Large Eddy Simulation of Flow Past a Circular Cylinder

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Abstract

Results are presented of large eddy simulations for smooth subcritical flow past a stationary circular cylinder, at a Reynolds number of 3900 where the wake is fully turbulent but the cylinder boundary layers remain laminar. Results obtained using a spectral element discretisation are compared with those obtained with a finite volume multiblock method and published experimental results.

Introduction

In order to find more general use for simulation of turbulent flows in industrial applications, large eddy simulation (LES) techniques must be married with spatial discretisation methods that can deal with arbitrary geometries. Here we compare results from two alternative applicable discretisation techniques, spectral element and multiblock finite volume, applied to simulation of the flow past a circular cylinder with $Re = U_{\infty}D/\nu = 3900$, where U_{∞} is the freestream flow speed, D is the cylinder diameter and ν is the (molecular) kinematic viscosity of the fluid.

Large eddy simulation is based on the idea of partitioning a turbulent flow into resolved-scale and modelled turbulent fields through convolution with a filter kernel: this operation produces the resolved-scale velocity field \overline{u} from the underlying complete turbulent field u. The resolved-scale field \overline{u} is that which is explicitly represented on the computational mesh, i.e. the filtering operation is assumed to be implicit in the formulation. Filtering the incompressible Navier–Stokes equations leads to

$$\partial_t \overline{\boldsymbol{u}} + \nabla \overline{\boldsymbol{u}} \overline{\boldsymbol{u}} = -\nabla \overline{P} + \nu \nabla^2 \overline{\boldsymbol{u}},\tag{1}$$

where $\overline{P} = \overline{p}/\rho$. As in conventional turbulence modelling, the nonlinear terms are not closed, because the filtered dyad \overline{uu} is not available in terms of the resolved components \overline{u} . To overcome this problem, the concept of a subgrid-scale stress (SGSS) τ is introduced, such that $\overline{uu} = \overline{u} \,\overline{u} + \tau$. Then the momentum equation becomes

$$\partial_t \overline{\boldsymbol{u}} + \nabla \cdot \overline{\boldsymbol{u}} \,\overline{\boldsymbol{u}} = -\nabla \overline{P} + \nu \nabla^2 \overline{\boldsymbol{u}} - \nabla \cdot \boldsymbol{\tau},\tag{2}$$

and the turbulence modelling task is to predict τ from the resolved velocity field \overline{u} .

The model adopted here is an eddy-viscosity type based on the "mixing length" approach, i.e. the Smagorinsky model. The eddy-viscosity assumption is that the anisotropic part of τ is related to the resolved strain rate tensor \overline{S} through a scalar eddy viscosity,

$$\boldsymbol{\tau} - \operatorname{tr}(\boldsymbol{\tau})\mathbf{1}/3 = -2\nu_t \overline{\boldsymbol{S}} = -\nu_t \left[\nabla \overline{\boldsymbol{u}} + (\nabla \overline{\boldsymbol{u}})^T\right], \quad (3)$$

with the isotropic (scalar) part of τ subsumed in \overline{P} . Then

the Smagorinsky model obtains the turbulent eddy viscosity from the strain rate magnitude,

$$\nu_t = (C_S \Delta)^2 |S| = (C_S \Delta)^2 \operatorname{tr}(2\overline{\boldsymbol{S}}^2)^{1/2}; \qquad (4)$$

the mixing length is the product of C_S , a model constant, and Δ , a measure of the local grid length scale; here $\Delta = (\Delta x \Delta y \Delta z)^{1/3}$.

There are few comparable experimental results for these Reynolds numbers, since while experimental correlations for the variation with Reynolds number of global flow measures such as Strouhal number and base pressure coefficient have been published [5], there have been comparatively few studies of the velocity field of the cylinder near wake. A cylinder-surface pressure distribution for a nearby Reynolds number (Re = 3000) is available [4]. Specifically for Re = 3900, Ong and Wallace [6] carried out a hotwire traverse study of the wake flow, but to avoid bias problems associated with flow reversal, they restricted attention to distances greater then 3D downstream of the cylinder centreline. Another study [3], reported by Beaudan and Moin [1], utilised PIV methods to collect near-wake data, although these appear to be somewhat unreliable, as they often display unacceptable deviations from expected wake-centreline symmetries.

Computational Methods

Our numerical methods have been previously described [2, 7, 8]; here we provide a brief review. The spectral element approximation employs a standard Gauss-Legendre-Lobatto collocation on two-dimensional elements, with the extension to three dimensions made by Fourier expansions in the cylinder-axis direction. The code employs a mixed explicit-implicit time integration scheme of second order, and is parallel across two-dimensional data planes—each process carries a subset of the total number of data planes. The block-structured finite volume calculation uses a centred second-order approximation for evaluation of all terms. Time-integration for the finite-volume method employs a second-order fullyimplicit technique based on the SIMPLE algorithm, and is parallelised across blocks; each process carries a block of data. Direct comparison of the two codes on problems with identical geometries and numbers of nodes have shown very similar ratios of CPU time to flow evolution time, but since the spectral element discretisation is higher-order, it would be expected to display greater spatial accuracy in a comparison of this kind.

Mesh outlines are shown in Figure 1. The overall dimensions of the meshes are somewhat different, but the key values of mesh cross-flow and axial extents are identical; $y = \pm 7D$ and $z = \pi D$ respectively. The spectral element mesh used 480 elements, each with 64 collocation points, providing each plane of data with 30,720 inde-



Figure 1: Simulation meshes (two-dimensional projections); (a), spectral element mesh, 480 elements; (b) finite volume mesh, 84 blocks. For both meshes, cross-flow and axial extents are $y = \pm 7D$, $z = \pi D$.

Source	St	C_{pb}	C_d
Norberg [4, 5]	0.210	-0.88	0.99
Spectral element LES	0.218	-0.93	1.01
Finite volume LES	0.242	-1.10	1.07

Table 1: Global flow parameters from simulations compared to experimental correlations.

pendent mesh nodes. There were 48 planes of data in the cylinder-axis direction and the total number of mesh points was 1,474,560. The finite volume mesh had 84 blocks, 32 axial planes of data, and a total of 855,040 control volumes. While the spectral element mesh has a greater number of mesh points, its streamwise extent is 48D as opposed to 19D for the finite-volume mesh, so the near-wake resolution for the two meshes may be considered comparable.

The turbulence model for both simulations used the same Smagorinsky constant $C_S = 0.1$, chosen primarily on the basis of previous LES calculations, which have used values in the range 0.065–0.1. For the spectral-element simulation, van-Driest type wall damping [2] was also incorporated to reduce the computed mixing lengths to zero as the cylinder wall is approached.

Results

The simulations were run until an approximately statistically-steady state had been achieved, after which collection of statistics was initiated. In the statistics presented below, the resolved velocity field is decomposed into ensemble-average and fluctuating components: $\overline{u} = U + u'$ (all components are normalised by the freestream value U_{∞}). The streamwise, crossflow and spanwise (x, y, z) components of \overline{u} are respectively $\overline{u}, \overline{v}$ and \overline{w} , and the coordinate system origin is located at the centre of the cylinder.

The most basic global parameter for the flow past a circular cylinder is the Strouhal number $St = fD/U_{\infty}$, where f is the centre frequency of cross-flow wake velocity and the fluctuating lift on the cylinder. Figure 2 illustrates the time series of spanwise-averge lift coefficient derived from the spectral element simulations. Two other global



Figure 2: Time series of coefficient of spanwise-average lift resulting produced by the spectral element simulation. The Strouhal number $St = fD/U_{\infty} = 0.218$.



Figure 3: Time-mean pressure coefficient on the cylinder surface as a function of angular position. —, spectral element simulation; -, finite volume simulation; \bullet , experimental results for Re = 3000 [4].

parameters of interest are the base pressure coefficient and the drag coefficient. Table 1 shows the values of these three coefficients from the two simulations, compared to experimental correlations of Norberg [4, 5]. For all three parameters the values from the spectral element simulation are closer to the experimental correlations than those from the finite volume simulation.

Next we examine the time-mean pressure distribution on the cylinder surface, presented in terms of the pressure coefficient $C_p(\theta) = 2[P(\theta) - P_{\infty}]/U_{\infty}^2$. A comparison between coefficients derived from the two simulations and experimental measurements conducted at Re = 3000 [4] is presented in Figure 3. All three sets of data show pressure minima at $\theta \approx 70^{\circ}$, and mean separation point near $\theta = 90^{\circ}$, however the spectral element results are closer to the experiental values than those derived from the finite volume simulations, in agreement with values presented in table 1.

In turning to the velocity-based statistics, we may compare to results from the Re = 3900 experiments of Lourenco and Shih [3] and Ong and Wallace [6]. The first point of comparision is the distribution of time-mean streamwise velocity component U on the wake centreline, shown in Figure 4. Where the experimental measurements overlap near x/D = 4, they are in reasonable agreement with one another, and with results from both simulations. Further upstream, where only Lourenco and Shih's results are available, the simulation results differ



Figure 4: Time-mean streamwise velocity on wake centreline. —, spectral element simulation; − –, finite volume simulation; •, experimental results [3]; ■, experimental results [6].

from one another, and the experimental values. Note that the experimental results show a positive streamwise velocity right at the base of the cylinder, which cannot be valid. The mean flow-reversal length is $x/D \simeq 1.7D$ for the experiments, 1.8D for the finite volume simulation and 2.1D for the spectral element simulation (which is similar to the Smagorinsky-model results of Beaudan and Moin [1]). Time-mean streamlines of the near-wake flows are presented in Figure 5, where the difference in streamwise extent of the recirculation bubbles reflects the difference in zero-crossing locations in Figure 4.

The streamwise distributions of the velocity component variances are presented in Figure 6. In this case, no directly comparable experimental results are available. Results from the two simulations are similar, although some differences are evident. The peak values for the spectral element simulation results occur somewhat further downstream than they do for the finite volume results, which is very likely related to the difference in mean recirculationbubble lengths. An interesting detail here is the fact that the crossflow and spanwise velocity fluctuations retain finite values until a quite thin boundary layer is entered just near the base of the cylinder.

Time-mean velocity component profiles at two streamwise locations are shown in Figure 7. In the more upstream location (x/D = 1.54), comparison can be made only with a single set of experimental results [3]. The differences in the minima of the streamwise velocity component U can be related to the differences in the centreline values observed in Figure 4. The experimental results here suggest a thicker wake than predicted by either of the simulations. The cross-flow velocities are somewhat different in detail at this location—also, note the lack of symmetry in the experimental values. At x/D = 3.00we can compare also to Ong and Wallace's hotwire measurements [6], but we note that (as is the case for most statistics at this overlap location) the two sets of experimental values are not in particularly good agreement.

We present in Figure 8 velocity variance and covariance profiles at x/D = 1.54 and 3.00. In general we expect more variability than for the previous comparisons, as



Figure 5: Streamlines of the time-mean flow field; (a), spectral element simulation; (b) finite volume simulation.



Figure 6: Variances of velocity components on wake centreline. (a), $\langle u'u' \rangle$, streamwise velocity component; (b), $\langle v'v' \rangle$, crossflow velocity component; (a), $\langle w'w' \rangle$, spanwise velocity component. —, spectral element simulation; – –, finite volume simulation.

these are higher-order statistics. At the more upstream of the two locations, many of the differences between the two sets of simulation and single set of experimental results can again be related to differences in the length of the mean recirculation bubble. At x/D = 3.00, the two sets of simulation results are generally in better agreement with one another than the two sets of experimental results, with the exception of the crossflow variance $\langle v'v' \rangle$, in which case the two sets of simulations, but there is a distinct difference between experiment and simulation.



Figure 7: Time-mean velocity profiles at two downstream locations, x/D = 1.54 and x/D = 3.00. —, spectral element simulation; –, finite volume simulation; •, experimental results [3]; •, experimental results [6].

Discussion and Conclusions

It is difficult to decide which of the simulations or experiments examined here provides the most reliable statistics for this flow. The apparent lack of agreement between the two sets of experimental measurements used as bases for comparison of wake velocity data is rather surprising, but this flow is notorious for its sensitivity to variations in Reynolds number, boundary conditions and turbulence in the oncoming flow. Reliable experimental results will greatly assist future validation studies.

On the basis of the generally better agreement of the results from the spectral element simulation to the experimental correlations for global parameters such as Strouhal number, it appears likely that the spectral element simulation is somewhat more reliable than the finite volume discretisation, at least with the spatial resolutions and turbulence model employed here. Simulation results presented in [1] suggest some sensitivity to the value of C_S employed, and we plan to investigate this aspect in future work. In the final analysis, the results presented suggest that both of the simulation methodologies presented hold promise for large eddy simulation of turbulent flows in complex geometries.

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Figure 8: Velocity (co-)variance profiles at x/D = 1.54 and x/D = 3.00. —, spectral element simulation; – –, finite volume simulation; •, experimental results [3]; •, experimental results [6].