Central Bank Communication and Expectations Stabilization*

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Abstract

The value of communication is analysed in a model in which agents' expectations need not be consistent with central bank policy. Without communication, the Taylor principle is not sufficient for macroeconomic stability: divergent learning dynamics are possible. Three communication strategies are contemplated to ensure consistency between private forecasts and monetary policy strategy: i) communicating the precise details of policy; ii) communicating only the variables on which policy decisions are conditioned; and iii) communicating the inflation target. The former strategies restore the Taylor principle as a sufficient condition for anchoring expectations. The latter strategy, in general, fails to protect against expectations-driven fluctuations.

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> A central bank that is inscrutable gives the markets little or no way to ground these perceptions [about monetary policy] in any underlying reality — thereby opening the door to expectational bubbles that can make the effects of its policies hard to predict. (Blinder, 1999)

1 Introduction

One potential benefit from successful implementation of inflation targeting is the anchoring of expectations, with its stabilizing effect on macroeconomic activity. Failing to anchor expectations might result in undesired fluctuations and economic instability. Given the role of expectations, a central bank's communication strategy is a crucial ingredient of inflation targeting. Yet despite its importance, relatively little formal analysis in the context of dynamic stochastic general equilibrium models has been done on the mechanisms by which communication might prove beneficial. The analysis here addresses this hiatus, providing examples of how communication might prove beneficial in a model for monetary policy evaluation.

Motivated by Friedman (1947, 1968), stabilization policy is conducted in the presence of two informational frictions. First, the central bank has imperfect information about the current state of the economy and must forecast the current inflation rate and output gap when setting the nominal interest rate in any period; new information is responded to with a delay.[1] Second, households and firms have an incomplete model of the macroeconomy, knowing only their own objectives, constraints and beliefs. They do not have a model of how aggregate state variables, including nominal interest rates, are determined. They forecast exogenous variables relevant to their decision problems by extrapolating from historical patterns in observed data.

This specification of subjective beliefs, which can differ from the objective probabilities implied by the economic model, permits defining meaningfully the notion of anchored expectations as those beliefs consistent with the monetary policy strategy of the central bank. The possibility of beliefs being inconsistent with monetary policy strategy and, therefore, unanchored, presents a challenge for stabilization policy and permits examination of the role of communication in policy design.[2]

[1] Policy is implementable in the sense of McCallum (1999).

[2] This contrasts with a rational expectations analysis in which expectations are anchored by construction. Beliefs must be consistent with monetary policy regardless of whether policy induces a determinate or indeterminate rational expectations equilibrium.

Communication is modeled as providing agents with certain types of information about how the central bank determines its nominal interest-rate setting. This information serves to simplify agents' forecasting problem and to co-ordinate expectations about various macroeconomic variables in a desirable way. Worth underscoring is that uncertainty about the path of nominal interest rates is only one of several sources of uncertainty present in this economy. Agents are similarly unsure about how aggregate output and inflation are determined. The central question is whether uncertainty about the determination of interest rates is an especially important source of uncertainty and whether additional knowledge about their future evolution helps anchor expectations, assisting macroeconomic stabilization.

Three communication strategies are considered which successively reduce the information conveyed to agents. This information is used by agents to restrict the class of maintained forecasting models. The central bank communicates either: i) The policy rule employed to set nominal interest rates. Agents construct policy-consistent forecasts. Expectations are anchored; ii) The set of variables upon which nominal interest rates are conditioned. Agents have no knowledge of the policy rule parameters. This strategy might reflect partial central bank credibility: agents use available data to verify the policy reaction function; or iii) The desired average outcomes for inflation, nominal interest rates and the output gap.

Central results are: In the case of no communication, policy rules that implement policy under rational expectations fail to anchor expectations and frequently lead to divergent learning dynamics. In contrast to a rational expectations analysis, an aggressive response to inflation expectations, as adherence to the Taylor principle prescribes, does not guarantee stability. Instability arises because expectations are inconsistent with the monetary policy strategy of the central bank, and, therefore, unanchored. Aggressive policy leads to greater macroeconomic uncertainty.

In contrast, communicating the entire policy decision process mitigates instability by stabilizing expectations. Accurate information about the systematic component of current and future monetary policy decisions anchors expectations by ensuring subjective beliefs of agents are consistent with monetary policy strategy. This promotes macroeconomic stability. These stabilization benefits can also be fully captured by a communication strategy that only conveys the set of endogenous variables upon which monetary policy decisions are conditioned, as proposed by the second communication strategy.

Announcing an inflation target and the associated average long-run values of the nominal interest rate and output gap frequently leads to divergent learning dynamics. In an

economy with persistent shocks, the conditions for convergence are identical to those for the benchmark no communication case where these quantities must be learned. Hence, in such economies, communicating the inflation target does little to help stabilize expectations. This is because no information is given on how the central bank will achieve this objective. Communication helps by providing information about the systematic component of policy, and, importantly, by giving information on how the central bank intends to achieve its announced objectives.

Related literature: The analysis builds on Cukierman and Meltzer (1986) and more recently Faust and Svensson (2001, 2002). These papers consider models in which the central bank has an idiosyncratic employment target which is imperfectly observed by the public. Fluctuations in this target lead to central bank temptation to deviate from pre-announced inflation goals. Transparency is desirable as it provides a commitment mechanism.[3] This literature assumes rational expectations on the part of the central bank and the public. Here we assume that the central bank does not have complete information on private sector expectations formation and cannot manipulate agents' beliefs to its own advantage. Strategic interaction between the central bank and the private sector is excluded. Furthermore, in our model, agents have *incomplete information about the policy reaction function*, unlike the papers above where agents have imperfect information about specific variables that appear in the reaction function.

A recent literature focuses on the question of whether transparency of central bank forecasts of exogenous state variables is desirable. The public correctly understands central bank preferences but has imperfect information about the central bank's forecast of the aggregate state. Building on Morris and Shin (2002), the papers Amato and Shin (2006), Hellwig (2002) and Walsh (2006), among others, show that full transparency about the central bank forecast is not always desirable because private agents may over react to noisy public signals and under react to more accurate private information. More generally, Geraats (2002) argues that models based on diverse private information often have the property that pronouncements by the central bank may lead to increased economic volatility. However, Roca (2006) shows that some of these conclusions depend on the postulated objectives of the central bank. Similarly, Svensson (2006) and Woodford (2005) argue that the conclusions

[3] Svensson (1999) further argues on the ground of this result that for inflation targeting central banks it is generally desirable to publish detailed information on policy objectives, including forecasts. Such transparency enhances the public's understanding of the monetary policy process and raises the cost to a central bank from deviating from its stated objectives.

of Morris and Shin (2002) depend on implausible parameter assumptions.[4]

Our analysis departs from this literature by analyzing the value of communicating information about current and future nominal interest-rate decisions of the central bank. Like Walsh (2006), our analysis considers a theory of price setting that is consistent with recent New Keynesian analyses of monetary policy. We propose a fully articulated dynamic stochastic general equilibrium model, and, unlike Walsh, assume that the central bank and private agents have symmetric information about the kinds of disturbances that affect the economy. The asymmetry instead lies in knowledge about how nominal interest rates are determined — that is, monetary policy strategy. This permits a tractable analysis of communication about endogenous decision variables of the central bank rather than announcements about exogenous state variables.[5]

Finally, the paper is related to Orphanides and Williams (2005) and Preston (2006). The former presents a reduced-form model in which announcing the inflation target achieves a better inflation-output trade-off. Because it reduces the amplitude of macroeconomic fluctuations the announcement of the inflation target is welfare enhancing. However, in their model, regardless of whether or not the inflation target is announced, expectations are well anchored: divergent learning dynamics cannot arise. The improvement in welfare results from agents having a more accurate forecast of future policy decisions. Moreover, learning about the monetary policy reaction function is not explicitly modeled. In contrast, this paper presents a model in which divergent learning dynamics emerge even if the inflation target is announced and credible, and, in which, knowledge about the central bank's policy rule proves crucial for expectations stabilization.

The latter incidentally demonstrates that knowledge of the central bank's policy rule can improve stability under learning dynamics. The present analysis extends this work by systematically evaluating a range of examples in which communication might prove beneficial. In particular, it considers a more general class of model in which: decisions are made one-period in advance; agents have more general forecasting models; and the central bank employs a variety of communication strategies. This provides new insights on the desirability of forecast-based instrument rules — in contrast to Preston (2006), more

[4] See also Woodford (2005) and Geraats (2002) for a review of the benefits of central bank communication and transparency.

[5] Rudebusch and Williams (2008) present an analysis that is similar in spirit, but in which expectations are anchored by assumption, analyzing the consequences of asymmetric information about future policy actions. One of the contributions of our paper is to build on their analysis by developing microfoundations which imply asymmetric information about the economy.

aggressive responses to inflation expectations might be destabilizing — and a considerably richer understanding of the potential benefits of communication.

2 A Simple Model

This section describes the aggregate implications of a model similar in spirit to Goodfriend and King (1997), Rotemberg and Woodford (1999) and Svensson and Woodford (2005). A continuum of households faces a canonical consumption allocation problem and decides how much to consume of available differentiated goods and how much labor to supply to firms for the production of such goods. A continuum of monopolistically competitive firms produces differentiated goods using labor as the only input and faces a price-setting problem of the kind proposed by Rotemberg (1982).[6] The major difference is the incorporation of non-rational beliefs, delivering an anticipated utility model. The analysis follows Marcet and Sargent (1989a) and Preston (2005), solving for optimal decisions conditional on current beliefs. Details of the assumed microfoundations, derivations and log-linear approximation are provided in a technical appendix.[7]

Aggregate demand and supply are described by

$$x_t = \hat{E}_{t-1} \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta) x_T - \beta (i_T - \pi_{T+1}) + \beta r_T^e \right] \tag{1}$$

and

$$\pi_t = \frac{\gamma_1 \xi}{(1-\gamma_1 \beta)} \hat{E}_{t-1} \sum_{T=t}^{\infty} (\gamma_1 \beta)^{T-t} \left[(1-\gamma_1 \beta) (x_T + \mu_T) + \pi_T \right] \tag{2}$$

where x_t is the output gap, π_t the inflation rate, i_t the nominal interest rate and both r_t^e and μ_t are exogenous disturbance terms satisfying

$$r_t^e = \rho_r r_{t-1}^e + \varepsilon_t^r \quad \text{and} \quad \mu_t = \rho_\mu \mu_{t-1} + \varepsilon_t^\mu$$

where $0 < \rho_r,\ \rho_\mu < 1$ and $(\varepsilon_t^r,\ \varepsilon_t^\mu)$ are independently and identically distributed random

[6] An analysis of price setting of the kind proposed by Calvo (1983), as implemented by Yun (1996), would lead to similar conclusions.

[7] Various mechanisms of persistence, such as habit formation, price indexation and inertial monetary policy are abstracted from. This provides sharp, perspicuous analytical results. It is also motivated by Milani (2007) and Eusepi and Preston (2008b) which suggest that purely forward-looking business-cycle models with learning dynamics provide a superior characterization of various U.S. macroeconomic time series than do rational expectations models with various persistence mechanisms. An earlier version of this paper, Eusepi and Preston (2008a), demonstrates that our conclusions regarding the value of communication in policy design remain pertinent in models with such modifications.

variables, with autoregressive coefficients known to households and firms.[8] All variables are properly interpreted as log-deviations from steady-state values. The household's discount factor satisfies $0 < \beta < 1$. The degree of nominal rigidities is indexed by $\xi \equiv (\theta - 1)\bar{Y}/\psi > 0$, where $\bar{Y}$ is steady state output, θ is the degree of substitutability between differentiated goods in the underlying microfoundations and ψ measures the cost of adjusting prices. Larger values of ξ imply smaller costs of adjustment — prices are more flexible. The parameter γ_1, described in the technical appendix, satisfies the restrictions $0 < \gamma_1 < 1$ and $\xi = (1 - \gamma_1\beta)(1 - \gamma_1)\gamma_1^{-1}$. In a model with Calvo price adjustment, γ_1 would denote the probability of not re-setting the price. Finally, $\hat{E}_t$ denotes the assumption of possibly non-rational expectations — to be discussed — and represents the aggregation of household beliefs. Because households and firms in the underlying microfoundations only know their own objectives, constraints and beliefs they cannot compute aggregate probability laws. As a result, $\hat{E}_t$ does not satisfy the law of iterated expectations.

The model is closed with the monetary policy rule

$$i_t = i_t^* + \phi\hat{E}_{t-1}\pi_t + \phi_x\hat{E}_{t-1}x_t. \tag{3}$$

The policy parameters satisfy $\phi, \phi_x \equiv \phi\lambda_x/\xi > 0$ where i_t^* is a stochastic intercept term and $\lambda_x > 0$ can be interpreted as the weight given to output gap stabilization in an optimal policy problem in which the central bank minimizes a quadratic loss in inflation and the output gap. The technical appendix demonstrates that under rational expectations appropriate choice of i_t^* in (3) permits implementing optimal policy under discretion and commitment.

All agents make decisions in period t based on $t - 1$ information. In the case of monetary policy, this is appealing: a principle challenge in monetary policy is estimation of the current state. The assumption is also consistent with vector autoregression evidence on the response of spending and pricing decisions to a monetary policy shock — see Rotemberg and Woodford (1997).

3 Learning and Central bank Communication

Agents do not know the true structure of the economic model determining aggregate variables. To forecast state variables relevant to their decision problems, though beyond their

[8] This assumption can be dispensed. Because these shocks are exogenous and assumed to be observed by agents, estimating a first-order process for each shock will recover the true autoregressive coefficient, with probability going to one, as the sample size goes to infinity.

control, agents make use of atheoretical regression models. The regression model is assumed to contain the set of variables that appear in the minimum-state-variable rational expectations solution to the model. Each period, as additional data become available, agents re-estimate the coefficients of their parametric model.

In this paper, we assume stabilizing expectations to mean that learning dynamics converge to the model's predictions under rational expectations. Convergence is assessed using the notion of expectational stability outlined in Evans and Honkapohja (2001). We refer to a situation in which expectations fail to converge to rational expectations equilibrium as "expectations-driven fluctuations" or "divergent learning dynamics". A property of learning models is that dynamics are self-referential: beliefs affect the data generating process and vice versa. This permits analyzing the role of communication in stabilizing expectations. In a rational expectations analysis, expectations are pinned down by construction of the equilibrium and are necessarily consistent with the adopted policy rule. By analyzing a model that permits beliefs to become unanchored from rational expectations and possibly be inconsistent with the monetary policy strategy of the central bank, the value of certain types of information regarding the monetary policy process in stabilizing expectations can be clearly and fruitfully evaluated.

Beliefs of agents in our benchmark analysis of no communication are now outlined. As additional information is communicated to households and firms, the structure of beliefs will change accordingly. These modifications will be noted as they arise, with an illustrative example given below. The agents' estimated model at date $t-1$ can be expressed as

$$Z_t = \begin{bmatrix} x_t \\ \pi_t \\ i_t \\ \hat{\mu}_t \\ \hat{r}_t^e \end{bmatrix} = \omega_{0,t-1} + \omega_{1,t-1} Z_{t-1} + \bar{e}_t \tag{4}$$

where ω_0 denotes the constant, ω_1 is defined as

$$\omega_1 = \begin{bmatrix} \omega_{xx} & \omega_{x\pi} & \omega_{xi} & \omega_{x\mu} & \omega_{xr} \\ \omega_{\pi x} & \omega_{\pi\pi} & \omega_{\pi i} & \omega_{\pi\mu} & \omega_{\pi r} \\ \omega_{ix} & \omega_{i\pi} & \omega_{ii} & \omega_{i\mu} & \omega_{ir} \\ 0 & 0 & 0 & \rho_\mu & 0 \\ 0 & 0 & 0 & 0 & \rho_r \end{bmatrix}$$

and $\bar{e}_t$ represents an i.i.d. estimation error. Agents are assumed to know the autocorrelation coefficients of the shocks but estimate remaining parameters (with time subscripts being dropped for convenience). Absent knowledge of the monetary policy rule, the forecasts of $\{x_t, \pi_t, i_t\}$ implied by (4) need not satisfy (3). Note that agents include a larger set of variables than just those appearing in the minimum-state-variable solution to the model. This is done for generality.

Communication is modeled as information about the dynamics of nominal interest rates. As an example, suppose the central bank credibly announces that monetary policy will be conducted so that inflation, output and nominal interest rates will on average be zero in deviations from steady state. Agents know this with certainty and impose this restriction on their regression model. Hence, $\omega_{0,t-1} = 0$: agents need only learn a subset of coefficients relevant to the reduced-form dynamics of macroeconomic aggregates. This captures well the idea that communicating characteristics of the monetary policy strategy is an attempt to manage the evolution of expectations.

At the end of period $t-1$, agents form their forecast about the future evolution of macroeconomic variables, given their current beliefs about reduced-form dynamics. Given the vector Z_{t-1}, expectations $T+1$ periods ahead are calculated as

$$\hat{E}_{t-1} Z_{T+1} = (I_5 - \omega_{1,t-1})^{-1} \left(I_5 - \omega_{1,t-1}^{T-t+2}\right) \omega_{0,t-1} + \omega_{1,t-1}^{T-t+2} Z_{t-1}$$

for each $T > t-1$, where I_5 is a (5×5) identity matrix. Substituting agents' expectations into (1)–(3) provides the evolution of the output gap, inflation and the nominal interest rate as a function of agents' beliefs and Z_{t-1},

$$Z_t = T\left(\omega_{0,t-1}, \omega_{1,t-1}\right) Z_{t-1} + i.i.d \text{ shocks.}$$

The T-map and the notion of expectational stability are detailed in the technical appendix.

4 Preliminary Foundations

4.1 Benchmark Properties

To ground the analysis, and provide a well-known comparative benchmark, the stability properties of the model under rational expectations are as follows.

Proposition 1 *Under rational expectations, the model given by equations (1), (2) and (3) has a unique bounded solution if $\phi > 1$.*

This is an example of the Taylor principle. If nominal interest rates are adjusted to ensure appropriate variation in the real rate of interest, then determinacy of rational expectations equilibrium obtains. This feature, along with other robustness properties noted by Batini and Haldane (1999) and Levin, Wieland, and Williams (2003), has led to advocacy of forecast-based instrument rules for the implementation of monetary policy. Indeed, such policy rules appear in a number of central bank forecasting models — see, for instance, the Bank of Canada. Furthermore, Clarida, Gali and Gertler (1998, 2000) adduce empirical evidence for interest-rate reaction functions that respond to one-period-ahead forecasts.

Learning dynamics yield strikingly different predictions for the evolution of household and firm expectations. Lack of communication can lead to instability, independently of the central bank response to inflation. Following Evans and Honkapohja (2001), the evolution of agents' beliefs — around the rational expectations equilibrium — are represented by the ordinary differential equation

$$\frac{d\left(\omega_0,\omega_1\right)}{d\tau}=T\left(\omega_0,\omega_1\right)-\left(\omega_0,\omega_1\right) \tag{5}$$

which describes the interaction between the true model $T\left(\omega_0,\omega_1\right)$ and agents' model (4), parameterized by $\left(\omega_0,\omega_1\right)$, in 'notional time' τ.[9] An REE that is locally stable under (5) is said to be "expectationally stable" or "E-stable". It is known that the property of E-stability governs local stability under least squares learning. To delineate key properties, consider a simple case where the discount rate $\beta\approx 1$. The evolution of the constant dynamics are approximated by the linear ODE

$$\begin{bmatrix}\dot{\omega}_{x,0}\\ \dot{\omega}_{\pi,0}\\ \dot{\omega}_{i,0}\end{bmatrix}=J^*\begin{bmatrix}\omega_{x,0}\\ \omega_{\pi,0}\\ \omega_{i,0}\end{bmatrix} \tag{6}$$

where

$$J^*=\begin{bmatrix}0 & \beta/\left(1-\beta\right) & -\beta/\left(1-\beta\right)\\ 1-\gamma_1 & 0 & 0\\ \phi\lambda_x/\xi & \phi & -1\end{bmatrix}$$

is the Jacobian of (10) evaluated at the rational expectations equilibrium. Learning dynamics are stable if all eigenvalues of the Jacobian have negative real parts. In this simple case instability occurs if $\lambda_x<\xi\left(1-\gamma_1\right)$: the Taylor principle is not sufficient for stability.[10]

[9] This is the E-Stability mapping described in the technical appendix. Given a sufficiently large sample of data, the evolution of agents' belief coefficients under least-squares learning are arbitrarily well-described by (10).

[10] The Taylor principle is, however, a necessary condition for stability under learning.

To give insight to this result, Figure 1 shows the dynamics of both agents' beliefs and actual variables implied by (5) when learning does not converge to the rational expectations equilibrium. The economy is assumed to be initially in the deterministic steady state with no shocks occurring in the simulation. We perturb the beliefs of private agents, making the initial estimate of the inflation coefficient, $\omega_{\pi 0}$, higher than its rational expectations value. This can be interpreted as an increase in inflation expectations or equivalently an expectational error on the part of agents.[11] Only the response of the estimates $\omega_{x,0}$, $\omega_{\pi,0}$ and $\omega_{i,0}$ are considered as they correspond to expectations of the output gap, inflation and the nominal interest rate.

Beliefs respond with a delay to changes in actual variables. Output increases on impact followed by an increase in inflation and nominal interest rate. The increase in the nominal interest rate has a weak effect on output, as the expected interest rate adjusts gradually. The latter is the result of not communicating the policy rule — agents fail to project an increase in future real interest rates. As a result, the central bank responds to the economy too much and too late. The nominal interest rate continues to rise with expected inflation, even when output begins to moderate. Agents gradually adjust upwards expectations about future interest rates and reduce their spending below steady state. The converse then occurs as the monetary authority lowers the policy rate to stimulate spending, which further destabilizes the economy.

A sufficiently strong response to output avoids instability, by allowing the central bank to move *ahead* of inflation expectations. To see this, the evolution of inflation expectations is determined from (6) as

$$\dot{\omega}_{\pi,0} = (1 - \gamma_1)\, \omega_{x,0}.$$

Assuming $\phi > 1$, if $\lambda_x > \xi\,(1 - \gamma_1)$, the model is stable, and the central bank response to the output gap is stronger than the impact of the expected output gap on future inflation expectations. As a result, actual and expected interest rates adjust more rapidly to inflation, driving aggregate demand and inflation back to steady-state levels. Indeed, combining the dynamics of $\dot{\omega}_{\pi,0}$ and $\dot{\omega}_{i,0}$ in (6) gives

$$\dot{\omega}_{i,0} = \phi \left(\omega_{\pi,0} + \frac{\lambda_x}{\xi\,(1 - \gamma_1)} \dot{\omega}_{\pi,0} \right).$$

[11] The response are obtained by choosing $\beta = 0.99$, $\xi = 0.06$, $\phi = 2$ and $\lambda_x = 0.005$, which implies $\phi_x = 0.16$, similar to the output gap coefficient in a Taylor type rule estimated on quarterly non-annualized inflation.

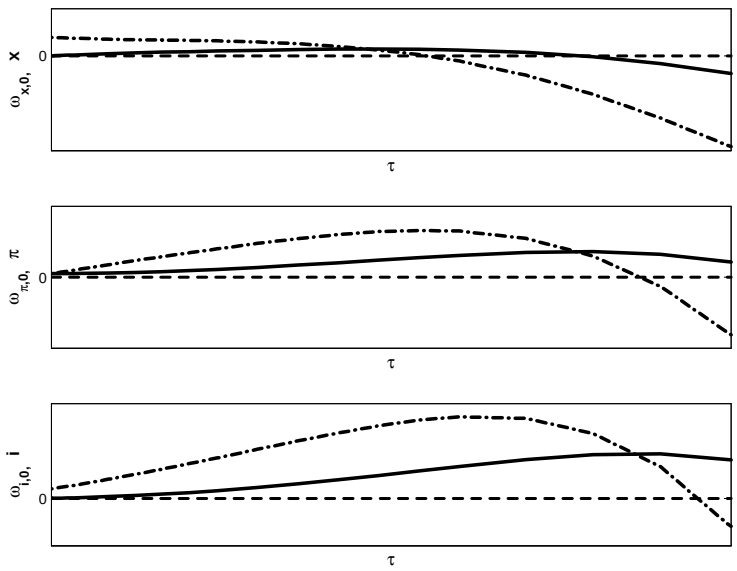

Figure 1: Instability under no communication. The dashed-dotted line denotes actual variables. The solid lines denotes expectations.

If the aforementioned restriction holds, the central bank responds relatively more to expected output which is a more 'leading' indicator of inflation, in the sense that in the neighborhood of rational expectations it is proportional to changes in expected inflation. Alternatively, the monetary authority responds more strongly to expected current inflation, which is a 'lagging' indicator, as it depends on average past inflation. Policy is then destabilizing as the timing of the response is not synchronous with the business cycle, magnifying economic fluctuations instead of dampening them.

Notice that it is the response to output gap *relative* to inflation that matters. Increasing ϕ would increase both coefficients in the policy rule, leaving unchanged the relative importance of the forward-looking component. In this example an increase in ϕ would only translate into higher frequency fluctuations, without affecting stability. In the general case with $\beta \approx 1$, an increase in ϕ can make the equilibrium *less* stable. This is briefly discussed below. This finding contrasts with Ferrero (2007) and Orphanides and Williams (2005) which argue that under learning policy should be more aggressive in response to inflation. The difference in conclusion stems from different assumptions about the central bank's knowledge of the state of the economy, and its ability to manipulate current demand through appropriate choice of the contemporaneous interest rate. In this model, the central bank has incomplete knowledge about the current state of the economy. Because agents' decisions are predetermined, current

interest-rate decisions are less effective in shaping aggregate demand.[12] That forecast-based policy rules might lack robustness across model environments has also been noted by Evans and Honkapohja (2008). They show simple rules can have poor stabilization properties when agents forecast using constant-gain algorithms rather than decreasing-gain algorithms as considered here.

The following propositions summarize the above discussion by providing stability results for the general model.

Proposition 2 *Consider the economy under learning dynamics where the central bank does not communicate the policy rule and $\phi > 1$.*

1. *The REE is unstable under learning provided*

$$\bar{M}_I \equiv \frac{\beta\phi}{\xi}\left[\lambda_x - (1-\gamma_1)\,\xi\right] - M\left(\gamma_1, \beta\right) < 0$$

where

$$M\left(\gamma_1, \beta\right) = \frac{\gamma_1\left(1-\beta\right)\left(1-\gamma_1\right)}{\left(1-\gamma_1\beta\right)}\beta - \frac{1-\gamma_1\beta+\gamma_1\left(1-\beta\right)}{\left(1-\gamma_1\beta\right)^2}\left(1-\beta\right)^2\gamma_1 \gtreqless 0.$$

Hence:

2. *If $\xi \to \infty$ then the REE is unstable for all parameter values.*
3. *If $\beta \to 1$ and $\lambda_x < (1-\gamma_1)\,\xi$, then the REE is unstable for all parameter values.*
4. *If $\beta \to 0$, then the REE is stable under learning for all parameter values.*

Proposition 3 *Consider the economy under learning dynamics where the central bank does not communicate the policy rule. Assume an arbitrarily small degree of nominal rigidities so that $\xi \to \infty$. If $\phi > 0$ and λ_x increases to ensure λ_x/ξ is equal to some constant $\bar{\lambda}_x = \lambda_x/\xi$, then there exists $\bar{\lambda}_x^*$ such that $\bar{\lambda}_x > \bar{\lambda}_x^*$ implies $\bar{M}_I > 0$ guaranteeing E-stability for all parameter values.*

For many reasonable parameter values, $\bar{M}_I < 0$, rendering the economy prone to divergent learning dynamics. Standard parameterizations invariably take the household's discount rate to be near unity, which implies instability under learning for a sufficiently low weight on the output gap. Conversely, as β becomes small, $\bar{M}_I > 0$, guaranteeing stability of

[12] These observations underscore the differences to stability results for a similar class of policies discussed in Preston (2006). That paper argued that stability would obtain only for extremely aggressive responses to inflation expectations.

the equilibrium. Intuitively, as β increases, current consumption plans become more sensitive to expectations, and a correct prediction of the future path of the nominal interest rate, together with predictions about the output gap and inflation, becomes crucial for stability. Analogously, as the degree of nominal rigidity declines, goods prices become more sensitive to expectations about future marginal cost conditions. As $\xi \to \infty$, the flexible price limit, instability obtains for all parameter values. Both features emphasize the importance of stabilizing long-term expectations.

As mentioned above, an increase in ϕ can lead to instability. This is the case for β positive but sufficiently low such that $M(\gamma_1, \beta) < 0$. If the response to output gap is weak ($\lambda_x < (1-\gamma_1)\xi$), an increase in ϕ could make $\bar{M}_I$ negative. To help intuition, a more aggressive policy towards inflation would magnify the oscillatory convergence back to the equilibrium. The central bank tends to over react to changes in expected inflation, amplifying expansions and recessions. This particular result depends on the assumption that consumption and pricing decisions happen with a delay. As a result, current changes in the nominal interest rate do not have, no matter how large, immediate effect on aggregate demand and prices.

Returning to the role of λ_x, the monetary authority can guarantee the stability of expectations, but at a cost. Under rational expectations, for empirically plausible degrees of nominal rigidity, the optimal response to the output gap is near zero — see Rotemberg and Woodford (1997).[13] Hence, co-ordinating expectations under learning likely comes at a cost of lower welfare. More generally, the observation that policies giving greater weight to output gap stabilization are less likely to be prone to instability has relevance for recent debate on the merits of simple policy rules. For example, Schmitt-Grohe and Uribe (2005) demonstrate, in a medium-scale model of the kind developed by Smets and Wouters (2003), that optimal monetary policy can be well-approximated by a simple nominal interest-rate rule that responds to contemporaneous observations of inflation. Policies that respond to output are undesirable, since over-estimating the optimal policy coefficient by even small amounts can lead to sharp deterioration in household welfare. What the above result demonstrates is that, in a world characterized by small departures from rational expectations, the policy maker may face a trade-off: strong responses to the output gap may reduce welfare, but they may protect against even more deleterious consequences from divergent learning dynamics.

[13] This also follows from the optimal policy rule derived in the technical appendix, for which standard parameterizations of this model deliver a near zero coefficient.

Finally, these results provide a striking counter example to McCallum (2007) which shows determinacy implies E-stability in a broad class of linear rational expectations models. Related counter examples are found in Eusepi and Preston (2008c) and Preston (2006). Conclusions differ because optimal decisions conditional on beliefs are considered — forecasts about macroeconomic conditions are made into the indefinite future, rather than just one-period ahead. Bullard and Eusepi (2007) discuss the connection between determinacy and learnability in a general class of models under both approaches to modeling learning.

4.2 Eliminating Policy Delays

This instability result naturally raises the question of how can expectations be managed more effectively in the pursuit of macroeconomic stabilization. The model has two key information frictions. First, the central bank responds to information about the true state of the economy with a delay. This is an implication of the forecast-based monetary policy rule. Second, households and firms have an incomplete model of the macroeconomy and need to learn about the reduced-form dynamics of aggregate prices. It follows that agents are faced with statistical uncertainty about the true data generating process describing the evolution of nominal interest rates. Resolving these informational frictions may mitigate expectations-driven instability.

In regards to the policy maker's uncertainty, suppose the central bank has perfect information about current inflation and the output gap. It can then implement the policy rule

$$i_t = i_t^* + \phi\left(\pi_t + \frac{\lambda_x}{\xi} x_t\right) \tag{7}$$

which is closer in spirit to the policy proposed by Taylor (1993). The following result obtains.

Proposition 4 *Consider the economy under learning dynamics and $\phi > 1$. If the central bank implements monetary policy with the rule (7) without communication then expectational stability obtains for all parameter values under maintained assumptions.*

Timely information about the state of the economy stabilizes expectations. Comparing this result to proposition 2 underscores that instability stems from the interaction between the two sources of information frictions in the model. Given that central banks are unlikely in practice to have complete information about the current state of the economy, it is worth considering other approaches to effective management of expectations. The remainder of the paper explores the role of communication.

To presage results in the sequel, communication can be an effective stabilization tool. Moreover, it is precisely when the central bank is uncertain about the state and other features of the structural economy that communication is effective. When the central bank faces a difficult prediction problem regarding the state of the economy, the benefits of communication are high. By announcing the monetary policy strategy, the central bank can better control the economy even though the near-term evolution of the economy is highly uncertain.

5 The Value of Communication

Communication is modeled in a very direct and simple way. Under learning dynamics, households and firms are uncertain about the true data generating process characterizing the future path of nominal interest rates, the output gap and inflation. We can therefore ask what kinds of information about the monetary policy strategy assist in reducing the forecast uncertainty that emerges from having a misspecified model. Three communication strategies are considered which successively reduce the information made available to the public, providing insight as to what kinds of information are conducive to macroeconomic stabilization.

5.1 Strategy 1

This communication strategy discloses all details of the monetary policy decision process. The central bank announces the precise reaction function used to determine the nominal interest-rate path. Agents know which variables appear in the policy rule and all relevant coefficients. As a result, agents need not forecast the path of nominal interest rates independently — they need only forecast the set of variables upon which nominal interest rates depend. An alternative, but equivalent strategy, is the central bank announces in every policy cycle t its conditional forecast path for the nominal interest rate, $\{E_{t-1} i_T\}_{T \geq t}$. This might arguably characterize current practice by the Norges Bank and the Reserve Bank of New Zealand — see Norges Bank (2006). These forecasts can be used directly by agents when making spending and pricing decisions. Since they are by construction consistent with the adopted policy rule, if agents base decisions directly on these announced forecasts, it must be equivalent to them knowing the policy rule and constructing the forecast path of nominal interest rates independently, subject to the caveats now noted.

To keep the analysis as simple as possible, we assume that the private sector and the central bank share the same expectations about the future evolution of the economy. Analyzing a model in which the central bank communicates its reaction function but in which there is disagreement about the forecasts is feasible though beyond the scope of this paper.[14] Regardless of how this communication strategy is implemented, we assume that the central bank is *perfectly credible*, in the sense that the public fully incorporates announced information in their forecasts without verification. Issues related to cheap talk, as analyzed by Stein (1989) and Moscarini (2007), are not considered. The central bank is able to communicate its reaction function without noise so the market fully understands its policy goals and strategy, both in the current period and into the indefinite future.

Imposing knowledge of the policy rule on households' and firms' forecasting models is equivalent to substituting this equilibrium restriction into the aggregate demand equation to give

$$x_t = \hat{E}_{t-1} \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta) x_T - \beta (i_T^* + \phi \pi_T + \phi \frac{\lambda_x}{\xi} x_T - \pi_{T+1}) + \beta \hat{r}_T^e \right].$$

The remaining model equations are unchanged with the exception of beliefs. Beliefs about the evolution of output and inflation take the form

$$x_t = \omega_{x,0} + \omega_{xx} x_{t-1} + \omega_{x\pi} \pi_{t-1} + \omega_{xi} i_{t-1} + \omega_{xr} r_{t-1}^n + \omega_{x\mu} \mu_{t-1} + e_t^x$$

$$\pi_t = \omega_{\pi,0} + \omega_{\pi x} x_{t-1} + \omega_{\pi\pi} \pi_{t-1} + \omega_{\pi i} i_{t-1} + \omega_{\pi r} r_{t-1}^n + \omega_{\pi\mu} \mu_{t-1} + e_t^\pi$$

with policy-consistent forecasts constructed from knowledge of (3). Compactly, agents' forecasts are determined by

$$\begin{bmatrix} \tilde{Z}_t \\ i_t \end{bmatrix} = \begin{bmatrix} \Omega_0 \\ \Phi\Omega_0 \end{bmatrix} + \begin{bmatrix} \Omega_z & \Omega_i \\ \Phi\Omega_z & \Phi\Omega_i \end{bmatrix} \begin{bmatrix} \tilde{Z}_{t-1} \\ i_{t-1} \end{bmatrix} \tag{8}$$

where

$$\tilde{Z}_t = \begin{bmatrix} x_t & \pi_t & \mu_t & \hat{r}_t^e \end{bmatrix}' \text{ and } \Phi = \begin{bmatrix} \phi\lambda_x/\xi & \phi & \phi_\mu & 1 \end{bmatrix}$$

and Ω_j are matrices of estimated coefficients appropriately defined and the statistical properties of $\{\mu_t, \hat{r}_t^e\}$ continue to be known.[15] The second block of the beliefs, describes agents'

[14] See Honkapohja and Mitra (2005) for an analysis of a New Keynesian model in which only one-period-ahead forecasts matter and conditions under which heterogeneous forecasts deliver the same stability results. This paper, however, does not study a model which requires agents to forecast nominal interest rates.

[15] The matrix Ω_z is of dimension 4×4, while the matrix Ω_i is of dimension 4×1.

policy-consistent interest-rate forecasts, where knowledge of the policy rule (3) is imposed through the coefficient vector Φ. This model is then used to generate expectations as in the case of no communication.

Under these assumptions, uncertainty about the model concerns only the laws of motion for inflation and output, which are affected by other features of the model beyond monetary policy decisions. Perfect knowledge about the central bank's policy framework does not guarantee that the agents' learning process converges to the rational expectations equilibrium, since market participants do not fully understand the true model of the economy. However, it does tighten the connection between the projected paths for inflation and nominal interest rates. This property proves fundamental.

Proposition 5 *Assume the bank communicates under perfect credibility the interest-rate forecast* $\left\{E_{t-1}^{CB} i_T\right\}_{T=t}^{\infty}$ *or, equivalently, the policy rule (3) and* $\phi > 1$. *Then the REE is stable for all parameter values under maintained assumptions.*

Even though the central bank and the private sector have incomplete information about the state of the economy, communication of the policy rule completely mitigates instability under learning dynamics. Communication has value precisely when the central bank is uncertain about the current state — compare proposition 4. The result shows how communicating the reaction function helps shape beliefs about future policy, making it possible for agents to anticipate future policy better. As an example, suppose inflation expectations increase. Under full communication, agents' conditional forecasts of inflation and nominal interest rates are co-ordinated according to (3). Agents, therefore, correctly anticipate that higher inflation leads to a higher path for nominal interest rates — one that is sufficient to raise the projected path of the real interest rate. As a result, output decreases, leading to a decrease in inflation, which in turn mitigates the initial increase in expectations, leading the economy back to equilibrium. In absence of communication, an agents' conditional forecasts for nominal interest rates and inflation give rise to projected falls in future real interest rates, generating instability by validating the initial increase in inflation expectations — recall proposition 2.

5.2 Strategy 2

Now suppose the central bank only announces the set of variables relevant to monetary policy deliberations so that agents do not know the precise restriction that holds between nominal interest rates, inflation and the output gap. Furthermore, suppose that while agents do

not know the policy coefficients, they do know that nominal interest rates are set according to a linear function of these variables. By limiting knowledge of private agents about the monetary policy process relative to the benchmark full-information analysis several aspects of central bank communication can be captured. First, uncertainty about parameters and forecasts can be interpreted as a constraint on the communication ability of the central bank. This reflects the fact that the policy decision is the outcome of a complex process, the details of which are often too costly to communicate — see Mishkin (????). Second, the central bank might face credibility issues, leading the private sector to want to verify announced policies. Third, complete announcement might not be an optimal strategy for a central bank, given the agent's learning process.[16]

Partial information about the policy process can be incorporated by agents in the following two-step forecasting model. First, using the history of available data, agents run a regression of nominal interest rates on expected inflation and the output gap

$$i_t = \psi_{0,t-1} + \psi_{\pi,t-1}\hat{E}_{t-1}\pi_t + \psi_{x,t-1}\hat{E}_{t-1}x_t + e_t.$$

This yields estimates of the coefficients of the policy rule.[17] In the technical appendix, we consider the more general case where the model is estimated using a recursive instrumental variable method, allowing for the possibility that private sector and central bank forecasts are different, or that the central bank can communicate its expectations with noise.

As a second step, conditional on these estimates, agents proceed in the same manner as strategy 1: they forecast the future paths of the output gap and inflation rate and then use the estimated policy rule to construct a set of nominal interest-rate forecasts as described by (8).

Proposition 6 *If households and firms understand the variables upon which nominal interest-rate decisions are conditioned and $\phi > 1$, then the REE is stable under learning for all parameter values under maintained assumptions.*

The central bank need not disclose all details of the monetary policy strategy. It is sufficient that information be given regarding the endogenous variables relevant to the determination of policy and the functional form of the rule — but not its parameterization.

[16] A discussion of the optimal policy under learning is left for further research.

[17] There is an important subtlety in specifying this regression. We assume that private agents include a fixed constant and do not explicitly allow for a stochastic constant as in (3). Hence, the regression is misspecified, though the misspecifiction vanishes as $\rho_r, \rho_\mu \to 0$. This assumption avoids multicollinearity problems in the case of convergent learning dynamics, given the presence of only two shocks. An alternative approach, that yields that same results, is to add an additional shock to the model and allow for a stochastic constant.

Credible public pronouncements of this kind, combined with a sufficient history of data, provide agents with adequate information to verify the implemented rule. And despite the estimation uncertainty attached to the policy coefficients, local to the rational expectations equilibrium of interest, expectations are nonetheless well-anchored relative to the no communication case.[18] This communication strategy is equally useful in protecting against instability from expectations formation as strategy 1 in which agents know the true policy coefficients. Of course, the out-of-equilibrium dynamics would differ across these two strategies — the estimation uncertainty affects the true data generating process of macroeconomic variables — which in turn has welfare implications. Analyzing such implications is beyond the scope of this paper.

5.3 Strategy 3

Over the past two decades numerous countries have adopted inflation targeting as a framework for implementing monetary policy. A central part of this monetary policy strategy has been the clear articulation of a numerical target for inflation. As a final exercise, consider a communication strategy that conveys the desired average outcome for inflation and the associated values for nominal interest rates and the output gap. Given that our analysis is in deviations from steady state, these three values are zero. As discussed in section 3, given this knowledge, agents need not estimate a constant in their regression model, leading to more accurate forecasts of the future path of nominal interest rates.

Proposition 7 *Assume the central bank communicates only the inflation target $\bar{\pi} = 0$ and the associated values for the output gap and nominal interest rates, $\bar{x} = \bar{\imath} = 0$.*

1. *Define $\rho \equiv \max\left(\rho_\mu, \rho_r\right)$ and let $\rho \to 1$. Then the REE is unstable under learning if condition one of Proposition 2 holds.*
2. *Let $\xi \to \infty$. Then there exists $\rho^* < 1$ such that if $\rho \geq \max\left[0, \rho^*\right]$ then instability obtains for all parameter values.*

Economies subject to persistent shocks may be prone to expectations-driven instability. Indeed, the instability conditions for the no communication case obtain for cost-push or efficient-rate disturbance processes having roots near unity. This result nicely demonstrates a fundamental insight of rational expectations analysis: it is not enough to announce an

[18] Formally, this means that agents cannot hold initial beliefs about the policy cofficients that are too different from the true values. Analyzing this possibility would require a global analysis of the model which is well beyond the scope of this paper.

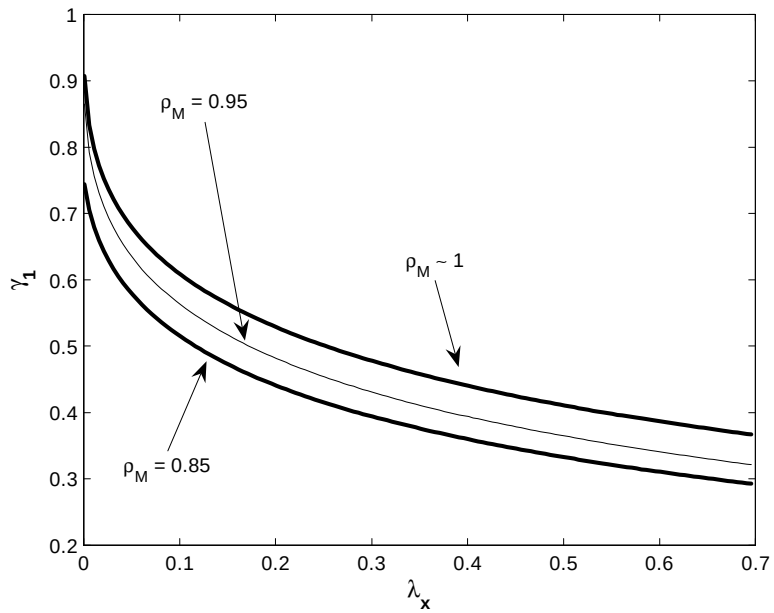

Figure 2: Instability under the announcement of an inflation target

inflation target — one must also announce how one will achieve this target.[19] Only by providing information regarding the systematic component of monetary policy can expectations be effectively managed when shocks are persistent. In contrast, as $\rho \to 0$, there is no persistence in macroeconomic aggregates. Information about the systematic component is less important as the economy has no intrinsic dynamics, making household and firm forecasting problems less complex. The result also underscores another difference to a rational expectations analysis of the model: the precise details of how exogenous disturbances are specified matters for expectational stability. This is not true for determinacy of rational expectations equilibrium.

To further interpret this condition a graphical analysis is useful. The model is calibrated with $\beta = 0.99$, $\phi = 2$ and $\rho_r = 0.2$. Figure 2 plots three contours demarcating stability and instability regions, above and below respectively, as functions of the parameters $(\gamma_1,\ \lambda_x)$. Plotting values of γ_1 assists interpretation and comparison with the literature. Each contour is indexed by the maximum autoregressive coefficient, denoted $\rho_\mu = \rho^M$. It is immediate that as the maximum eigenvalue increases the set of parameter values for which expecta-

[19] Note that if only the inflation target was announced without declaring the associated values of the long-run interest rate and output gap targets, then the stability properties can only be worse since agents must learn a greater number of coefficients.

tions are stabilized narrows. For a given degree of price stickiness, as the persistence in exogenous disturbances rise, a stronger response to the output gap is required. Similarly, for a given weight on output gap stabilization, only in economies with less flexible prices does learnability of rational expectations equilibrium obtain.

The degree of nominal rigidity in price setting has important implications for stabilization policy under learning dynamics. Economies with greater rigidity tend to be conducive to expectations stabilization — current prices are less sensitive to expectations about future macroeconomic conditions. Because prices move little, inflation expectations display low volatility, promoting macroeconomic stability. This is not a property of the model under rational expectations: expectations are well-anchored so long as the Taylor principle is satisfied, regardless of the degree of nominal friction.[20]

6 Discussion

The results of the paper have been derived in a model that makes a number of specific assumptions. For the most part, this has been to facilitate analytical results. The following offers general remarks on why it should be expected that communication, as modeled, will continue to be relevant in more richly specified models. More specific remarks then follow, addressing issues relating to alternative monetary policy rules and central bank knowledge of agent expectations.

Taking a broad perspective, any model in which agents are required to forecast the future path of nominal interest rates without knowledge of the central bank's policy rule will have the property that beliefs will be inconsistent monetary policy strategy, at least during the learning transition. That is, beliefs about future values of various macroeconomic objects relevant to their decisions will not satisfy the restriction implied by that rule. This is true regardless of whether determinacy and E-stability conditions are identical or not. Because of this, communication can play a role by permitting agents to make policy-consistent forecasts. And because communication limits the number of equilibrium restrictions that agents are attempting to learn, it should always be expected to improve the efficacy of stabilization policy.

Of course, we develop a model in which determinacy and E-stability conditions dif-

[20] To quantitatively evaluate the result, keep in mind that even very small values of λ_x might imply large values in terms of the response of output gap in the policy rule, ϕ_x, depending on the degree of nominal rigidity.

fer. On the basis of the authors' research experience, models of the kind proposed in this paper invariably have E-stability conditions that are more stringent than those required for determinacy of rational expectations equilibrium. Communication will only ever relax these conditions — making them closer to those for determinacy — as the information provided increases agents' knowledge of the underlying rational expectations equilibrium to be learned. This makes communication an interesting and substantive issue to explore given the emphasis on E-stability.

To our knowledge, this is the first paper to emphasize consequences of belief structures not disciplined by the assumption of rational expectations for communication and monetary policy. Importantly, it gives substance to what is meant by "unanchored expectations", a central concern of modern monetary policy. It has antecedents in Orphanides and Williams (2005) and Preston (2006). It differs from the former paper by explicitly modeling the transmission mechanism and positing a specific interest-rate rule, rather than assuming a reduced-form model in which the central bank directly controls inflation. Moreover, the Orphanides and Williams (2005) paper is always E-stable. The present analysis underscores that communication matters for stability analysis. But even in models in which communication does not affect the requirements for E-stability, it will qualitatively always matter if beliefs are inconsistent with policy outside of rational expectations equilibrium. It differs from the latter by considering a model in which agents make decisions one period in advance, and by exploring a broader range of communication strategies. However, it has in common with that paper the property that communicating more information about monetary policy improves stabilization policy in the sense of relaxing the requirements for E-stability. And by providing analytical results, Preston (2006) gives some assurance that the insights of this paper hold in other environments.

One reason that these ideas have not been explored in the learning literature is that most analyses — see, for example, Bullard and Mitra (2002) and Evans and Honkapohja (2003) — have confined attention to models where only one-period-ahead forecasts matter to spending and pricing decisions. Importantly, the adopted models do not require forecasts of nominal interest rates. The question of whether beliefs are consistent with policy is moot.

Two additional specific remarks are offered. First, in principle there are infinitely many rules that could be studied. This paper examines one particular class of rule that has received attention in the monetary policy literature. While policy details will affect the precise conditions for determinacy of equilibrium and stability under learning dynamics,

they are less relevant for the question of communication. What matters is agents' knowledge about any adopted rule. If agents have imperfect knowledge of monetary policy strategy, then there will be a role for communication because beliefs are inconsistent with policy objectives out-of-rational-expectations equilibrium. For example, Bullard and Mitra (2007) and Preston (2008) demonstrate inertial rules that respond to lagged endogenous variables, in addition to contemporaneous measures of inflation and the output gap, tend to increase the likelihood of E-stability. However, in the case of the latter (the former being a model in which communication can play no role), this does not imply identical conditions for determinacy and E-stability. Communication can play a role. An earlier version of this paper, Eusepi and Preston (2008a), provides numerical examples demonstrating results are similar for inertial policy rules and policy rules that respond to expectations of future inflation in models with habit formation and price indexation.

Second, there exist policy strategies that might obviate the need for communication, at least from an E-stability perspective, if the central bank has complete knowledge of household and firm behavior, including the structure of beliefs. Preston (2006) proposes an approach in which the central bank implements forecast-based instrument rules based on internal forecasts — as opposed to observed agent forecasts — of inflation and output. Internal forecasts are constructed by exploiting the true structure of the economy and internalizing the effects of evolving beliefs on the future path of interest rates. This approach restores the Taylor principle as a necessary and sufficient condition for E-stability. Even though E-stability conditions could not be improved under this approach, communication might still matter for welfare by influencing out of equilibrium dynamics. Alternatively, if policy is implemented assuming the central bank: i) employs targeting rules of the kind proposed by Giannoni and Woodford (2002); ii) has complete knowledge of household and firm behavior; and iii) conditions nominal interest-rate decisions on all future expectations about output, inflation and nominal interest rates, then communication may also not matter. This approach is detailed in Preston (2008) in a model closely related to that considered here.[21] Such policy strategies have the property that regardless of the expectations held by agents, expectations will be consistent with the monetary policy strategy of the central bank both in and out of rational expectations equilibrium — see Woodford (2005) for further discussion. Hence, in the terminology of this paper: expectations are anchored regardless of whether

[21] Evans and Honkapohja (2003) explore such strategies in a model where only one-period-ahead forecasts matter to decisions. However, commnunication has no role in their framework because nominal interest rate forecasts affect neither spending nor pricing decisions.

the central bank communicates its strategy or not. Preston (2008) shows in a related model that such policies ensure determinacy of equilibrium and stability under learning for all parameter values. Given that these two approaches demand much knowledge of about agents behavior, communication appears to have value.

7 Conclusion

Using a simple dynamic stochastic general equilibrium model this paper provides theoretical examples of how central bank communication might prove beneficial for monetary policy strategy. Under no communication, policy fails to stabilize macroeconomic dynamics, promoting expectations-driven fluctuations. However, by announcing the details of the policy process, or only the variables upon which policy is conditioned, stability is restored. Communication permits agents to construct more accurate forecasts, engendering greater stability in observed output, inflation and nominal interest rates. Finally, if the central bank only announces the desired inflation target, economies with persistent shocks are prone to expectations-driven fluctuations. This makes clear that it is not sufficient to announce only desired objectives — one must also announce the systematic component of policy which describes how these objective will be achieved.

A Appendix

A.1 Expectational Stability

Substituting for the expectations in the equations for the output gap, inflation and the nominal interest rate, permits writing aggregate dynamics of the economy as

$$Z_t = \Gamma_0 \left(\omega_{0,t-1}, \omega_{1,t-1}\right) + \Gamma_1 \left(\omega_{1,t-1}\right) Z_{t-1} + \Gamma_2 \bar{\varepsilon}_t \tag{9}$$

with obvious notation and where $\bar{\varepsilon}_t$ is a (5×1) vector of zeros with final two elements ε_t^{μ} and ε_t^{r}. This expression captures the dependency of observed dynamics on agents' beliefs about the future evolution of the economy. Moreover, it implicitly defines the mapping between agents' beliefs and the actual coefficients describing observed dynamics as

$$T\left(\omega_{0,t-1},\ \omega_{1,t-1}\right) = \left(\Gamma_0 \left(\omega_{0,t-1}, \omega_{1,t-1}\right),\ \Gamma_1 \left(\omega_{1,t-1}\right)\right). \tag{10}$$

A rational expectations equilibrium is a fixed point of this mapping. For such rational expectations equilibria we are interested in asking under what conditions does an economy with

learning dynamics converge to each equilibrium. Using stochastic approximation methods, Marcet and Sargent (1989b) and Evans and Honkapohja (2001) show that conditions for convergence are characterized by the local stability properties of the associated ordinary differential equation

$$\frac{d\left(\omega_0,\omega_1\right)}{d\tau}=T\left(\omega_0,\omega_1\right)-\left(\omega_0,\omega_1\right), \tag{11}$$

where τ denotes notional time. The rational expectations equilibrium is said to be expectationally stable, or E-Stable, when agents use recursive least squares if and only if this differential equation is locally stable in the neighborhood of the rational expectations equilibrium.[22]

B Proofs

B.1 Some useful properties of γ_1

Aggregate inflation is determined by

$$\pi_t=\frac{\gamma_1\xi}{(1-\gamma_1\beta)}\hat{E}_{t-1}\pi_t+\frac{\gamma_1\xi}{(1-\gamma_1\beta)}\hat{E}_{t-1}\sum_{T=t}^{\infty}\left(\gamma_1\beta\right)^{T-t}\left[\left(1-\gamma_1\beta\right)\left(\hat{s}_T+\hat{\mu}_T\right)+\gamma_1\beta\pi_{T+1}\right]$$

with the derivation found in the technical appendix to this paper. The parameter $0<\gamma_1<1$ is an eigenvalue from underlying microfoundations. The following properties of the eigenvalue γ_1 in the limit $\xi\to\infty$ (which corresponds to the neighborhood of flexible price equilibrium) are used in the proofs. The eigenvalue is given as

$$\gamma_1\left(\xi\right)=\frac{1}{2\beta}\left[\xi+1+\beta-\sqrt{\left(\xi+1+\beta\right)^2-4\beta}\right].$$

The following limits are immediate:

1. $\lim_{\xi\to\infty}\gamma_1\left(\xi\right)=0$, where the notation $\gamma_1\left(\xi\right)$ means that γ_1 is a function of ξ. Let

$$\gamma_1=\frac{1}{2\beta}\frac{\left[\xi+1+\beta-\sqrt{\left(\xi+1+\beta\right)^2-4\beta}\right]\left[\xi+1+\beta+\sqrt{\left(\xi+1+\beta\right)^2-4\beta}\right]}{\left[\xi+1+\beta+\sqrt{\left(\xi+1+\beta\right)^2-4\beta}\right]}$$

[22] Standard results for ordinary differential equations imply that a fixed point is locally asymptotically stable if all eigenvalues of the Jacobian matrix $D\left[T\left(\omega_0,\omega_1\right)-\left(\omega_0,\omega_1\right)\right]$ have negative real parts (where D denotes the differentiation operator and the Jacobian understood to be evaluated at the relevant rational expectations equilibrium).

which gives

$$\gamma_1 = \frac{1}{2\beta} \frac{4\beta}{\left[\xi + 1 + \beta + \sqrt{(\xi + 1 + \beta)^2 - 4\beta}\right]}$$

and thus $\lim_{\xi \to \infty} \gamma_1(\xi) = 0$.

2. $\lim_{\xi \to \infty} \gamma_1(\xi)\, \xi = 1$. We have

$$\begin{aligned} lim_{\xi \to \infty} \gamma_1(\xi)\, \xi &= \frac{1}{2\beta} \frac{4\beta}{\left[1 + \xi^{-1} + \xi^{-1}\beta + \sqrt{\xi^{-2}\left((\xi + 1 + \beta)^2 - 4\beta\right)}\right]} \\ &= \frac{1}{2\beta} \frac{4\beta}{2} = 1. \end{aligned}$$

B.2 Proof of Proposition 2

Expectational stability is determined by the eigenvalues of the ODE (11). The local dynamics of this system can be decomposed into four independent sub-systems. The first

$$\dot{\omega}_0 = \left(J_{\omega_0^*} - I_3\right) \omega_0 \tag{12}$$

characterizes the dynamics of the estimated constants, where $J_{\omega_0^*}$ is the Jacobian evaluated at the rational expectations equilibrium of interest. The second and the third describe the evolution of the coefficients on the exogenous shocks are given by

$$\dot{\omega}_\mu = \left(J_{\omega_\mu^*} - I_3\right) \omega_\mu \tag{13}$$

$$\dot{\omega}_r = \left(J_{\omega_r^*} - I_3\right) \omega_r \tag{14}$$

where $\omega_\mu = \begin{pmatrix} \omega_{x\mu} & \omega_{\pi\mu} & \omega_{i\mu} \end{pmatrix}'$ and $\omega_\mu = \begin{pmatrix} \omega_{xr} & \omega_{\pi r} & \omega_{ir} \end{pmatrix}'$. The final subsystem characterizes the coefficients on the endogenous variables and is given by

$$vec\left(\dot{\omega}_e\right) = \left(J_{\omega_e^*} - I_9\right) vec\left(\omega_e\right) \tag{15}$$

where $\omega_e = \begin{pmatrix} \omega_{xx} & \omega_{x\pi} & \omega_{xi} \\ \omega_{\pi x} & \omega_{\pi\pi} & \omega_{\pi i} \\ \omega_{\pi x} & \omega_{\pi\pi} & \omega_{\pi i} \end{pmatrix}$. Subsequent proofs employ the same structure.

To prove the instability result, we show that the system (12) is locally unstable. The associated Jacobian matrix is

$$\left(J_{\omega_0^*} - I_3\right) = \begin{bmatrix} 0 & \beta/(1-\beta) & -\beta/(1-\beta) \\ \gamma_1\xi + \frac{\gamma_1^2\beta\xi}{(1-\gamma_1\beta)} & \frac{\gamma_1\xi}{(1-\gamma_1\beta)} + \frac{\gamma_1^2\beta\xi}{(1-\gamma_1\beta)^2} - 1 & 0 \\ \frac{\phi\lambda_x}{\xi} & \phi & -1 \end{bmatrix}.$$

Necessary and sufficient conditions for stability under learning are

$$\text{Trace}\left(J_{\omega_0^*} - I_3\right) < 0 \tag{16}$$

$$\text{Determinant}\left(J_{\omega_0^*} - I_3\right) < 0 \tag{17}$$

and

$$\bar{M} = -\text{Sm}\left(J_{\omega_0^*} - I_3\right) \cdot \text{Trace}\left(J_{\omega_0^*} - I_3\right) + \text{Determinant}\left(J_{\omega_0^*} - I_3\right) > 0, \tag{18}$$

where Sm denotes the sum of all principle minors. Satisfaction of these three conditions ensure that all three eigenvalues are negative.

Evaluating the trace gives

$$(1-\gamma_1\beta)^{-1}\left[\gamma_1\xi + \frac{\gamma_1^2\beta\xi}{1-\gamma_1\beta} - 2\left(1-\gamma_1\beta\right)\right]$$

which can be simplified to

$$-2 + \frac{1-\gamma_1}{1-\gamma_1\beta} < 0$$

where we use $\xi = (1-\gamma_1)(1-\gamma_1\beta)/\gamma_1$. The determinant is

$$-\frac{(-\phi\lambda_x\gamma_1\xi + \phi\lambda_x - 2\phi\lambda_x\gamma_1\beta + \phi\lambda_x\gamma_1^2\beta^2 + \gamma_1\xi^2\phi - \gamma_1^2\xi^2\phi\beta - \gamma_1\xi^2 + \gamma_1^2\xi^2\beta)}{\xi(1-\beta)(1-\gamma_1\beta)^2}\beta$$

and can be rearranged to obtain the following condition

$$-\left[\xi\left(\phi-1\right) + \frac{\phi\lambda_x}{\xi}\left(1-\beta\right)\right] < 0$$

which holds if $\phi > 1$. Finally, the third condition is

$$\left\{-\frac{\gamma_1\xi}{(1-\gamma_1\beta)^2} + 1 + \frac{\beta}{(1-\beta)}\left[-(\gamma_1\xi + \frac{\gamma_1^2\beta\xi}{(1-\gamma_1\beta)}) + \frac{\phi\lambda_x}{\xi}\right]\right\}(2 - \frac{1-\gamma_1}{(1-\gamma_1\beta)}) +$$
$$-\beta\frac{(-\gamma_1\xi^2 + \gamma_1^2\xi^2\beta + \gamma_1\xi^2\phi - \gamma_1^2\xi^2\phi\beta - \phi\lambda_x\gamma_1\xi + \phi\lambda_x - 2\phi\lambda_x\gamma_1\beta + \phi\lambda_x\gamma_1^2\beta^2)}{\xi(1-\beta)(1-\gamma_1\beta)^2}$$

which on simplification and multiplying by $(1-\beta)$ gives

$$\bar{M}_I \equiv \frac{\beta\phi}{\xi}\left[\lambda_x - (1-\gamma_1)\xi\right] + \beta\left(1-\gamma_1\right) - M\left(\xi,\beta\right) \tag{19}$$

where

$$M\left(\xi,\beta\right) = \left(2 - \frac{1-\gamma_1}{(1-\gamma_1\beta)}\right)\left(\frac{(1-\gamma_1)(1-\beta)}{(1-\gamma_1\beta)} - (1-\beta) + \beta(1-\gamma_1)\right).$$

$\bar{M}_I$ may be positive or negative. This gives property 1 in the proposition.

In the limit $\beta \to 1$ this expression simplifies to

$$\frac{\beta\phi}{\xi}\left[\lambda_x - (1-\gamma_1)\,\xi\right] + \beta\,(1-\gamma_1) = \frac{\phi}{\xi}\left(\lambda_x - (1-\gamma_1)\,\xi\right).$$

Hence for $\lambda_x < (1-\gamma_1)\,\xi$ instability obtains for all parameter values, establishing property 3 of the proposition. Now consider the limit $\xi \to \infty$ which implies $\gamma_1 \to 0$. In this case the condition simplifies to $-\phi < 0$ for all finite $\{\phi, \lambda_x\}$. This establishes property 2 of the proposition. It is also immediate that as $\beta \to 0$, $\bar{M}_I = \gamma_1\,(1+\gamma_1)$ which guarantees stability (property 4).

B.3 Proof of Proposition 3

From Proposition 2, for $\xi \to \infty$, and implicitly allowing λ_x to increase to ensure $\bar{\lambda}_x = \lambda_x/\xi$ is a given constant,

$$\bar{M}_I = \beta\left[\phi\bar{\lambda}_x - \phi\right], \tag{20}$$

so that a large enough choice of $\bar{\lambda}_x$, say $\bar{\lambda}_x^I$, provides $\bar{M}_I > 0$, ensuring convergence of the intercept. The stability of the shock coefficients depend on the matrix

$$J_h - I_3 = \begin{bmatrix} \frac{\beta+\beta(1-\beta)\rho_h}{1-\beta\rho} & \frac{\beta\rho_h}{1-\beta\rho_h} & -\beta - \frac{\beta^2\rho_h}{1-\rho_h} \\ \gamma_1\xi + \frac{\gamma_1^2\beta\xi\rho_h}{1-\gamma_1\beta\rho_h} & J_1 & 0 \\ J_2 & J_3 & -1 - \frac{\phi\lambda_x}{\xi}\beta - \frac{\phi\lambda_x\beta^2\rho_h}{\xi(1-\beta\rho_h)} \end{bmatrix}$$

for $h = \{r, \mu\}$ where

$$\begin{aligned} J_1 &= -1 + \frac{\gamma_1\xi}{1-\gamma_1\beta} + \frac{\gamma_1^2\xi\beta\rho_h}{(1-\gamma_1\beta)\,(1-a\beta\rho_h)} \\ J_2 &= \phi\gamma_1\xi + \frac{\phi\gamma_1^2\xi\beta\rho_h}{1-a\beta\rho_h} + \phi\lambda_x\xi(1-\beta) + \frac{\phi\lambda_x\beta\xi(1-\beta)\rho_h}{1-\beta\rho_h} \\ J_3 &= \frac{\phi\xi\gamma_1}{1-\gamma_1\beta} + \frac{\phi\xi\gamma_1^2\beta\rho_h}{(1-\gamma_1\beta)(1-\gamma_1\beta\rho_h)} + \frac{\phi\lambda_x\beta\rho_h}{\xi\,(1-\beta\rho_h)}. \end{aligned}$$

The trace is

$$\frac{\gamma_1\xi}{(1-\gamma_1\beta)} + \frac{\gamma_1^2\xi\beta\rho_h}{(1-\gamma_1\beta)(1-\gamma_1\beta\rho_h)} - \beta - 2 + \frac{\beta(1-\beta)\rho_h}{(1-\beta\rho_h)} - \frac{\phi\lambda_x}{\xi}\beta - \frac{\phi\lambda_x\beta^2\rho_h}{\xi\,(1-\beta\rho_h)}$$

$$\Longrightarrow \quad \frac{\xi\gamma_1}{(\gamma_1\beta\rho_h - 1)\,(\gamma_1\beta - 1)} - \beta - 2 + \frac{\beta(1-\beta)\rho_h}{(1-\beta\rho_h)} - \frac{\phi\lambda_x}{\xi}\beta - \frac{\phi\lambda_x\beta^2\rho_h}{\xi\,(1-\beta\rho_h)}$$

$$\Longrightarrow \quad \frac{(1-\gamma_1)}{(1-\gamma_1\beta\rho_h)} - \beta - 2 + \frac{\beta(1-\beta)\rho_h}{(1-\beta\rho_h)} - \frac{\phi\lambda_x}{\xi}\beta - \frac{\phi\lambda_x\beta^2\rho_h}{\xi\,(1-\beta\rho_h)} < 0.$$

The determinant is

$$\beta\frac{(\xi\gamma_1^2\beta^2\rho_r^2-\xi\gamma_1^2\beta^2\rho_r-\phi\lambda_x\gamma_1^2\beta^2\rho_r-\gamma_1^2\xi^2\beta\rho_r+\phi\gamma_1^2\xi^2\beta-\xi\gamma_1\beta\rho_r^2+\phi\lambda_x\gamma_1\beta\rho_r)}{(1-\gamma_1\beta)\xi(1-\beta\rho_r)(1-\gamma_1\beta\rho_r)}$$
$$+\beta\frac{(\xi\gamma_1\beta+\beta\phi\lambda_x\gamma_1-\phi\gamma_1\xi^2+\gamma_1\xi^2+\phi\lambda_x\gamma_1\xi+\xi\rho_r-\xi-\phi\lambda_x)}{(1-\gamma_1\beta)\xi(1-\beta\rho_r)(1-\gamma_1\beta\rho_r)}$$

which simplifies to

$$-\beta\left[\gamma_1\xi\phi-(\rho_h-\gamma_1)(1-\gamma_1\beta\rho_h)+\frac{\phi\lambda_x}{\xi}\gamma_1(1-\beta\rho_h)\right]<0$$

provided the intercept converges. Denote the sum of principle minors as $\bar{M}(\rho_h)$ and consider the case where $\xi\to\infty$. Then $\bar{M}(\rho_h)$ becomes

$$\frac{(3\beta\rho_h^2-4\beta\rho_h+\beta-\phi-\rho_h+\phi\beta\rho_h+\phi_x\beta+\phi_x+1-2\phi\bar{\lambda}_x\beta\rho_h)}{(1-\beta\rho_h)^2}\beta$$
$$\Longrightarrow\quad\frac{3\beta\rho_h^2-4\beta\rho_h+\beta-\phi-\rho_h+\phi\beta\rho_h+\phi\bar{\lambda}_x(1+\beta-2\beta\rho_h)+1}{(1-\beta\rho_h)^2}\beta$$
$$\Longrightarrow\quad\frac{3(\rho_h-1)[\beta\rho_h-\beta(1-\rho_h)-(1-\beta\rho_h)]-\phi(1-\beta\rho_h)+\phi\bar{\lambda}_x[1-\beta\rho_h+\beta(1-\rho_h)]}{(1-\beta\rho_h)^2}\beta \tag{2F}$$

Hence for large enough $\bar{\lambda}_x$, say $\bar{\lambda}_x^S$, stability of the shock coefficients can always be guaranteed. The local stability of the coefficients on lagged variables are determined by a nine dimensional system. The Jacobian matrix $(J_e^*-I_9)$ implies that the dynamics of the following coefficients are independent and converge for all parameter values:

$$\begin{aligned}\dot{\omega}_{x\pi} &= -\beta\omega_{x\pi};\quad \dot{\omega}_{xi}=-\beta\omega_{xi};\quad \dot{\omega}_{\pi x}=\left(-\gamma_1+\frac{\gamma_1(1-\gamma_1)\beta}{(1-\gamma_1\beta)}\right)\omega_{\pi x}\\ \dot{\omega}_{\pi i} &= -\gamma_1\omega_{\pi i};\quad \dot{\omega}_{i\pi}=-\omega_{i\pi};\quad \dot{\omega}_{ix}=-\gamma_1\omega_{ix}.\end{aligned}$$

The remaining parameters evolve according to

$$\begin{bmatrix}\dot{\omega}_{xx}\\ \dot{\omega}_{\pi\pi}\\ \dot{\omega}_{ii}\end{bmatrix}=\begin{bmatrix}-\beta & 0 & -\beta\\ \gamma_1\xi & \frac{\gamma_1\xi}{1-\gamma_1\beta}-1 & 0\\ -\lambda_x & \phi & -1\end{bmatrix}.$$

Again, consider the case where $\xi\to\infty$. In this limit the trace is $-\beta-1<0$, and the determinant $-\beta\phi<0$. Finally, the sum of principle minors is $\beta(-\phi+\phi\bar{\lambda}_x(1+\beta)+1+\beta)$ which is positive for sufficiently high response to the output gap, say $\bar{\lambda}_x^L$. We can now define $\bar{\lambda}_x^*=max\left(\bar{\lambda}_x^I,\bar{\lambda}_x^S,\bar{\lambda}_x^L\right)$ and the proposition is proved.

B.4 Proof of Proposition 4

Stability is again determined by four subsystems analogous to (12), (13), (14) and (15). Local stability of the intercept is determined by the Jacobian matrix

$$\left(J_{\omega_0^*} - I_3\right) = \begin{bmatrix} 0 & \frac{\beta}{1-\beta} & -\frac{\beta}{1-\beta} \\ \frac{\gamma_1\xi}{1-\gamma_1\beta} & \frac{\gamma_1\xi-1+2\gamma_1\beta-\gamma_1^2\beta^2}{(1-\gamma_1\beta)^2} & 0 \\ -\phi\frac{(-\lambda_x+\lambda_x\gamma_1\beta-\gamma_1\xi^2)}{\xi(1-\gamma_1\beta)} & -\frac{(-\gamma_1\xi^2+\gamma_1\xi^2\beta-\lambda_x\beta+2\lambda_x\beta^2\gamma_1-\lambda_x\beta^3\gamma_1^2)\phi}{\xi(1-\gamma_1\beta)^2(1-\beta)} & -\frac{\phi\lambda_x\beta+\xi-\xi\beta}{\xi(1-\beta)} \end{bmatrix}$$

which can be shown to have one eigenvalue equal to -1. Noting that $\omega_{i,0} = \phi\omega_{\pi,0} + \phi\lambda_x/\xi\omega_{x,0}$, make a change of variables so that local stability can be analyzed by computing the eigenvalues of the two dimensional matrix

$$\tilde{J}_{\omega_0^*} - I_2 = \begin{bmatrix} -\frac{\beta\phi\lambda_x}{\xi}(1-\beta)^{-1} & \beta\frac{\phi-1}{1-\beta} \\ 1-\gamma_1 & \frac{\gamma_1\xi-1+2\beta\gamma_1-\beta^2\gamma_1^2}{(1-\gamma_1\beta)^2} \end{bmatrix}.$$

Local stability requires that $\text{trace}(\tilde{J}_{\omega_0^*} - I_2) < 0$ and $\det(\tilde{J}_{\omega_0^*} - I_2) > 0$. The trace is

$$-1 + (\frac{\gamma_1}{(1-\gamma_1\beta)} + \frac{\gamma_1^2\beta}{(1-\gamma_1\beta)^2})\xi - \frac{\beta}{(1-\beta)}\lambda_x\phi\xi^{-1} = -1 + \frac{(1-\gamma_1)}{(1-\gamma_1\beta)} - \frac{\beta}{1-\beta}\lambda_x\phi\xi^{-1} < 0$$

The determinant is

$$\frac{(-\phi\lambda_x\gamma_1\xi + \phi\lambda_x - 2\phi\lambda_x\gamma_1\beta + \phi\lambda_x\gamma_1^2\beta^2 + \gamma_1\xi^2\phi - \gamma_1^2\xi^2\phi\beta - \gamma_1\xi^2 + \gamma_1^2\xi^2\beta)}{\xi(1-\beta)(1-\gamma_1\beta)^2}\beta$$

which on rearranging gives

$$\xi(\phi-1) + \frac{\phi\lambda_x}{\xi}(1-\beta) > 0. \tag{22}$$

The Jacobian associated with both (13) and (14) takes the form

$$\left(J_{\omega_h^*} - I_3\right) = \begin{bmatrix} -\frac{(1-\rho_h)\beta}{1-\beta\rho_h} & \frac{\beta\rho_h}{1-\beta\rho_h} & -\frac{\beta}{1-\beta\rho_h} \\ \frac{\gamma_1\xi}{1-\gamma_1\beta\rho_h} & \frac{\gamma_1\xi}{(1-\gamma_1\beta\rho_h)(1-\gamma_1\beta\rho_h)} - 1 & 0 \\ c_1 & c_2 & c_3 - 1 \end{bmatrix}$$

where $h = \{r, \mu\}$ and c_j are convolutions of model parameters. Again, one eigenvalue is equal to -1. A change of variables implies stability depends on the eigenvalues of the two-dimensional matrix

$$\left(\tilde{J}_{\omega_h^*} - I_2\right) = \begin{bmatrix} -\beta\frac{\left(1-\rho_h+\frac{\phi\lambda_x}{\xi}\right)}{1-\beta\rho_h} & \beta\frac{\phi-\rho_h}{1-\beta\rho_h} \\ \frac{\gamma_1\xi}{1-\gamma_1\beta\rho_h} & \frac{\gamma_1\xi-1+\beta\gamma_1\rho_h+\beta\gamma_1-\beta^2\gamma_1^2\rho_h}{(1-\gamma_1\beta)(1-\gamma_1\beta\rho_h)} \end{bmatrix}$$

which has trace equal to

$$-\beta-\beta\frac{\phi\lambda_x}{\xi}+(\beta(1-\beta)-\beta^2\frac{\phi\lambda_x}{\xi})\frac{\rho_h}{1-\beta\rho_h}+\frac{\gamma_1\xi}{(1-\gamma_1\beta)}+\frac{\gamma_1^2\xi\beta\rho_h}{(1-\gamma_1\beta)(1-\gamma_1\beta\rho_h)}-1$$

$$\Longrightarrow\quad -1-\beta+\beta\frac{(1-\beta)\rho}{1-\beta\rho}-\frac{\beta}{1-\beta\rho}\frac{\phi\lambda_x}{\xi}+\frac{1-\gamma_1}{(1-\gamma_1\beta\rho)}<0.$$

The determinant is

$$\frac{(-\gamma_1\xi^2+\xi-\xi\gamma_1\beta+\xi\gamma_1^2\beta^2\rho_h-\phi\lambda_x\gamma_1\xi+\phi\lambda_x-\phi\lambda_x\gamma_1\beta\rho_h-\phi\lambda_x\gamma_1\beta+\phi\lambda_x\gamma_1^2\beta^2\rho_h-\rho_h\xi)}{\xi(1-\beta\rho_h)(1-\gamma_1\beta)(1-\gamma_1\beta\rho_h)}\beta$$
$$+\frac{(\rho_h^2\xi\gamma_1\beta-\rho_h^2\xi\gamma_1^2\beta^2+\gamma_1\xi^2\phi-\gamma_1^2\xi^2\phi\beta+\gamma_1^2\xi^2\beta\rho_h)}{\xi(1-\beta\rho_h)(1-\gamma_1\beta)(1-\gamma_1\beta\rho_h)}\beta$$

which simplifies to

$$\gamma_1\xi\phi-(\rho_h-\gamma_1)(1-\gamma_1\beta\rho_h)+\frac{\phi\lambda_x}{\xi}\gamma_1(1-\beta\rho_h). \tag{23}$$

This term is greater than zero if (22) obtains.

Finally, the stability of the coefficients of the lagged variables depends on the matrix (15) which is nine-dimensional. The following six subsystems are independent and it is immediate that the dynamics of these coefficients are stable for all parameter values under maintained assumptions:

$$\begin{aligned}\dot\omega_{x\pi} &= -\beta\omega_{x\pi};\quad \dot\omega_{xi}=-\beta\omega_{xi};\quad \dot\omega_{\pi x}=\left(\frac{1-\gamma_1}{1-\gamma_1\beta}-1\right)\omega_{\pi x}\\ \dot\omega_{\pi i} &= \left(\frac{1-\gamma_1}{1-\gamma_1\beta}-1\right)\omega_{\pi i};\quad \dot\omega_{i\pi}=-\gamma_1\omega_{\pi i};\quad \dot\omega_{ix}=-\gamma_1\omega_{ix}.\end{aligned}$$

The final three coefficients have local dynamics described by the system

$$\begin{bmatrix}\dot\omega_{xx}\\ \dot\omega_{\pi\pi}\\ \dot\omega_{ii}\end{bmatrix}=\begin{bmatrix}-\beta & 0 & -\beta\\ \gamma_1\xi & \frac{\gamma_1\xi}{1-\gamma_1\beta}-1 & 0\\ c_1 & c_2 & c_3\end{bmatrix}\begin{bmatrix}\omega_{xx}\\ \omega_{\pi\pi}\\ \omega_{ii}\end{bmatrix}$$

where c_j for $j=\{1,2,3\}$ is a convolution of model parameters. Again one eigenvalue is equal to -1 so that stability is determined by the two-dimensional matrix

$$\tilde J=\begin{bmatrix}-\beta\left(1+\frac{\phi\lambda_x}{\xi}\right) & -\beta\phi\\ \gamma_1\xi & \frac{\gamma_1\xi}{(1-a\beta)}-1\end{bmatrix}.$$

The trace of $\tilde J$ is

$$-\beta-\beta\frac{\phi\lambda_x}{\xi}+(1-\gamma_1)-1<0$$

and the determinant

$$\beta\frac{(-\xi^2\gamma_1+\xi-\beta\xi\gamma_1-\phi\lambda_x\gamma_1\xi+\phi\lambda_x-\phi\lambda_x\gamma_1\beta+\phi\gamma_1\xi^2-\phi\gamma_1^2\xi^2\beta)}{\xi\left(1-\gamma_1\beta\right)}=\gamma_1\beta(\frac{\phi\lambda_x}{\xi}+\phi\xi+1)>0.$$

This completes the proof.

B.5 Proof of Proposition 5

Expectational stability is determined by the eigenvalues of the associated ODE which can again be decomposed into four independent subsystems describing the stability properties of the constant, efficient rate shock, cost-push shock and lagged coefficient dynamics. Consider the stability of the constant dynamics for output and inflation. The Jacobian matrix

$$J_{\omega_0^*}-I_2=\begin{bmatrix} -\beta\frac{\phi\lambda_x}{\xi}\left(1-\beta\right)^{-1} & -\beta\frac{(\phi-1)}{1-\beta} \\ 1-\gamma_1 & \frac{\gamma_1\xi-1+2\beta\gamma_1-\beta^2\gamma_1^2}{(1-\gamma_1\beta)^2} \end{bmatrix}$$

has trace

$$-1+\frac{(1-\gamma_1)}{(1-\gamma_1\beta)}-(1+\frac{\beta}{(1-\beta)})\beta\lambda_x\phi\xi^{-1}<0$$

and determinant

$$\xi\left(\phi-1\right)+\frac{\phi\lambda_x}{\xi}\left(1-\beta\right)>0.$$

The shock coefficients have the following Jacobian matrix

$$\left(\tilde{J}_{\omega_h^*}-I_2\right)=\begin{bmatrix} -\beta\frac{\left(1-\rho_h+\frac{\phi\lambda_x}{\xi}\right)}{1-\beta\rho_h} & \beta\frac{\phi-\rho_h}{1-\beta\rho_h} \\ \frac{\gamma_1\xi}{1-\gamma_1\beta\rho_h} & \frac{\gamma_1\xi-1+\beta\gamma_1\rho_h+\beta\gamma_1-\beta^2\gamma_1^2\rho_h}{(1-\gamma_1\beta)(1-\gamma_1\beta\rho_h)} \end{bmatrix}$$

for $h=\{r,\mu\}$ which displays trace and determinant as in the previous proposition. For the six coefficients on the endogenous variables, the Jacobian can be expressed as

$$\tilde{J}\otimes I_3 \text{ where } \tilde{J}=\begin{bmatrix} -\beta\left(1+\frac{\phi\lambda_x}{\xi}\right) & -\beta\phi \\ \gamma_1\xi & \frac{\gamma_1\xi}{(1-a\beta}-1 \end{bmatrix}.$$

The trace is

$$-\beta-\beta\frac{\phi\lambda_x}{\xi}+(1-\gamma_1)-1<0$$

and determinant

$$\beta\frac{(-\xi^2\gamma_1+\xi-\beta\xi\gamma_1-\phi\lambda_x\gamma_1\xi+\phi\lambda_x-\phi\lambda_x\gamma_1\beta+\phi\gamma_1\xi^2-\phi\gamma_1^2\xi^2\beta)}{\xi\left(1-\gamma_1\beta\right)}$$

which can be rearranged to give

$$\gamma_1\beta(\frac{\phi\lambda_x}{\xi}+\phi\xi+1)>0.$$

Hence all six eigenvalues on lagged coefficients are less than zero. This completes the proof.

B.6 Proof of Proposition 6

Similarly to the previous proposition, the agents' forecasts is determined by :

$$\begin{bmatrix} \tilde{Z}_t \\ i_t \end{bmatrix} = \begin{bmatrix} \Omega_0 \\ \hat{\Psi}\Omega_0 + \hat{\psi}_0 \end{bmatrix} + \begin{bmatrix} \Omega_z & \Omega_i \\ \hat{\Psi}\Omega_z & \hat{\Psi}\Omega_i \end{bmatrix} \begin{bmatrix} \tilde{Z}_{t-1} \\ i_{t-1} \end{bmatrix} \tag{24}$$

where

$$\tilde{Z}_t = \begin{bmatrix} x_t & \pi_t & \hat{\mu}_t & \hat{r}_t^e \end{bmatrix}' \text{ and } \hat{\Psi} = \begin{bmatrix} \hat{\psi}_x & \hat{\psi}_\pi & 0 & 0 \end{bmatrix}.$$

Notice that we consider a policy rule with a fixed constant, to avoid multicollinearity problems. The evolution of $\hat{\psi}_t = \left(\hat{\psi}_{0,t}, \hat{\psi}_{x,t}, \hat{\psi}_{\pi,t}\right)'$ is described by

$$\hat{\psi}_t = \hat{\psi}_{t-1} + \gamma_t \tilde{R}_{t-1}^{-1} \begin{bmatrix} 1 \\ r_{t-1}^n \\ \mu_{t-1} \end{bmatrix} \left(\hat{\imath}_t - \hat{\psi}_{t-1}' \begin{bmatrix} 1 \\ \hat{E}_{t-1} x_t \\ \hat{E}_{t-1} \pi_t \end{bmatrix} \right)$$

where we assume agents use a Recursive Instrumental Variable estimator, to encompass the case of noise in the announced forecast:[23]

$$\tilde{R}_t = \tilde{R}_{t-1} + \gamma_t \left(\begin{bmatrix} 1 \\ r_{t-1}^n \\ \mu_{t-1} \end{bmatrix} \begin{bmatrix} 1 \\ \hat{E}_{t-1} x_t \\ \hat{E}_{t-1} \pi_t \end{bmatrix}' - \tilde{R}_{t-1} \right)$$

where the instrument is $\begin{bmatrix} 1 & r_{t-1}^n & \mu_{t-1} \end{bmatrix}$ so we can substitute for the correct coefficients

$$\hat{\psi}_t - \hat{\psi}_{t-1} = \gamma_t \tilde{R}_{t-1}^{-1} \begin{bmatrix} 1 \\ r_{t-1}^n \\ \mu_{t-1} \end{bmatrix} \left(\psi' - \hat{\psi}_{t-1}' \right) \begin{bmatrix} 1 \\ \hat{E}_{t-1} x_t \\ \hat{E}_{t-1} \pi_t \end{bmatrix} = \gamma_t \tilde{R}_{t-1}^{-1} \begin{bmatrix} 1 \\ r_{t-1}^n \\ \mu_{t-1} \end{bmatrix} \begin{bmatrix} 1 \\ \hat{E}_{t-1} x_t \\ \hat{E}_{t-1} \pi_t \end{bmatrix}' \left(\psi - \hat{\psi}_{t-1} \right)$$

where $\psi = (0, \phi\lambda_x/\xi, \phi)$ and

$$\tilde{R}_t = \tilde{R}_{t-1} + \gamma_t \left(\begin{bmatrix} 1 \\ r_{t-1}^n \\ \mu_{t-1} \end{bmatrix} \begin{bmatrix} 1 \\ \hat{E}_{t-1} x_t \\ \hat{E}_{t-1} \pi_t \end{bmatrix}' - \tilde{R}_{t-1} \right)$$

Taking limits we have

$$\dot{\hat{\psi}} = \tilde{R}^{-1} M \left(\Omega, \hat{\psi} \right) \left(\psi - \hat{\psi} \right)$$

[23] The gain sequence γ_t has the properties $0 < \gamma_t < 1$, $\lim_{t\to\infty} \gamma_t = 0$ and $\sum_{i=0}^{\infty} \gamma_i = \infty$. See Evans and Honkapohja (2001).

and

$$\dot{\tilde{R}} = M\left(\Omega, \hat{\psi}\right) - \tilde{R}.$$

Given that the rational expectations equilibrium delivers a stationary process, for Ω and $\hat{\psi}$ sufficiently close their rational expectations values, we have that

$$M\left(\Omega, \hat{\psi}\right) = E\left(\begin{bmatrix} 1 \\ r_{t-1}^{n} \\ \mu_{t-1} \end{bmatrix} \begin{bmatrix} 1 \\ \hat{E}_{t-1} x_t \\ \hat{E}_{t-1} \pi_t \end{bmatrix}'\right)$$

is finite, where E denotes the unconditional expectations operator.[24] Hence: $\tilde{R} \to M\left(\Omega, \hat{\psi}\right)$, and therefore $\hat{\psi} \to \psi$. The stability conditions are then the same as for the case of full communication.

B.7 Proof of Proposition 7

Communication implies the constants are known to be zero in rational expectations equilibrium. Therefore, only the stability properties of the efficient rate shock, cost push shock and lagged coefficient dynamics need to be examined. Proposition 3 delivers the conditions for the trace and determinant of the shock matrix. The final condition for stability comes from the sum of principle minors, (18), and delivers a complicated expression of model parameters. Letting $\bar{M}(\rho_h)$ be the implied expression, it can be shown that $lim_{\rho_h \to 1}\left(\bar{M}(\rho_h) - \bar{M}_I\right) = 0$ for $h = \{r, \mu\}$. Hence instability arises under the same conditions as in Proposition 2 as asserted in part one of the proposition.

To prove the final part of the proposition, consider $\xi \to \infty$. Moreover, in this case let $\phi_x \equiv \frac{\phi \lambda_x}{\xi} \to 0$ as ξ increases (that is we consider the optimal targeting rule with finite λ_x). Then $\bar{M}(\rho_h)$ becomes

$$\frac{3(\rho_h - 1)\left[\beta\rho_h - \beta(1-\rho_h) - (1-\beta\rho_h)\right] - \phi(1-\beta\rho_h)}{(1-\beta\rho_h)^2}\beta. \tag{25}$$

Finally, determine $0 \leq \rho_h^* \leq 1$ that gives instability. Evaluating the numerator of (25) at $\rho_h = 1$ we get $-\beta(\phi - 1)(1-\beta)^{-1} < 0$. For $\rho_h = 0$, $[3(\beta+1) - \phi]\beta$. If negative, so that $\phi > 3(1+\beta)$, the proof is complete: there is instability for every ρ_h. If positive $\phi < 3(1+\beta)$ and the derivative of (25) with respect to ρ_h is $\beta(\phi\beta - 4\beta + 6\beta\rho_h - 1)$. Evaluating the slope of the parabola at $\rho_h = 0$ assuming $3(\beta+1) > \phi$ yields a gradient smaller than

$$\beta\left[3(\beta+1)\beta - 4\beta - 1\right] = 3\beta^2 + 3\beta - 4\beta - 1 < 0.$$

[24] See Evans and Honkapohja (2001), p.234 for example.

Hence, from the numerator of (25) there exists a $\rho_h^* < 1$ satisfying

$$3\beta^2 \left(\rho_h^*\right)^2 + \beta(\phi\beta - 1 - 4\beta)\rho_h^* + \beta(\beta - \phi + 1) = 0$$

such that for $\rho_h > \rho_h^*$ instability occurs. Solving the quadratic equation gives

$$\rho_h^* = \frac{\beta(1 + 4\beta - \phi\beta) - \sqrt{\beta^2(1 + 4\beta - \phi\beta)^2 - 12\beta^3(\beta - \phi + 1)}}{6\beta^2}.$$

This completes the proof of Proposition 7.

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