, including a direct link between U.S. and Canadian monetary policy decisions may better capture international comovement. In this spirit, a log-linearized alternative specification for Canadian monetary policy is

$$i_t = \rho_i i_{t-1} + (1 - \rho_i) [\phi_{i^*} i_{t-1}^* + \phi_\pi \pi_t + \phi_y y_t + \phi_{\Delta y} \Delta y_t] + \varepsilon_{m,t}$$

where there is now an explicit dependence on lagged realizations of U.S. interest rates. All remaining modeling equations are unchanged.¹⁸ With a posterior mode estimate for ϕ_{i^*} of 0.04, it is not surprising that the variance decompositions are largely unchanged (column 7). Identical conclusions obtain even when policy responds to contemporaneous U.S. interest rates.

Finally, we consider a specification that first independently estimates the foreign block of the model.¹⁹ We then impose very tight priors (with dispersion 0.01) around the posterior estimates of the foreign block, and choose the prior persistence and volatilities of domestic innovations to match closely the moments observed in Canadian data. While in clear violation of specifying prior beliefs before looking at the data, this model is quite informative on which foreign disturbances are responsible for comovement a priori, a feature later exploited in section 8. The last column in table 4 evidences that this specification cannot account for the influence of foreign shocks either.

¹⁸The prior for ϕ_{i^*} is normal with mean 0.3 and dispersion 0.2, allowing it to take negative values.

¹⁹Priors are as in the baseline specification, although for robustness we impose uniform priors [1e-4,10] on the innovation standard deviations and a Beta density with mean 0.5 and dispersion 0.25 for the persistence parameters.

7 Common Shocks

The benchmark model assumes that all shocks in the U.S. and Canada are independent. However, the empirical evidence presented in section 2 is consistent with both spillovers from U.S.-specific disturbances and the existence of common shocks affecting both countries. This section presents alternative model specifications that accommodate the latter. Such specifications are unusual in the new open-economy macroeconomics literature. Notable exceptions are Adolfson et al. (2007) and de Walque et al. (2005) which include a common stochastic trend in neutral technology.

7.1 Specification

Common shocks are introduced by expressing the Canadian disturbances in the model as the sum of two orthogonal shocks. The first one is shared with the *same* type of disturbance in the U.S. block and referred to as the common shock. The second component affects only the domestic block and is labelled a country-specific shock. There is still no spillover from the Canadian to the U.S. economy given the small open-economy assumption.

As an illustration, when modeling a common shock in neutral technology this disturbance in Canada is written as $a_t = a_t^* + a_t^d$ where the common shock, a_t^* , and country-specific shock, a_t^d , evolve as independent AR(1) processes. The common shock is the corresponding structural disturbance in the U.S. block. Its share of variability in Canadian neutral technology, $Var(a^*)/Var(a)$, and implied correlation, $corr(a, a^*)$, can be readily computed.

In this way, common components are also introduced between Canadian disturbances to preferences, labor disutility, home-goods inflation and monetary policy, and their respective counterparts in the U.S. An advantage of this specification relative to the direct estimation of the correlations, $corr(a, a^*)$, is that it allows for a clean decomposition of the variance of all series attributed to each component. Note that this can be viewed as a DSGE structural approximation to the decomposition into common and idiosyncratic components using reduced-form dynamic factor models, as in Kose et al. (2003, 2008).

Given the emphasis on technology shocks in the international RBC literature, a natural starting point for adding common shocks would be to introduce a common unit root in neutral technology. A difficulty with this approach is strong evidence against a common stochastic trend in U.S. and Canadian output, at least in our sample. Tests for cointegration between log output per-capita in both countries do not reject the null hypothesis of no cointegration,

regardless of the specification of lags and deterministic components.²⁰ Similarly, the null of a unit root in the difference in levels of these two series cannot be rejected, while the null of stationarity is rejected.²¹ These results accord well with a persistent gap in labor productivity across these two countries; a topic that has been the subject of substantial research and policy discussion in Canada — see Eldridge and Sherwood (2001) and references therein.²²

7.2 Posterior variance shares with common shocks

For each U.S. shock and Canadian counterpart we re-estimate the model when common components are introduced one at a time. This permits identifying which common disturbances can help match the comovement in the data. A specification with a common component in all shocks is also presented. Priors are as in the baseline model with one exception. For both common and country-specific shocks we specify the same density: a $B(0.6, 2)$ for the autoregressive coefficients, and an IG for their standard deviations equal to that of the corresponding U.S. shock in table 1. Common and country-specific disturbances are on equal footing.²³ Results would be very similar using the prior from the baseline specification.²⁴

Panel A in table 5 reports posterior variance shares for specifications with a single common shock. We report the horizon with the largest share for output. Comparing these results with the baseline variance decomposition — reproduced in column 1 — yields several interesting findings.

Introducing a common component in neutral technology alone does little to alter the contribution of U.S. shocks, except for hours (column 2). Spillovers in neutral technology here play a small role in reproducing comovement. The intuition for this finding is that

²⁰We use both the trace and maximum eigenvalue tests, allowing for 1-6 lags while also varying the presence/absence of an intercept in the VAR or the cointegrating relationship, gauging relative fit using both the BIC and AIC. For each lag length, both information criteria prefer a specification with an intercept in the VAR and cointegrating equation (as expected) in which case the null of no cointegration cannot be rejected with either test (for all lags considered). The p-values for the null of no cointegration are never below 0.2 and close to 0.5 if the preferred lag lengths are used.

²¹The null of a unit root is not rejected at the 10 percent significance level when using the test of Elliot et al. (1996) or any of the test statistics proposed by Ng and Perron (2001), both with automatic lag selection. The null of stationarity under the KPSS tests is rejected at the 5 percent level.

²²Labor productivity is an observed state in our model since we are using data on output and hours for each country. The filtered series matches labor productivity from Statistics Canada (Table 383-0012).

²³The implied prior distribution of the correlation coefficient between the aggregate Canadian disturbance and its common component, is quite dispersed with a mean and median of roughly 0.7, standard deviation of 0.23 and 5-95% bands covering 0.08 to 0.99. This is also the prior correlation with the country-specific part of the shock, e.g. $\text{corr}(a, a^d)$. By construction the sum of these two squared correlations equals 1.

²⁴In this case with the tilt towards the foreign block, the mean and median prior variance shares of the U.S. shocks would have jumped to 90% or above. Also, for each composite disturbance the median prior correlation with its common component would have been tightly centered around 0.95. Nonetheless, the variance shares are only 1 to 3 percentage points higher with this alternative, extreme prior.

in our model U.S. neutral technology shocks induce a negative comovement between output and hours within the foreign block, as documented in closed-economy models by Gali (1999), Ireland (2004) and Gali and Rabanal (2004). There is a tension between having technology shocks as a source of international comovement and fitting the large positive hours-output comovement observed in U.S data.

A common shock to the disutility of labor only (column 3) has a negligible effect on the variance decomposition of output, inflation and interest rates, but helps improve the foreign share in wages. Cost-push shocks (column 4) only bump up the foreign contribution to inflation variability. Meanwhile, with a common shock only in the monetary policy rules (column 5), the fraction of the variance in Canadian interest rate and output attributable to foreign shocks climbs to 23 and 10 percent. The largest increase in the share of output variability explained by U.S. disturbances occurs with a common component in preference shocks (column 5), but even in this case only 11 percent of output fluctuations are accounted for by all foreign shocks.

Panel B reports shares at various horizons in a specification with common components in all U.S. shocks and their respective Canadian counterparts. The fraction of variation explained by all foreign disturbances is now larger than in the baseline, particularly for output, hours and interest rates. These results show that the comovement observed in the data can be partly reproduced by correlating domestic disturbances with all of their U.S. counterparts.

The last row in each panel of the table reports the marginal data density, computed using the modified harmonic mean. Most specifications achieve a lower fit than the baseline (-1003.3), even when all common shocks are added simultaneously (-1010.6). The fit is almost indistinguishable from the baseline when either monetary policy or cost-push shocks are correlated across countries. While caution is warranted when comparing these marginals due to differences in priors, this suggests that neither of the common shocks specifications substantially improves the model's fit, relative to the baseline without common shocks.

There are at least three further reasons why these specifications with common shocks should not be viewed as panacea for the model's failure in accounting for the influence of foreign disturbances. First and foremost, while shares in panel B align well with those from the SUR at a 1 quarter horizon, for longer horizons the SUR posterior bands do not encompass the smaller DSGE estimates — compare table 2. Second, some of the common components are hard to rationalize on structural grounds. Recall that neither a specification of the Canadian policy rule including a direct response to the exchange rate nor to U.S. interest rates could explain comovement, unless shocks are correlated. This suggests that cross-equation

restrictions prevent the model from structurally explaining the cross-correlation in these two series. Third, panel B demonstrates an extreme manifestation of the exchange rate disconnect analyzed by Devereux and Engel (2002). Fluctuations in the real exchange rate and the terms of trade are completely disconnected from the U.S. block and for the most part from the real domestic variables as well.²⁵ In summary, even with common shocks there is ample scope to improve the model’s ability to capture the influence of foreign disturbances.

8 On the Source of Model Failure

The results so far have documented an important model failure with little said about its determinants. This section provides several insights on model and data features which limit comovement. We first show that the unaccounted correlation seen in the data translates into correlated innovations — in violation of the maintained assumption of orthogonality. This information, together with insights into which U.S. disturbances are a priori responsible for comovement, guide a set of exercises attempting to understand which shocks, transmission mechanisms and data series pose difficulties for the model.

8.1 Where Does the Correlation Go?

The adopted likelihood-based procedure provides estimates of structural parameters and unobserved shocks that perfectly match the data. The large cross-country correlations in the observables which are not explained by the model get reflected in correlated innovations, a clear indication of model misspecification. This correlation is not picked up in the various exercises conducted in this paper as, with the exception of the common shocks models of the previous section, the disturbances are assumed to be orthogonal. This is the standard assumption in empirical DSGE models.

Table 6 reports the cross-correlation between the supposedly orthogonal two-sided U.S. and Canadian innovations to the exogenous shocks, in the baseline specification. Seven of those correlations are statistically different from zero (in bold). Interestingly, in five of these seven cases, the correlation occurs across disturbances of different type (e.g. U.S. preference and Canadian technology shocks). This helps explain why the common shock models estimated in the previous section — which allow for correlation only amongst disturbances of the same

²⁵Risk-premium and import cost-push shocks account for roughly 90 and 85 percent of the variance of the real exchange rate and the terms of trade, respectively, at all horizons, while explaining only about 5 percent of the variability in domestic output, real wages and hours.

type — could not fully reconcile model and data. A more complex set of interactions across disturbances is revealed.

We next turn to a prior and posterior comparison that provides evidence on why these correlations cannot be captured by the transmission mechanisms embedded in the model.

8.2 The Transmission Role of Disturbances and Data

Identifying the mechanisms limiting comovement is a challenging task. It requires assessing numerous cross-equation restrictions, the prior and posterior properties of a model with 47 estimated parameters and the statistical covariance properties of 12 observable time series. A clean narrative would ideally attribute model failure to one cross-equation restriction or one particular time series. Unfortunately matters are not so simple in a model of this dimension. Having said that, we conduct additional exercises suggesting a few culprits which serve to guide future research on this topic.

The prior-posterior comparisons here are based on the model discussed in section 6 which pre-estimates and then “calibrates” the U.S. block of the economy. This is done for the following reasons. First, in contrast to the benchmark prior, this “prior” is highly informative as to which shocks are most relevant for dynamics in the foreign block and therefore comovement. In particular, U.S. preference shocks explain the bulk of U.S. output fluctuations and a substantial portion of interest rate variability, especially at longer horizons. U.S. monetary policy shocks are also important for U.S. interest rates, while cost-push shocks drive U.S. inflation. This broadly highlights the same set of foreign disturbances suggested by the relative fit of the common shocks specifications in table 5 and the analysis of section 8.1.²⁶

Second, by fixing properties of the U.S. block, attention is focused on differences between the prior and posterior implications for the domestic block and its interaction with the foreign block, narrowing the scope of inquiry. Third, a priori the average variance share explained by all U.S. shocks is roughly 0.4 for all Canadian series — without recourse to correlated innovations — with prior bands covering the interval 0.1 to 0.7. Clearly, the prior suggests substantial comovement while, as documented in table 4, the posterior decomposition implies a negligible role for U.S. shocks in the domestic economy.

To understand why the posterior severs cross-country linkages, we analyze the international transmission of U.S. monetary policy, preference and cost-push shocks. Taking these

²⁶This variance decomposition for the foreign block is, not surprisingly, very similar to the posterior in the baseline specification.

disturbances jointly and in isolation, we ask what are their prior implications for model-implied cross-correlations and how do these square with the data. This uncovers a number of counterfactual prior implications which help explain why the posterior shuts down the role of U.S. shocks. We provide a brief summary of our findings, with additional details and graphs discussed in the on-line appendix.

- A priori output comovement can be captured by U.S. preference shocks. However, they induce a strong negative correlation between U.S. output and Canadian inflation and interest rates, as well as the terms of trade. In the data these correlations are positive, particularly in the case of the former two domestic series. Furthermore, U.S. preference shocks imply Canadian output is negatively correlated with both Canadian inflation and nominal interest rates. Again this is strongly counterfactual.
- Similarly, U.S. cost-push shocks a priori capture the comovement between U.S. inflation with Canadian inflation and nominal interest rates. However, these disturbances imply large positive correlations between U.S. and Canadian inflation with each of the terms of trade and real exchange rate. Instead, these correlations are negative in the data and small in absolute value.
- Monetary policy shocks in the U.S. generate comovement with Canadian interest rates and inflation. But their key counterfactual prediction is also a positive correlation between Canadian inflation and both the terms of trade and real exchange rate. Since large U.S. monetary policy shocks are required to match the substantial comovement between U.S. and Canadian nominal interest rates this exacerbates the inability to fit the terms of trade and real exchange rate.

Taken together these observations permit several insights. First, while foreign preference, cost-push and monetary policy shocks are important determinants of U.S. fluctuations, and in principle Canadian fluctuations, they have significant counterfactual predictions for various series vis-a-vis Canadian inflation, the terms of trade and real exchange rate, as well as amongst these three. Posterior inference reveals that once confronted with the data the ability of these shocks to generate comovement is severely limited.

Second, and related, is that the shifting importance of shocks across the prior and posterior implications of the model are consistent with exchange rate disconnect being a factor in the model's failure. Furthermore, posterior estimates from a model in which both series are

unobservable states improves the comovement properties to some degree — around 10 percent of variation in Canadian series is attributed to U.S. originating disturbances, without resorting to correlated shocks. Treating any other observable Canadian series (such as inflation) in the benchmark estimation as unobservable does not have similar implications.

The exchange rate disconnect hypothesis is provided further support given the importance of imported goods cost-push shocks and risk premium shocks as a source of variation for these two series, without a meaningful role for the remaining observables, domestic or foreign. This issue is discussed in greater detail in an earlier version of this paper — see Justiniano and Preston (2006).

To conclude we offer the following remarks on the model’s inability to explain the influence of foreign shocks. First, some correlation in exogenous disturbances appears warranted, despite the assumption of orthogonality being common practice in the DSGE literature.²⁷ Second, allowing for correlation across countries only amongst disturbances of the same type is unlikely to fully resolve the comovement problem. Third, prior-posterior comparisons point to deeper mechanisms, operative through particular cross-equation restrictions and evidenced by counterfactual implications for the terms of trade and the real exchange rate, particularly in regards to their link with domestic inflation. When confronted with the data, this tension between fitting some cross-country correlations and the model’s counterfactual prediction for other moments is resolved in favor of the latter by shutting down international linkages. As to which cross-equation restrictions and model features need to be re-considered to reconcile the model with the observed behavior of these three series is beyond the current exercise but clearly an exciting area of research.

9 Conclusion

This paper shows that an empirical semi-small open-economy model fails to account for one important dimension of Canadian data: the influence of U.S. disturbances. We initially assume uncorrelated shocks across countries, as it is done in almost all the empirical literature with this class of models. Variance decompositions reveal that the fraction of variation in Canadian series attributed to all shocks originating in the U.S. economy is negligible at all forecast horizons. Accordingly, the cross-country correlation functions implied by the model are close to zero. These findings contrast sharply with earlier work documenting strong linkages between these two countries and reduced-form evidence presented here.

²⁷An interesting recent contribution in the closed-economy setting in this regard is Curdia and Reis (2009).

Alternative specifications with common shocks can only partially resolve this problem. A model in which all U.S. shocks have a common component with the corresponding Canadian disturbance begins to reconcile the influence of foreign shocks in the model and the data. However, the variance shares explained by all U.S. disturbances still fall short of those observed in the data at medium and long horizons. While the empirical evidence is consistent with both common shocks and spillovers, there remains the question of what economic effects do these common shocks capture in the model. In particular, whether they correspond to purely exogenous disturbances or are instead simply capturing model misspecification. Finally, any gains with common shocks come at the expense of fully detaching fluctuations in the exchange rate and the terms of trade from the foreign block.

Analyzing the prior implications for the transmission of U.S. shocks indicates a role for exchange rate disconnect in the model's failure. In particular, we uncover a number of prior counterfactual correlations between U.S. and Canadian series, particularly involving inflation, the real exchange rate and the terms of trade.

Overall our findings suggest that additional work on the international transmission mechanisms of various shocks could improve the empirical performance of these models in this crucial dimension. An interesting exercise in this vein would be to alter the supply side of the model to account for cross-country linkages at multiple stages of production as in Huang and Liu (2007) and Burstein et al. (2008). Alternatively, expanding on international financial linkages and the role of asset prices might help explain the importance of U.S. disturbances abroad, as made evident by the current financial crisis.

10 Appendix A: Data

All series are downloaded from Haver Analytics. For the U.S., real per-capita GDP measures output, inflation corresponds to the log-difference in the GDP deflator, and the effective federal funds rate taken for interest rates. Nominal compensation per hour in the non-farm business sector divided by the GDP deflator measures wages. Total hours in the non-farm business sector is divided by population.

For Canada, real per-capita GDP is constructed with data from Statistics Canada (StatCan). The quarterly log difference in the consumer price index excluding food and energy (StatCan) measures overall inflation. The official discount rate published by the Bank of Canada corresponds to interest rates. Hours worked in the total economy (StatCan table 383-0012) is divided by population. From the same table we obtain total compensation and

convert it into real terms. In accordance with the model, the GDP deflator proxies for the price of home produced goods, while CPI inflation represents the aggregate price index.

For consistency, the log difference in the bilateral real exchange rate is constructed as the sum the log growth rates in the U.S. GDP deflator and the nominal exchange rate (Canadian dollars per U.S. dollars) minus Canadian aggregate inflation, as measured above. An earlier version of the paper used the bilateral real exchange rate constructed by the IMF with identical findings.

Finally, for the terms of trade we take the ratio of the deflator for imports to exports (Statcan) matching in our model, $\log(P_{F,t}/P_{H,t})$. According to Canada's national accounts data, this measure of the terms of trade would correspond more closely to $\log\left(P_{F,t}^*S_t/P_{H,t}\right)$, but this would not be consistent with the real exchange rate using aggregate U.S. inflation. As there is no perfect match between model variables we adopted the former measure for estimation. Inference with the latter interpretation does not affect our results.

11 Appendix B: SUR model

To match the reduced-form representation of the DSGE model we impose on a VAR the same zero restrictions of no feedback from Canada to the U.S. The resulting SUR model is estimated on the same sample of twelve series used for inference with the DSGE model.

Inference is substantially more involved than with a standard VAR, as the explanatory variables are not the same across all series. However, the estimation is feasible with the efficient block-recursive Gibbs algorithm proposed by Zha (1999), who documents the distortions to inference from not imposing the exclusion restrictions in a SUR between Canada and the U.S.

We simply outline the model and refer the readers to Zha (1999) for details. Partitioning the vector of observables y_t into U.S. and Canadian variables, y_t^{US} and y_t^{CN} , respectively, the two blocks of the SUR are given by

$$\begin{bmatrix} A_{US,US}(0) - A_{US,US}^p(L) & 0 \\ A_{CN,US}(0) - A_{CN,US}^p(L) & A_{CN,CN}(0) - A_{CN,US}^p(L) \end{bmatrix} \begin{pmatrix} y_t^{US} \\ y_t^{CN} \end{pmatrix} = \begin{pmatrix} \epsilon_t^{US} \\ \epsilon_t^{CN} \end{pmatrix}$$

for matrices of conformable size, where $A_{i,j}(0)$ corresponds to the impact matrix and $A_{i,j}^p(L)$ denotes a matrix of lag-polynomials, of order p , in the positive powers of L . The structural errors $[\epsilon_t^{US}, \epsilon_t^{CN}]'$ are orthogonal with unit variance.

Our goal is not to identify each of the structural disturbances but simply to compute the variance shares of the Canadian series attributed to the sum of all U.S. disturbances, ϵ_t^{US} . To this end, we impose a lower triangular structure in the impact matrices $A_{US,US}(0)$ and $A_{CN,CN}(0)$. This is equivalent to a Cholesky decomposition of the reduced-form SUR

covariance matrix. Results on the sum of all block-specific disturbances are invariant to ordering.

We report results with $p = 4$ in light of recent work by Fernandez-Villaverde et al. (2007) and Del Negro et al. (2007) who have brought attention to the issue of lag truncation in VARs as approximations to DSGE models. To deal with the large number of parameters relative to sample length, we use priors that shrink coefficients at distant lags. More specifically, we specify the priors $A_{ij}(1) \sim N(0.9, 0.2)$ for $i = j$ and $N(0, 0.4)$ for $i \neq j$.²⁸ There's no distinction between own and others' lags for $k > 1$ and assume a normal prior centered at zero with a dispersion equal to 0.2 for $k = 2$, 0.15 for $k = 3$, and 0.1 for $k = 4$. The lower triangular elements of the contemporaneous matrices are $A_{ij}(0) \sim N(0, 10)$. Results are largely insensitive to looser priors, and, when feasible (p is 1 or 2), pretty much identical to sampling a single block SUR with an uninformative Inverse-Wishart prior on the reduced-form covariance matrix.

The Gibbs sampler is initialized at random starting values from the prior (or a classical SUR with 2 lags) and we run 3 chains, discarding, for each, the first 40,000 draws, and retaining 1 in 10 of the remaining 50000. For each draw we compute the fraction of the variability in y_t^{CN} , explained by the sum of all five U.S. shocks, at different horizons.

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²⁸For the exchange rate and terms of trade the mean is 0.3, since these are expressed in log-differences.

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Table 1: SUR Posterior Variance Shares¹ of Canadian Series
Attributed to All U.S. Shocks

Median variance shares and [5,95] posterior bands for all U.S. shocks²

Series	1 quarter horizon		4 quarter horizon	
Output	0.22	[0.07 , 0.41]	0.44	[0.19 , 0.68]
Inflation	0.20	[0.06 , 0.39]	0.37	[0.14 , 0.63]
Interest Rate	0.14	[0.03 , 0.31]	0.37	[0.14 , 0.63]
Real Wages	0.13	[0.03 , 0.28]	0.34	[0.12 , 0.59]
Hours	0.07	[0.02 , 0.16]	0.25	[0.08 , 0.48]
Real Exchange Rate	0.07	[0.01 , 0.17]	0.17	[0.06 , 0.35]
Terms of Trade	0.12	[0.03 , 0.26]	0.22	[0.08 , 0.40]

Series	8 quarter horizon		Stationary Variance ³	
Output	0.52	[0.25 , 0.76]	0.76	[0.44 , 0.98]
Inflation	0.42	[0.20 , 0.69]	0.65	[0.33 , 0.95]
Interest Rate	0.47	[0.21 , 0.73]	0.71	[0.40 , 0.97]
Real Wages	0.49	[0.22 , 0.74]	0.79	[0.49 , 0.98]
Hours	0.42	[0.16 , 0.69]	0.75	[0.43 , 0.98]
Real Exchange Rate	0.26	[0.10 , 0.49]	0.62	[0.29 , 0.95]
Terms of Trade	0.29	[0.13 , 0.49]	0.57	[0.27 , 0.92]

Notes:

¹ Variance shares cover [0,1] interval. Hence 0.01 corresponds to 1 percent.

² Median of the sum of all five U.S. shocks computed for each draw of the SUR parameters obtained with a Gibbs simulator. Details of the SUR are given in Appendix B. Mean shares are very similar and if anything slightly higher in some cases.

³ Stationary refers to the long-horizon variance.

Table 2: Prior densities and posterior estimates for baseline model (domestic block)

Coefficient	Description	Prior			Posterior ²		
		Prior Density ¹	Mean	Std	Median	Std	[5 , 95]
φ	Inverse Frisch	N	1.00	0.30	1.27	0.27	[0.84 , 1.72]
σ	Intertemporal ES	N	1.00	0.40	1.43	0.30	[0.98 , 1.95]
α_H	Calvo domestic prices	B	0.60	0.10	0.86	0.04	[0.78 , 0.92]
α_F	Calvo import prices	B	0.50	0.20	0.42	0.06	[0.33 , 0.52]
α_W	Calvo wages	B	0.60	0.10	0.88	0.04	[0.79 , 0.93]
γ_H	Indexation domestic prices	B	0.50	0.20	0.42	0.12	[0.25 , 0.63]
γ_W	Indexation wages	B	0.50	0.20	0.22	0.11	[0.07 , 0.44]
h	Habit	B	0.50	0.10	0.64	0.06	[0.53 , 0.73]
τ	Openness	B	0.29	0.02	0.24	0.02	[0.21 , 0.27]
η	Elasticity H-F goods	N	0.90	0.10	0.86	0.09	[0.70 , 1.01]
θ_π	Taylor rule, inflation	N	1.80	0.30	2.00	0.26	[1.57 , 2.42]
θ_y	Taylor rule, output	G	0.25	0.13	0.21	0.08	[0.09 , 0.34]
θ_{dy}	Taylor rule, output growth	N	0.30	0.20	0.70	0.18	[0.42 , 1.00]
θ_{de}	Taylor rule, nominal exchange rate	G	0.30	0.20	0.31	0.10	[0.17 , 0.50]
θ_i	Taylor rule, smoothing	B	0.60	0.20	0.88	0.02	[0.84 , 0.91]
ρ_a	Technology	B	0.60	0.20	0.94	0.02	[0.91 , 0.96]
ρ_g	Preferences	B	0.60	0.20	0.92	0.02	[0.88 , 0.95]
ρ_L	Labor disutility	B	0.60	0.20	0.51	0.12	[0.30 , 0.70]
$\rho_{cp,F}$	Cost-push imports	B	0.60	0.20	0.92	0.03	[0.87 , 0.96]
ρ_{rp}	Risk premium	B	0.60	0.20	0.98	0.01	[0.96 , 0.99]
σ_a	sd technology	I	0.50	1.00	0.52	0.04	[0.46 , 0.59]
σ_i	sd monetary policy	I	0.15	1.00	0.21	0.02	[0.18 , 0.25]
σ_g	sd preferences	I	1.00	1.00	4.32	0.96	[3.05 , 6.10]
$\sigma_{cp,H}$	sd cost-push domestic	I	0.15	1.00	0.70	0.07	[0.61 , 0.83]
σ_L	sd labor disutility	I	2.00	1.00	3.51	1.79	[1.68 , 7.37]
$\sigma_{cp,F}$	sd cost-push imports	I	1.00	1.00	2.12	0.60	[1.37 , 3.32]
σ_{rp}	sd risk premium	I	1.00	1.00	0.31	0.03	[0.26 , 0.38]

(continued in the next page with the foreign block)

Table 2: Prior densities and posterior estimates for baseline model (foreign block)

Coefficient	Description	Prior			Posterior ²		
		Prior Density ¹	Mean	Std	Median	Std	[5 , 95]
φ^*	Inverse Frisch	N	1.00	0.30	1.19	0.27	[0.77 , 1.65]
σ^*	Intertemporal ES	N	1.00	0.40	0.99	0.27	[0.62 , 1.48]
α_{H^*}	Calvo prices	B	0.60	0.10	0.91	0.02	[0.86 , 0.94]
α_{W^*}	Calvo wages	B	0.60	0.10	0.87	0.03	[0.81 , 0.91]
γ_{H^*}	Indexation prices	B	0.50	0.20	0.58	0.12	[0.40 , 0.79]
γ_{W^*}	Indexation wages	B	0.50	0.20	0.29	0.16	[0.09 , 0.60]
h^*	Habit	B	0.50	0.10	0.56	0.07	[0.45 , 0.68]
λ^*	Elasticity foreign demand	N	1.50	0.50	0.54	0.11	[0.38 , 0.74]
θ_{π^*}	Taylor rule, inflation	N	1.80	0.30	1.76	0.26	[1.35 , 2.19]
θ_{y^*}	Taylor rule, output	G	0.25	0.13	0.19	0.06	[0.09 , 0.28]
θ_{dy^*}	Taylor rule, output growth	N	0.30	0.20	0.77	0.15	[0.54 , 1.02]
θ_{i^*}	Taylor rule, smoothing	B	0.60	0.20	0.85	0.02	[0.81 , 0.88]
ρ_{a^*}	Technology	B	0.80	0.15	0.93	0.02	[0.90 , 0.97]
ρ_{g^*}	Preferences	B	0.80	0.15	0.90	0.03	[0.85 , 0.94]
ρ_{L^*}	Labor disutility	B	0.80	0.15	0.37	0.08	[0.23 , 0.51]
σ_{a^*}	sd technology	I	1.00	2.00	0.47	0.04	[0.42 , 0.53]
σ_{i^*}	sd monetary policy	I	0.25	2.00	0.13	0.01	[0.11 , 0.15]
σ_{g^*}	sd preferences	I	2.00	2.00	2.55	0.44	[1.92 , 3.35]
σ_{g^*}	sd cost-push	I	0.25	2.00	0.22	0.02	[0.19 , 0.26]
σ_{L^*}	sd labor disutility	I	4.00	2.00	3.41	0.89	[2.33 , 5.22]

Relative to the text, the standard deviations of the innovations are scaled by 100 for the estimation, which is reflected in the prior and posterior estimates.

¹ N stands for Normal, B Beta, G Gamma and I Inverted-Gamma distribution

² Median and posterior percentiles from 4 chains of 100,000 draws generated using a Random walk Metropolis algorithm, where we discard the initial 50,000 and retain one in every 5 subsequent draws. For convergence we monitor trace plots as well as the potential scale reduction factors both for the variances and 90% posterior probability bands.

Table 3: Posterior Variance Shares¹ of Canadian Series
Attributed to All U.S. Shocks in Baseline DSGE

Median variance shares and [5,95] posterior bands for all U.S. shocks²

Series	1 quarter horizon		4 quarter horizon	
Output	0.01	[0.01 , 0.02]	0.01	[0.01 , 0.02]
Inflation	0.01	[0.01 , 0.02]	0.01	[0.01 , 0.02]
Interest Rate	0.03	[0.02 , 0.04]	0.02	[0.01 , 0.03]
Hours	0.00	[0.00 , 0.01]	0.00	[0.00 , 0.01]
Real Wages	0.01	[0.01 , 0.01]	0.01	[0.01 , 0.01]
Real Exchange Rate	0.03	[0.02 , 0.03]	0.03	[0.02 , 0.04]
Terms of Trade	0.04	[0.02 , 0.06]	0.04	[0.03 , 0.06]

Series	8 quarter horizon		Stationary Variance ³	
Output	0.01	[0.01 , 0.02]	0.01	[0.01 , 0.02]
Inflation	0.01	[0.01 , 0.02]	0.01	[0.01 , 0.02]
Interest Rate	0.01	[0.01 , 0.02]	0.01	[0.01 , 0.02]
Hours	0.00	[0.00 , 0.01]	0.01	[0.00 , 0.02]
Real Wages	0.01	[0.01 , 0.02]	0.01	[0.00 , 0.02]
Real Exchange Rate	0.03	[0.02 , 0.04]	0.03	[0.02 , 0.04]
Terms of Trade	0.05	[0.03 , 0.07]	0.05	[0.03 , 0.07]

Notes:

¹ Variance shares cover [0,1] interval. Hence 0.01 corresponds to 1 percent.

² Median of the sum of the shares for all five U.S. shocks computed with the posterior simulators of model parameters. We report means since domestic and foreign shares add up to one for each draw, but clearly given the tight posterior bands the medians are almost identical.

³ Stationary refers to the long-horizon variance.

Table 4: Variance Shares¹ of Canadian Series Attributed to All U.S. Shocks in Alternative Specifications

Specification	Baseline ²	Looser prior on volatilities and persistence ³	Maximum likelihood	Output and wages in first differences	RER and TOT in levels	Import cost- push shocks i.i.d	Response to foreign interest rate in policy rule ⁴	U.S. block estimated first ⁵
	Horizon 1	Horizon 1	Stationary	Horizon 12	Horizon 12	Horizon 1	Stationary	Horizon 12
Output	0.01	0.02	0.01	0.01	0.01	0.01	0.01	0.00
Inflation	0.01	0.01	0.00	0.01	0.02	0.00	0.02	0.01
Interest Rate	0.03	0.03	0.01	0.01	0.01	0.01	0.01	0.01
Real Wages	0.00	0.01	0.00	0.01	0.01	0.00	0.03	0.00
Hours	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00
Real Exchange Rate	0.03	0.03	0.02	0.03	0.05	0.03	0.03	0.04
Terms of Trade	0.04	0.05	0.03	0.04	0.14	0.00	0.06	0.06

Notes:

¹ Variance shares cover [0,1] interval. Hence 0.01 corresponds to 1 percent. These are computed at the mode of each specification. We select the horizon with the highest share for output. All shocks are independent from one another.

² Reproduced from table 3.

³ Prior for all domestic and foreign standard deviations is Uniform [1e-4,10] while for all persistence parameters, Beta with mean 0.5 and standard deviation 0.25.

⁴ Policy rule in Canada includes a direct response to lagged U.S. interest rates.

⁵ U.S. block is estimated first, using the same looser prior on volatilities and persistence described in note 3 to this table. For the foreign block a very tight "prior" is then specified around these posterior estimates (standard deviation 0.01) in order to estimate the domestic block, while selecting the "prior" for the volatilities of the domestic shocks in order to match the volatility observed in Canadian data. The "prior" for all remaining parameters in the domestic block is as in table 2.

Table 5: Variance Shares¹ of Canadian Series Attributed to All U.S. Shocks in Specifications with Common Shocks

Panel A. One Common Shock Only³						
	Baseline without common shocks ²	Technology	Labor Disutility	Cost-Push	Monetary Policy	Preference
Series \ Horizon	1 Period	1 Period	Stationary	1 Period	1 Period	1 Period
Output	0.01	0.02	0.02	0.02	0.10	0.11
Inflation	0.01	0.01	0.06	0.11	0.00	0.02
Interest Rate	0.03	0.03	0.04	0.05	0.23	0.04
Real Wages	0.00	0.00	0.13	0.04	0.00	0.01
Hours	0.01	0.21	0.01	0.01	0.06	0.08
Real Exchange Rate	0.03	0.02	0.04	0.02	0.01	0.04
Terms of Trade	0.04	0.04	0.08	0.03	0.01	0.06
<i>Marginal Data Density⁴</i>	-1003.3	-1033.5	-1037.8	-1002.6	-1005.8	-1037.2

Panel B. All Common Shocks Simultaneously⁵				
Series \ Horizon ⁵	1 Period	4 Periods	8 Periods	Stationary
Output	0.23	0.20	0.18	0.16
Inflation	0.11	0.13	0.13	0.13
Interest Rate	0.26	0.22	0.18	0.12
Real Wages	0.13	0.16	0.17	0.16
Hours	0.30	0.26	0.24	0.23
Real Exchange Rate	0.00	0.00	0.00	0.00
Terms of Trade	0.00	0.01	0.01	0.01
<i>Marginal Data Density⁴</i>	-1010.6			

Notes:

¹ Variance shares cover [0,1] interval. Hence 0.01 corresponds to 1 percent. Disturbances in Canada are given by the sum of two orthogonal components: a country-specific shock and a disturbance in common with the corresponding U.S. shock. These shares now include the variability attributed to the common component(s) of the corresponding Canadian composite disturbance.

² Reproduced from table 3.

³ Each specification has a common component in that disturbance only. These are computed at the mode. We report the horizon with the highest share for output.

⁴ Computed using the Modified Harmonic Mean

⁵ All 5 shocks from Panel A now have a common component with the corresponding U.S. disturbances. These variance shares are computed at the mode. We report the same horizons as in tables 1 and 3.

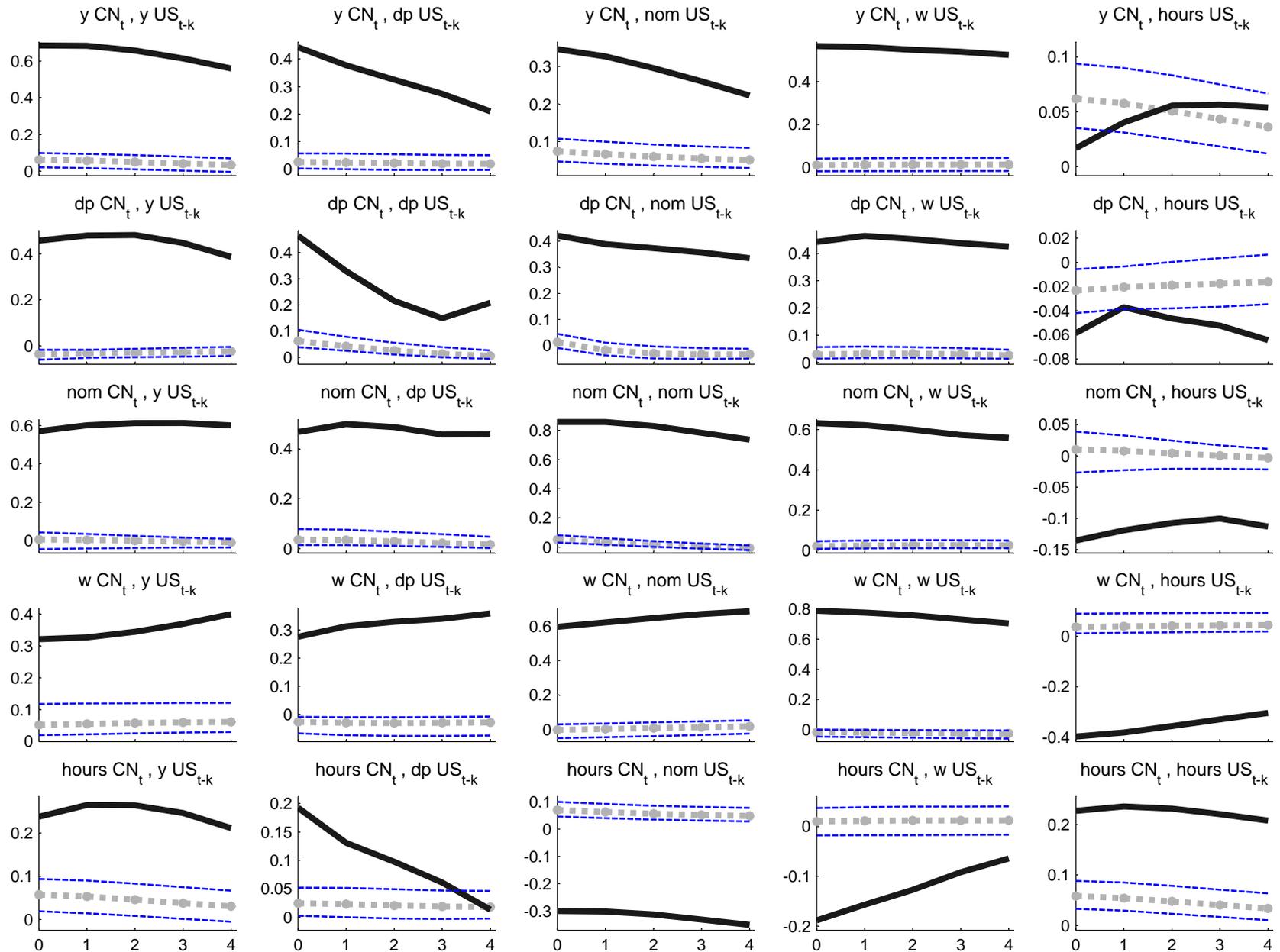
Table 6: Cross Correlation between U.S. and Canadian innovations in
baseline DSGE ¹

Innovations	U.S. Technology	U.S. Monetary Policy	U.S. Preference	U.S. Cost- Push	U.S. Labor Disutility
Technology	0.16	-0.12	0.30	0.06	0.11
Monetary Policy	-0.12	0.47	0.12	-0.07	0.06
Preference	-0.08	0.08	0.10	0.21	0.10
Cost Push Home Goods	-0.05	-0.01	0.06	0.39	-0.07
Labor Disutility	0.20	-0.06	0.21	0.01	0.11
Cost Push Foreign Goods	0.13	-0.04	0.08	-0.09	0.01
Risk Premium	0.12	-0.03	0.04	-0.29	-0.03

Notes:

¹ Cross-correlation of the smoothed innovations at the mode of the baseline model. Those in bold exceed in absolute value 1.96 times the inverse of the square root of the sample size.

Figure 1: Data and DSGE population cross-correlations Canada-U.S.



Legend: Data (solid dark), DSGE median (dotted grey) and [5,95] posterior probability bands (dashed blue)
 y (output), dp (inflation), nom (nominal interest rate), w (real wage) and hours.
 X-axis: k lags of U.S. variables, 0 through 4 (in quarters)