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Validation criteria for DNS of turbulent heat transfer in pipe flow

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Abstract

Direct numerical simulation (DNS) of turbulent flow and heat transfer involves directly solving the unsteady Navier-Stokes and thermal energy equations without considering any assumptions about the physics and resolve all the scales of the flow, including the energy and dissipation spectral peaks. Data from DNS of fully developed turbulent pipe flows subjected to a constant surface heat-flux are used to validate with the existing DNS database. The present validation process involves matching of first and second order turbulence statistics. However, accurate comparison of DNS data sets requires satisfying the conformity of necessary and sufficient computational parameters. This paper will address the influence of the computational conditions on the thermal statistics, which affect the comparison of the matching statistics from different DNS data sets. It was observed that apart from the governing parameters (Kármán and Prandtl numbers), computational mesh and domain size significantly influence the resulting thermal statistics, which must be considered while performing validation of DNS data.

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Keywords: Turbulent pipe flow; heat transfer; Prandtl number; Kármán number; domain length.

1. Introduction

Turbulence modelling possesses one of the main sources of inaccuracies in the numerical predictions of flow problems that are principally afflicted with errors, although the modelling of the near wall turbulence plays a dominant role in convective momentum, heat and mass transfer. Moreover, the current knowledge of the transport mechanism in the wall region is neither complete nor satisfactory, owing to various difficulties in measuring fluctuating turbulent quantities in the very thin boundary layer adjacent to the wall. Simulation (for example, DNS) databases are superior to experimental data as it enables thorough analysis of the turbulent flow structures with all the instantaneous flow variables [1]. They are also very useful for engineering turbulence modelling, since they provide quantitative data that cannot be obtained experimentally with confidence, e.g., each budget term in the transport equations of the Reynolds stress.

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Nomenclature

D	pipe diameter
$\boldsymbol{F}^+$	driving force vector
L	streamwise domain length
N	non-linear terms in the Navier-Stokes equation
N_x	number of nodes in streamwise direction
N_r	number of nodes in radial direction
N_θ	number of nodes in azimuthal direction
P'^+	fluctuating kinematic pressure normalized by fluid density and friction velocity
Pr	Prandtl number
q_w	constant heat flux
r	radial direction
Re_D	Reynolds number based on bulk velocity
Re_τ	Reynolds number based on friction velocity or Kármán number
t	(dimensionless) time normalized by friction velocity and pipe diameter
$\boldsymbol{u}^+$	velocity vector normalized by friction velocity
u_r	velocity in radia direction
u_x	velocity in streamwise direction
u_θ	velocity in azimuthal direction
x	streamwise direction
δ	pipe radius
θ	azimuthal direction
Θ^+	(dimensionless) temperature normalized by friction temperature

Code validation of a model is the process of determining the degree of accuracy of the mathematical assumptions and the effectiveness of the discretisation schemes. A model can only be validated for a specific range of applications for which there is availability of either experimental or highly accurate numerical data. Numerical *prediction* beyond the region of validity greatly depends on the successful validation of the model. Validation checks whether the computational models solved by the CFD code agree with real world observations. The aim is to identify and determine error and uncertainty in the simulation results. Moreover, the experiment data sets are not free from bias errors and random errors which need to be taken into account. Depending on the type of the application, the accuracy required in the validation activities can be varied.

Turbulent heat transfer in pipe flow, although essentially important in a large range of engineering applications, was not considered by many DNS investigations. Only Satake et al. [2], Piller [3] and Redjeem-Saad et al. [4] carried out DNS of turbulent heat transfer in pipe flows; however, any of them never put emphasis on code validation. There are few methods available to provide confidence in the DNS. Initially, the procedures follow a grid-independence test and after that, examination of the general characteristics of the flow and thermal statistics along with comparison of results in the open literature. The main objective of this paper is to present the criterion involved in validating DNS data with available numerical data.

2. Mathematical Model

The schematic diagram of a heated pipe along with cylindrical coordinate system is shown in Fig. 1. The fluid flow is considered to be turbulent and fully-developed, and the incompressible Newtonian fluid is heated with a uniform heat flux q_w imposed at the pipe wall. The fluid properties are assumed to be constant and the temperature is considered to be a passive scalar. The three-dimensional incompressible Navier-Stokes equations in cylindrical coordinates can be written in non-dimensional form as

$$\frac{\partial \boldsymbol{u}^+}{\partial t} + N\left(\boldsymbol{u}^+\right) = -\nabla P'^+ + \frac{1}{2Re_\tau}\nabla^2 \boldsymbol{u}^+ + \boldsymbol{F}^+, \tag{1}$$

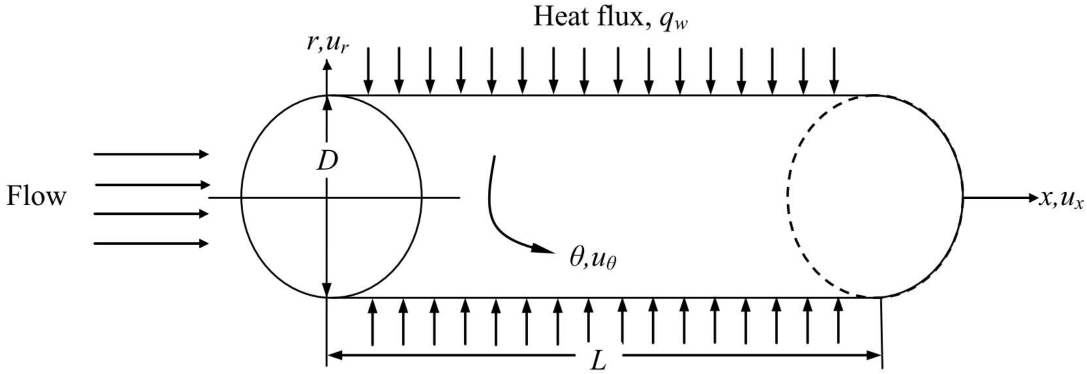

Fig. 1. Schematic of heated smooth wall pipe flow configuration.

$$\nabla \cdot \boldsymbol{u}^+ = 0. \tag{2}$$

Here $\boldsymbol{F}^+ = [4, 0, 0]$. In the present formulation, $N(\boldsymbol{u}^+)$ is implemented in skew-symmetric form for robustness,

$$N(\boldsymbol{u}^+) = 0.5\left[\boldsymbol{u}^+ \cdot \nabla \boldsymbol{u}^+ + \nabla \cdot \boldsymbol{u}^+ \boldsymbol{u}^+\right]. \tag{3}$$

The transport equation for passive scalar (temperature) is governed by the dimensionless energy equation for the thermal field in the form of an advection-diffusion problem

$$\frac{\partial \Theta^+}{\partial t} + \boldsymbol{u}^+ . \nabla \Theta^+ - 8\frac{Re_\tau}{Re_D} u_x^+ = \frac{1}{2Re_\tau Pr} \nabla^2 \Theta^+. \tag{4}$$

One needs to know Re_D in advance in order to integrate (4), i.e. the solution to the Navier-Stokes equations (1) needs to have reached statistically steady state in order to evaluate Re_D and maintain a constant ensemble-average scalar value at each point.

We use a cylindrical-coordinate spectral element/Fourier spatial discretization technique which combines the geometric flexibility of finite elements with the high accuracy of spectral methods. The present implementation is a modified version of the open source DNS code 'Semtex' written by Blackburn and Sherwin [5] which is able to solve time-varying Navier-Stokes problems along with passive scalar transport in both Cartesian and cylindrical coordinates using Fourier expansion functions for spatially-periodic directions. Parametrically mapped quadrilateral elements having tensor product Gauss-Lobatto-Legendre (GLL) Lagrange interpolants within each element are employed by this code to discretise the meridional semi-plane in order to achieve spectral accuracy. Fourier expansions are used in the azimuthal direction, which is possible because the domains are axisymmetric.

3. Results and Discussions

3.1. Dependence on Kármán numbers

It is important to maintain the essential parameters fixed while performing the validation of turbulent flow and heat transfer with other data sets. The first is the Kármán number, Re_τ, which significantly affects the flow and thermal statistics. Satake et al. [2] performed DNS of turbulent heat transfer inside a pipe for a range of Kármán numbers ($150 \leq Re_\tau \leq 1050$). For $Pr = 0.71$, they found that the mean and fluctuating temperature largely depended on the Kármán numbers. The rms of temperature fluctuation shifted away from the pipe center with increasing Kármán numbers. Hence, it is to maintain constant Kármán numbers in order to perform validation simulation.

3.2. Dependence on Prandtl numbers

The next essential parameter is the Prandtl number which is characterised by the thermo-physical properties of the flowing fluid. The change of Prandtl numbers within the range, $0.025 \leq Pr \leq 10$, are relevant to practical

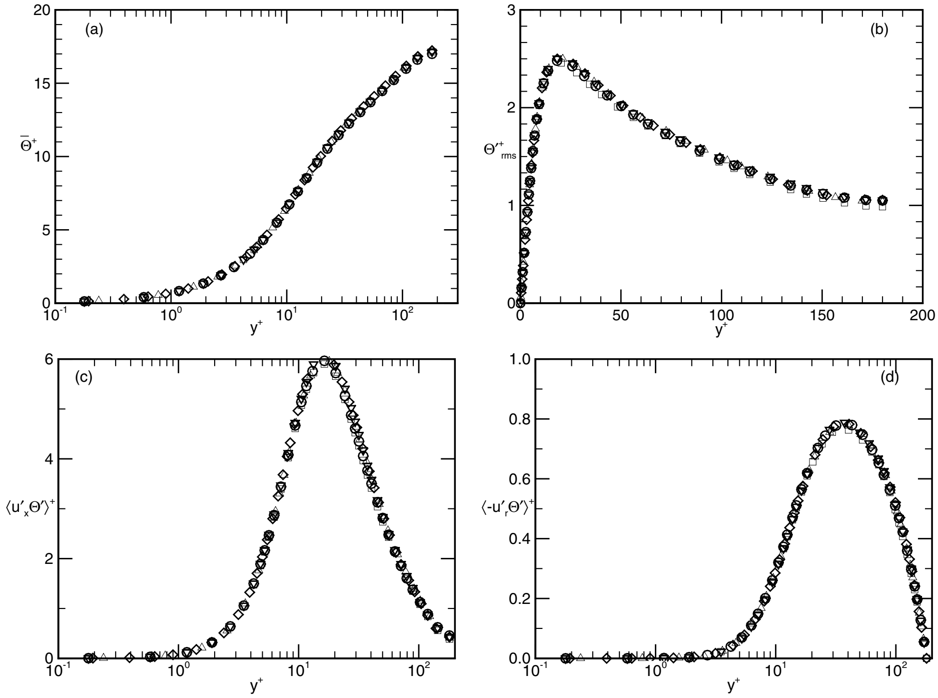

Fig. 2. Comparison of thermal statistics for different grid resolution. Symbols are as in table 1. Profiles of (a) mean temperature, (b) rms of fluctuating temperature, (c) streamwise turbulent heat flux, and (d) radial turbulent heat flux.

problems involving heat exchange in fluids such as liquid mercury (Pr = 0.025), air (Pr = 0.71), liquid CO_2 (Pr = 2.0) and water ($Pr \approx 5.0$ to 10.0 at $5 \sim 30^o$ C). Hence the properties of flowing fluid are significantly altered by the variation of Prandtl numbers. Using DNS of turbulent heat transfer in pipe flow, Redjeem-Saad et al. [4] performed a systematic study of the effect of Prandtl numbers ($0.026 \leq Pr \leq 1.0$) on thermal statistics and observed that mean and fluctuating temperature and turbulent heat fluxes were a strong function of Prandtl numbers. The peak value of fluctuating temperature and turbulent heat fluxes change rapidly with the change in Prandtl number and as a result, it is very important to maintain exact value of Pr for code validation.

3.3. Dependence on domain length

One of the sufficient parameters is the computational domain length which requires to be sufficiently large enough to achieve converged thermal statistics. Saha et al. [6] investigated the influence of pipe length on different thermal statistics computed from DNS data and concluded that the convergence of first and second order statistics relied on the selection of minimum pipe length for each case of Re_τ and Pr. Their investigation revealed that a minimum pipe length of $4\pi\delta$ is sufficient to achieve the convergence of first and second order thermal statistics for the range of parameters studied in the paper.

Table 1. Specification for the test of grid resolution at fixed pipe length of $4\pi\delta$, Re_τ = 180 and Pr = 0.71.

Case	Pr	Re_τ	L/δ	N_x	N_r	N_θ	Δx^+	Δy^+_{min}	Δy^+_{max}	$\Delta(\delta\theta)^+$	$\Delta t u_b D/(u_\tau L)$	Symbol
A	0.71	180	4π	151	81	128	14.9	0.178	5.589	8.8	40	□
B	0.71	180	4π	321	61	128	7.0	0.237	6.92	8.8	40	△
C	0.71	180	4π	321	81	128	7.0	0.178	5.589	8.8	40	○
D	0.71	180	4π	321	121	128	7.0	0.06	3.73	8.8	40	◊
E	0.71	180	4π	451	81	128	5.0	0.178	5.589	8.8	40	▽

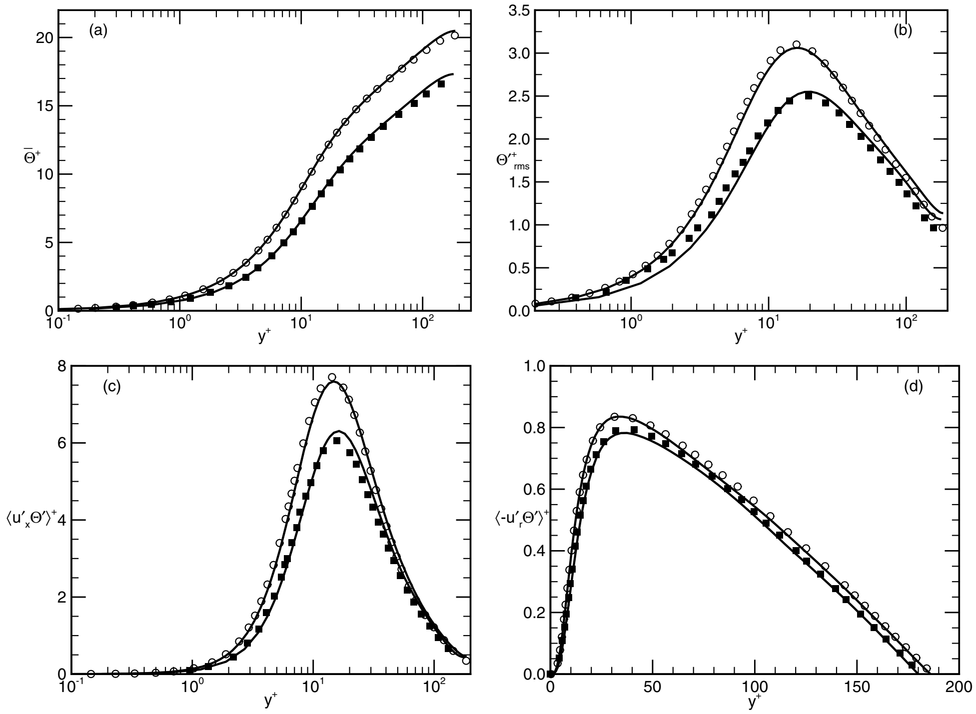

Fig. 3. Comparison of profile data for the smooth-walled pipe (solid lines) with DNS data sets of [2,4]. The symbols used are the same for three different cases: current (−), Satake et al. [2] (■) and Redjem-Saad et al. [4](○).

Table 2. Summary of computational condition and grid resolution of DNS data for turbulent heat transfer used for code validation.

Previous DNS	Re_τ	Pr	L/δ	N_x	N_r	N_θ	Δx^+	Δy^+	$\Delta(\delta\theta)^+$	Method
Satake et al. [2]	180	0.71	15	256	128	128	10.5	0.29 ∼ 1.04	8.84	FVM
Current	180	0.71	8π	501	81	256	9.0	0.178 ∼ 5.589	4.4	SEM
Redjem-Saad et al. [4]	186	1.0	15	257	129	129	10	0.01 ∼ 5	10	FDM
Current	186	1.0	15	301	121	128	9.3	0.061 ∼ 3.85	9.13	SEM

3.4. Dependence on grid resolution

In order to investigate the influence of spatial grid resolution on turbulence statistics, we select the computational length to be $4\pi\delta$ at $Re_\tau = 180$ and $Pr = 0.71$. Different cases considering the variation of grid resolution in streamwise and radial directions are summarised in table 1. Our reference case is C. Three cases having one coarse (case B) and one finer mesh (case D) of non-uniform distribution of grid points in the wall-normal direction are chosen here to compare the effect of grid resolution in radial direction. On the other hand, three cases A (coarse mesh), C and E (finer mesh) are employed to investigate the influence of streamwise grid resolution on the DNS data.

The mean temperature profiles for five different grid resolutions are shown in Fig. 2(a). The inner (in wall units, y^+) representation of mean temperature merge to a single curve indicating negligible influence of grid resolution in both streamwise and wall-normal directions. The data suggests that even a relatively coarse grid can produce accurate first order mean statistics from DNS. The second order statistics such as fluctuating temperature further show the grid resolution effects. Figure 2(b) present root-mean-square (rms) profiles of fluctuating temperature in linear wall-normal scaling. It is found that case A (coarse streamwise grid resolution) slightly fails to coincide with other cases in the outer-region. Moreover, the data generated from coarser grids in the streamwise direction underestimates the peak value of Θ'^{+}_{rms} compared with the results from finer grids. The turbulent heat flux profiles are plotted in Fig. 2. All these profiles agree closely to each other showing that the grid resolution has less effect for correct computation the first and second order statistics.

3.5. Validation with other DNS data

Due to lack of openly available experimental data, the validation of the present DNS data has been carried out by comparing the thermal statistics with those of other DNS as shown in table 2. There is a significant difference of the domain length and grid resolution between the current and previous DNS data (table 2) whereas there is a good match in the essential parameters, Re_τ and Pr. All previous DNS employed either finite volume method (FVM) or finite difference method (FDM) as the discretisation technique. In order to reveal the influence of a different discretisation method (SEM), we have further decided to compare the first and second order thermal statistics with DNS results of [4] by considering the same domain length $L = 15\delta$ along with a closely match grid resolutions.

The normalized mean temperature profiles $\bar{\Theta}^+$ for various Prandtl numbers are compared with existing DNS data as shown in Fig. 3(a). The temperature profiles at $Pr = 0.71$ show a similar trend for existing DNS data. For high Prandtl number ($Pr = 1.0$), the comparison of $\bar{\Theta}^+$ profiles between the present simulation and [4] clearly shows excellent agreement, as expected due to the similar computational conditions. The second order statistics such as rms of temperature fluctuation and turbulent heat fluxes are compared with previous DNS results in Figs. 3(b)-(d). Data from the present simulation agree well with DNS data available in the open literature, mostly near the peak location of statistics. The main distinctions of the peak value of radial heat flux at low Pr are due to the difference of the grid resolution and computational pipe length. Moreover, due to the different numerical schemes to overcome the effect of the singularity at the pipe centre, some discrepancies in the DNS results appear near the centre of the pipe.

4. Conclusion

The validation criteria for DNS of turbulent heat transfer in pipe flow reveals the accurate selection of computational condition. Parameters such as pipe length and mesh resolution should be sufficiently large enough to achieve convergence of the turbulent statistics. Moreover, it helps us to choose optimum DNS parameters in order to reduce the computational cost without affecting the accuracy of the model. The statistical results obtained from DNS of turbulent flow and heat transfer in smooth-wall pipe are seen to be in good agreement with data available in the open literature when the parameters are matched closely.

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