14th Australasian Fluid Mechanics Conference
Adelaide University, Adelaide, Australia
10-14 December 2001

# Spectral Element Based Simulation of Turbulent Pipe Flow

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## Abstract

We present results from direct and large eddy simulations of a turbulent pipe flow where both Cartesian and cylindrical formulations of the Navier–Stokes equations have been employed. For the large eddy simulations, a conventional eddy viscosity approach has been taken to sub-grid scale stress estimation, in conjunction with van Driest-type near-wall modification. The results obtained with both DNS and LES are in very good agreement with published experimental and numerical data.

## Introduction

Many industrial applications involve axisymmetric geometries (i.e. pipes, diffusers, cyclones) and complex three-dimensional flows with unsteady flow phenomena. Statistical turbulence models often fail to accurately simulate the turbulent motion and heat exchange in these kind of applications, because of the insensitivity of Reynolds-Averaged Navier–Stokes equations (RANS) to unsteady flow features. Modelling of higher correlations of the shear-rate and vorticity tensor can account for additional physical effects, such as streamline curvature, but are not able to alleviate general problems involved with this numerical method.

In contrast to a statistical approach, the simulation of turbulent flows with direct numerical simulation (DNS) and large eddy simulation (LES) has stimulated much interest, since they are able to capture unsteady flow patterns within complex flows, which are responsible for most of the energy and momentum transport in the flow. For this reason LES is a promising alternative to RANS for industrial applications at moderate Reynolds numbers.

## Simulation of Turbulence

DNS is an accepted numerical tool for the accurate study of turbulent flows. As this technique does not rely on any turbulence model, a complete three-dimensional data base can be obtained from a flow field and investigated in full detail, in order to construct turbulence models for LES/RANS or get information on the near-wall structures including the exact wall shear stress. Since all turbulent time and length scales from the integral (macro-scale) $L$ down to the Kolmogorov dissipation scale $\eta$ have to be captured by a DNS, requirements to memory and computing time are tremendous. Considering the ratio $L/\eta = Re_L^{3/4}$, the number of grid points required to carry out a DNS scale $N = Re_L^{9/4}$ summing up to a total cost $\sim Re^3$ (including time-integration). Therefore, DNS is restricted to fairly low Reynolds numbers and mainly used for validation purposes.

On the other hand, LES focuses on the largest length scales which determine the global flow behaviour. In this technique, only the large scales down to a cut-off wavenumber $k_c$ are explicitly resolved by the numerical grid while the smaller scales are represented by a subgrid-scale model. The motivation for this approach stems from the fact that the small-scale turbulence is expected to behave more isotropically, without any preferred orientation and should consequently be much easier to model than the whole spectrum of turbulence.

In terms of a mathematical representation of LES, the Navier–Stokes equations are filtered, leading to an unknown additional subgrid-stress $\boldsymbol{\tau} = \overline{\boldsymbol{u}\boldsymbol{u}} - \overline{\boldsymbol{u}}\,\overline{\boldsymbol{u}}$, which incorporates all interaction between the resolved and unresolved motion and has to be modelled. The governing evolution equation for LES reads

$$\partial_t \overline{\boldsymbol{u}} + \nabla\cdot\overline{\boldsymbol{u}}\,\overline{\boldsymbol{u}} = -\nabla\overline{p}/\rho + \nu\nabla^2\overline{\boldsymbol{u}} - \nabla\cdot\boldsymbol{\tau}\,. \qquad (1)$$

In the present work, closure of the unknown correlation $\boldsymbol{\tau} = -2\,\nu_t\,\overline{\boldsymbol{S}}$ is achieved by the Smagorinsky model

$$\nu_t = (C_s\overline{\Delta})^2\,|\overline{\boldsymbol{S}}|\,, \quad |\overline{\boldsymbol{S}}| = \mathrm{tr}(2\,\overline{\boldsymbol{S}}^2)^{1/2}\,, \qquad (2)$$

which is an algebraic mixing-length model, that includes a model constant $C_S$. This value has to be fixed *a priori* and depends on the flow configuration chosen.

Van Driest damping is incorporated to ensure that the mixing length in terms of the model vanishes at the solid walls;

$$C_S = C_{S_0}\left[1 - \exp(-(y^+/A^+)^3)\right]^{1/2}\,, \quad A^+ = 26\,. \qquad (3)$$

## Numerical Method

The existing spectral element scheme [1, 4, 5] employs a spatial discretisation with Fourier expansions in one geometrically homogeneous direction coupled with two-dimensional spectral elements in the remaining two coordinates. As the pipe flow features statistical homogeneity in the axial and azimuthal directions, the Fourier expansion can in turn be applied to each direction separately.

We refer to the discretisation that employs Fourier expansions in the pipe axis direction and spectral elements in the cross-section as the Cartesian formulation. The version that has Fourier expansions in the azimuth and spectral elements in the meridional semiplane as the cylindrical formulation. Part of the motivation for the recent work is to compare and cross-validate these two code formulations.

In this paper, the spectral element method is employed for a DNS and LES of the turbulent pipe flow, which can be regarded as a test case prior to attempting more complex axisymmetric flow configurations. The Reynolds number based on the bulk velocity $u_b$ and the pipe diameter $D$ was set to $Re_b = 4910$ and $Re_b = 16\,000$ for the DNS and LES respectively. For comparison, measurements of den Toonder & Nieuwstadt [2] and numerical DNS data of Eggels et al. [3] are available.

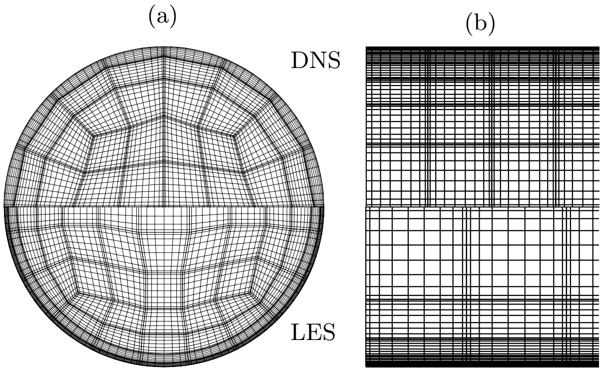


Figure 1: Computational mesh for DNS (top) and LES (bottom): (a), Cartesian formulation and (b), cylindrical formulation (detailed view of entire mesh).

#### Mesh Parameters

The domain length of the pipe is chosen to $L = 5D$ matching that of Eggels et al. [3]. For all meshes, the near-wall grid-spacing is kept under $y^+ < 1$ to account for recommendations in DNS and wall-resolving LES. The computational mesh for the Cartesian formulation is a cross-section of the pipe (see figure 1a) using 64 elements, each with 10th order Gauss–Lobatto–Legendre (GLL) tensor-product shape functions for the DNS and 105 elements with 8th order shape functions in case of the LES. In figure 1b the parts of the meshes are shown, which are used by the cylindrical formulation. The DNS grid consists of 150 elements (9th order shape functions) and the cylindrical LES mesh of 80 elements of the same order. Further grid details can be found in table 1.

| | DNS $Re_b = 4910$ | | LES $Re_b = 16\,000$ | |
|---|---|---|---|---|
| Grid type | Cart. | cyl. | Cart. | cyl. |
| No of Elements | 64 | 150 | 105 | 80 |
| Polyn. Order $N_P$ | 10 | 9 | 8 | 9 |
| $\Delta x^+$ | 0.34–4.85 | 7.62 | 0.81–20.65 | 36.07 |
| $\Delta y^+$ | 0.34–4.85 | 0.76 | 0.81–20.65 | 1.15 |
| Fourier direction $\Delta z^+$ | 11.22 | 0.48–11.22 | 34.0 | 2.18–34.00 |
| Nodes [$\times 10^{-6}$] | 1.23 | 1.17 | 1.29 | 0.62 |

Table 1: Grid details of the pipe flow computations.

#### Direct Numerical Simulation

The DNS at $Re_b = 4910$ were carried out for the validation of the spectral element scheme using either a Cartesian or a cylindrical formulation of the Navier–Stokes equation. In table 2, the global statistics are compared to DNS data from Eggels et al. [3]. Both present simulations compare well to Eggels' data; the main distinctions are due to the differences of the Reynolds number.

| | DNS-Cart. | DNS-cyl. | Eggels et al. [3] |
|---|---|---|---|
| $Re_c$ | 6485 | 6510 | 6950 |
| $Re_b$ | 4910 | 4910 | 5300 |
| $Re_\tau$ | 343 | 343 | 360 |
| $\overline{u}_c/\overline{u}_\tau$ | 18.91 | 18.98 | 19.31 |
| $\overline{u}_b/\overline{u}_\tau$ | 14.58 | 14.58 | 14.73 |
| $\overline{u}_c/\overline{u}_b$ | 1.297 | 1.302 | 1.31 |

Table 2: Pipe flow simulations: global statistics.

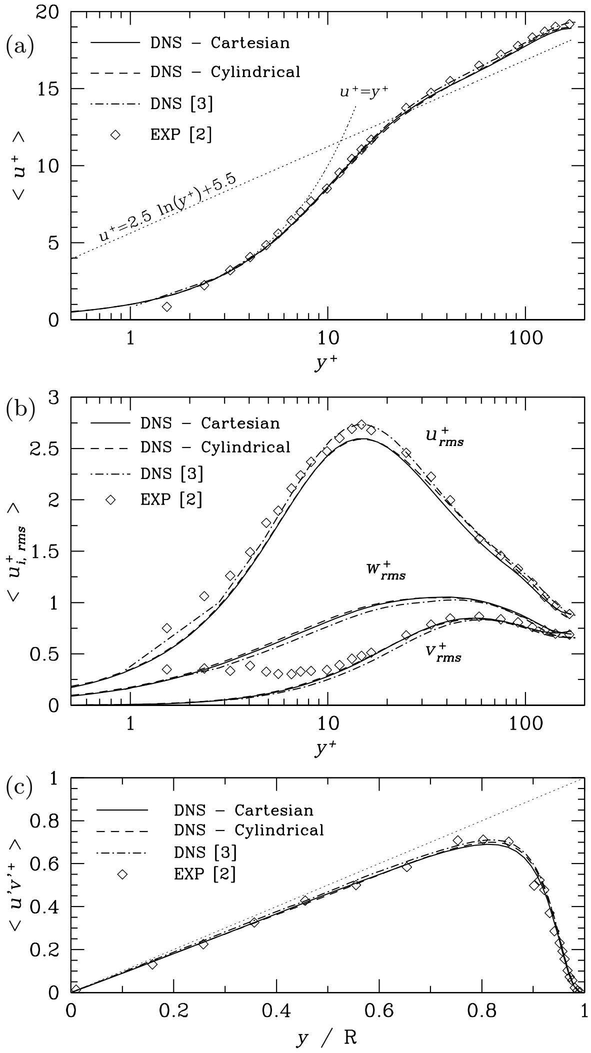


Figure 2: DNS: Comparison of Cartesian and cylindrical formulation: (a) mean velocities, (b) rms-values and (c) shear-stress distribution.

Both results are displayed in figure 2 along with experimental data from den Toonder & Nieuwstadt [2] for $Re_b = 4922$ ($Re_\tau = 334$) and DNS data [3]. The mean velocities (figure 2a) show excellent agreement with the measurements and demonstrate that the same quality of results can be obtained by both methods. Turning the attention to the rms-values of the velocity fluctuations (figure 2b), results from the two formulations are nearly identical, with slight variation in the outer layers. Predictions from the streamwise fluctuation ($u^+$) are slightly lower than the experimental values [2] and previous DNS [3]. Predictions for wall-normal fluctuations

($v^+$) again are slightly lower than the corresponding measurements. Note that the experiments are asymptotically incorrect in the approach to the wall. The shear-stresses (figure 2c) show no remarkable difference for all data sets.

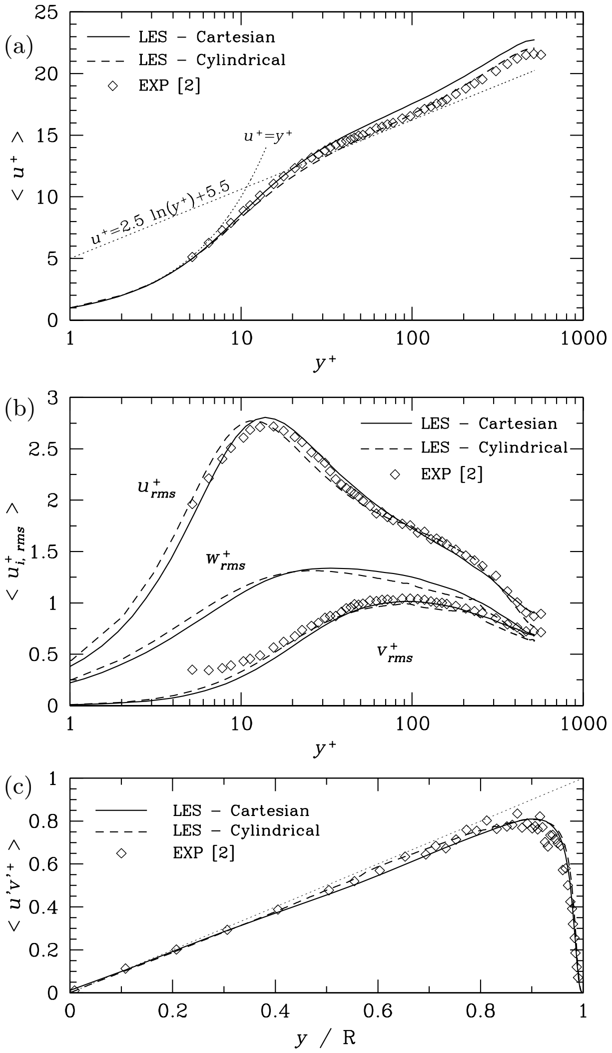


Figure 3: LES: Comparison of Cartesian and cylindrical formulation ($C_S = 0.05$) (a) mean velocities, (b) rms-values and (c) shear-stress distribution.

## Large Eddy Simulation

In figure 3, results of the LES at $Re_b = 16\,000$ are compared against experimental data [2], obtained at $Re_b = 17\,800$ ($Re_\tau = 1130$). The mean velocity profiles (figure 3a) display small differences between the Cartesian and cylindrical formulations, while the latter almost matches the experiments. The differences between the two formulations may result from different mesh resolutions. The velocity fluctuations (figure 3b) reveal a separation of both solutions in all components, which may be caused by a different grid resolution in the buffer layer. The stress profiles match well with the measurements, which appear to be very noisy in the region of maximum shear (figure 3c).

## Influence of Smagorinsky Constant

The constant of the Smagorinsky model is not a unique parameter and it strongly depends on the rate of shear acting on the flow. Therefore, the value has to be fixed according to the given test case as misadjustments can in turn lead to poor results. In figure 4, solutions based on three different constants $C_S = 0.05/0.10/0.15$ are presented for the cylindrical formulation. For the mean velocity (figure 4a), only $C_S = 0.05$ achieves a satisfactory agreement with [2], while higher values overpredict the velocities significantly. Considering the usual range of $C_S = 0.065 - 0.12$ found in literature, the present results show a strong sensitivity to variations of this parameter. The differences in the fluctuations (figure 4b) are even larger. While giving nearly the same level of streamwise velocity fluctuations, an increase of $C_S$ leads to a shift of the peak away from the wall. At the same time, the fluctuations in the remaining directions and the maximum shear stress (figure 4c) drops, as the model contribution in terms of higher values of $C_S$ rises.

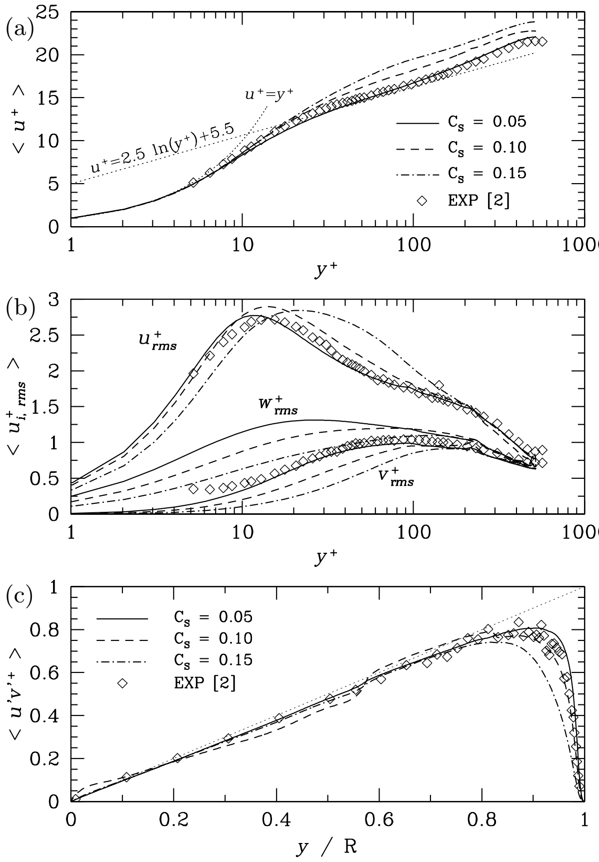


Figure 4: Influence of Smagorinsky constant, based on the cylindrical formulation: (a) mean velocities, (b) rms-values (experimental values are of $u^+_{rms}$ and $v^+_{rms}$ only) and (c) shear-stress distribution.

## Definition of Length Scale

The subgrid-scale model includes a length scale, which has to be defined. Since the general choice is somewhat arbitrary, usually a mean value of a mesh cell in uniform Cartesian grid $\Delta = [\Delta r \cdot r\,\Delta\Theta \cdot \Delta z]^{1/3}$ (referred herein as the volume formulation) is preferred. In a cylindrical coordinate system, this definition generates zero values of $\Delta$ at the pipe axis that can destablize the numerical procedure and lead to divergence. Applying a low-pass filter to dampen out the excessive growth of higher modes [6], can retain numerical stability. For this reason, the more appropriate length scale $\Delta = 1/3\,[(\Delta r)^2 + (r\,\Delta\Theta)^2 + (\Delta z)^2]^{1/2}$, (denoted as the hypotenuse formulation), has been adopted into the cylindrical formulation. This definition avoids zero length scales at the origin and does not require any kind of filtering or special treatment near the axis of symmetry.

In figure 5, results with both length-scale definitions based on $C_S = 0.05$ are given, indicating hardly any difference between them. Since the use of spectral filters to guarantee numerical stability is somewhat artificial, the hypotenuse formulation should be preferred in this framework. (All the preceding cylindrical LES results presented here employ the hypotenuse form.)

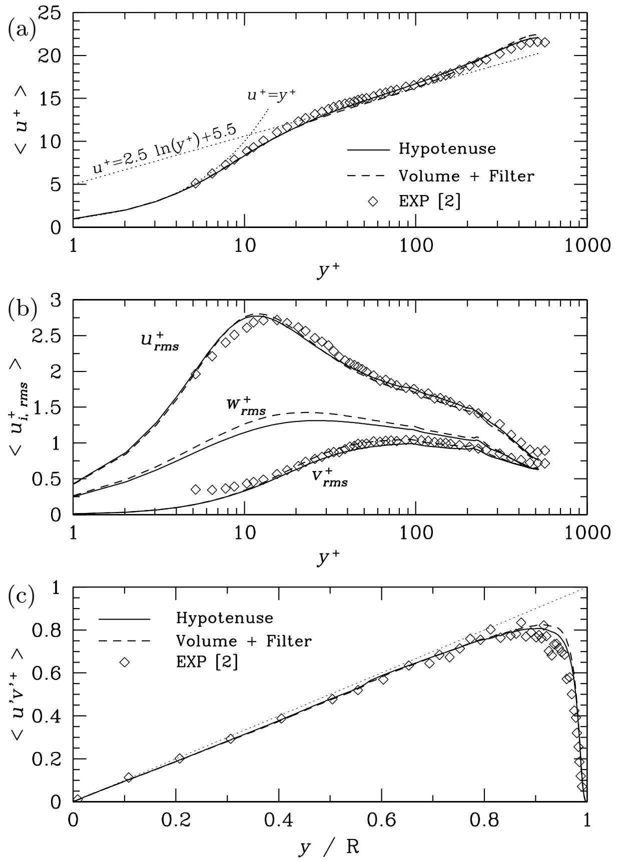


Figure 5: Comparison of hypotenuse and volume formulation ($C_S = 0.05$): (a) mean velocities, (b) rms-values and (c) shear-stress distribution.

## Conclusions

The results of these turbulent pipe flow simulations demonstrate the ability of the spectral element method to efficiently and accurately carry out DNS and LES of turbulent flows. The employed formulations of using either the axial or the azimuthal direction for the Fourier expansion makes this method applicable to a wide range of industry-related applications with greater geometrical complexity the pipe flow. The LES results suggest that there is good chance of achieving accurate results even at higher Reynolds numbers.

However, the Smagorinsky model (2) requires specification of the appropriate model parameter, as wrong values can in turn lead to an overestimation of the dissipation and consequently to false levels of kinetic energy coinciding with a misrepresentation of the stress anisotropy. Also, the Smagorinsky formulation requires adoption of van Driest modifications in the near-wall region (3), which requires that the mean wall shear stress be known in advance. This does not introduce a problem in the present application, as the time-mean wall shear stress can be determined from the Blasius correlation for turbulent pipe flow. In more general cases, however, this approach would be difficult to implement as $u_\tau$ would not be known in advance.

The definition of the length scale has given rise to numerical stability problems at the centreline, where the volume formulation gives zero values. A more appropriate definition based on the hypotenuse formulation avoids this singularity and should be adopted in the cylindrical formulation.

Future work will include the implementation of the dynamic model into the cylindrical formulation, which avoids the need to specify the wall shear stress or the parametrisation of the mesh length scale. This has already been accomplished for the Cartesian formulation [7].

## Acknowledgements

The computations were carried out on the Fujitsu VPP300 of the ANU Supercomputer Facility (ANUSF) and the Joint High Performance Computing and Communications Centre (HPCCC) of Bureau of Meteorology and CSIRO, both of whose support is gratefully acknowledged.

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