Modelling Paper Tensile Strength from the Stress Distribution along Fibres in a Loaded Network

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Tensile strength of paper

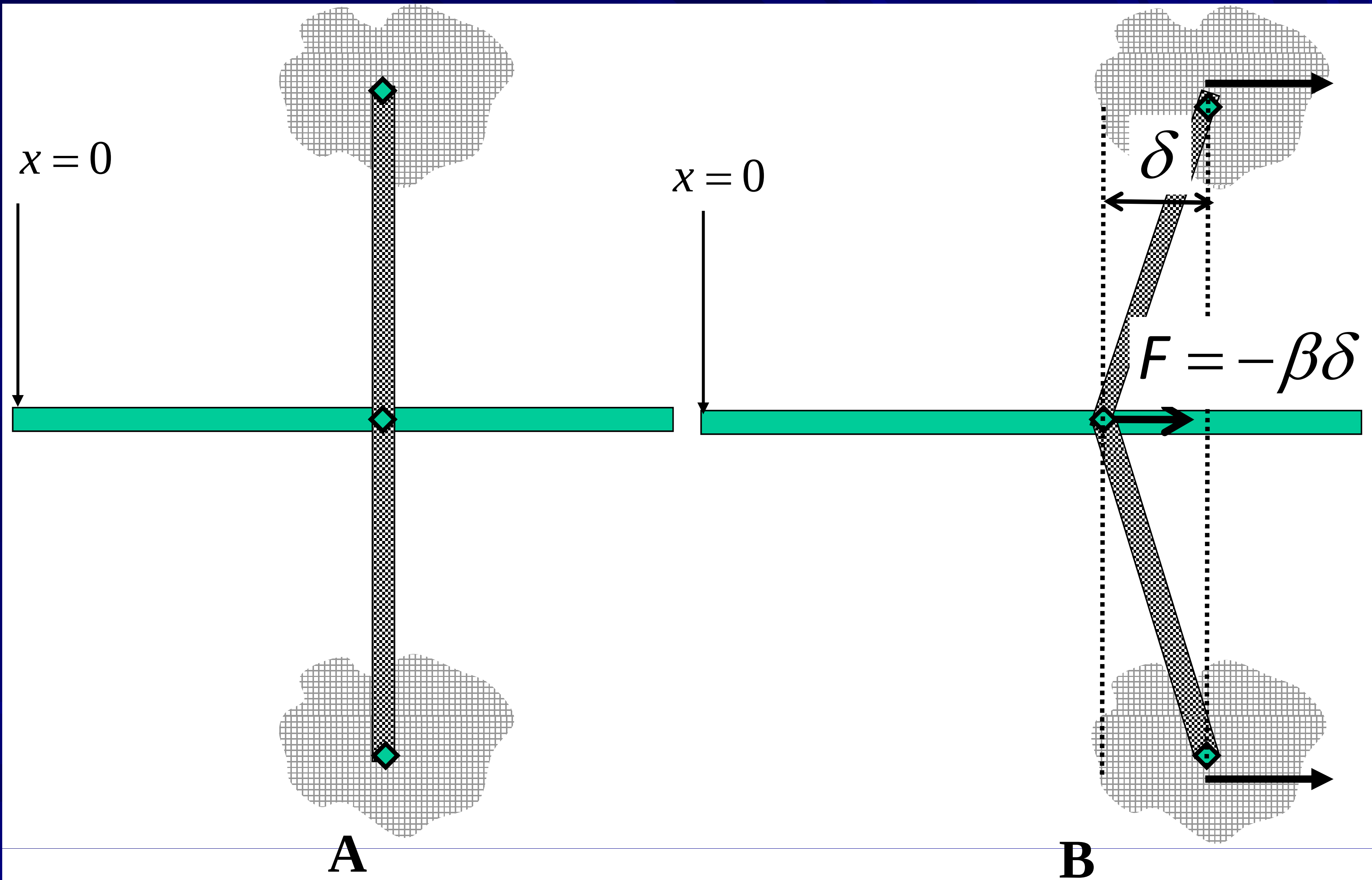
- Important to predict
- Approaches
  - Analytical models
  - Finite Element simulations of low density sheets

Problems with strength modelling

- Qualitative not quantitative predictions
- Many variables difficult to measure
- Unverified assumptions about fracture process
- Simplified models with average fibre properties

New approach presented here

- Model paper behaviour with single fibre stress development
  - Usually fibres in stress direction
- Micro-Macro approach
  - Single fibre of interest
  - Discrete contacts transferring stress from solid Matrix
  - Apply strain to matrix
  - Cox type shear lag assumption of stress transfer at each contact

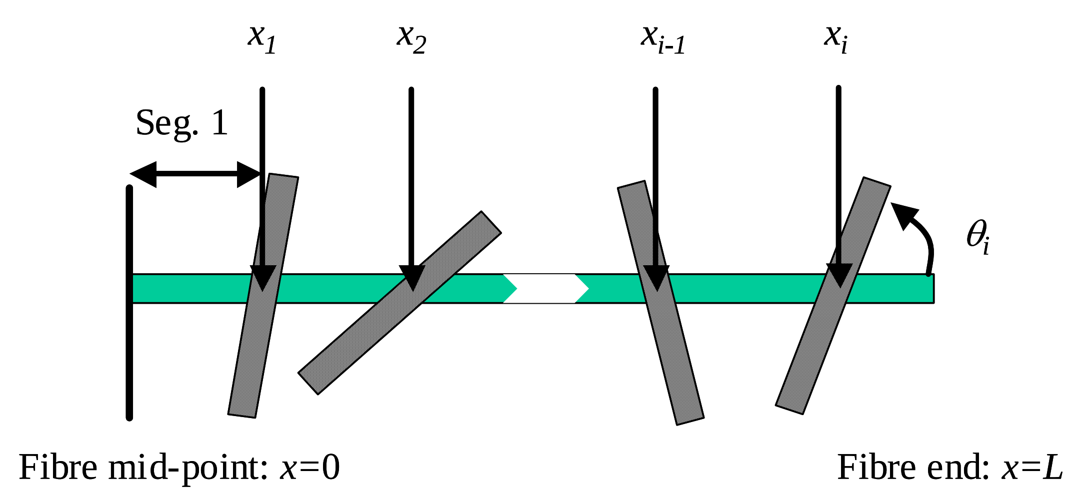


$x = 0$
$x = 0$
δ
$F = -\beta\delta$
A
B

The Problem!

- Consider steps: ONE contact with $F \propto \delta$
  – Strain Matrix
  – Displaces contact
  – Produces Force in fibre
  – Force reduces contact displacement
  – Reduces force etc etc..
- Equilibrium: force produces displacement required to generate force
- How do find displacements for multiple contacts??

Solution



$$\delta_{n-1} = \delta_n - \frac{(x_n - x_{n-1})}{E_n A_n}\left(\sum_{j=n}^{j=i} \beta_n \delta_n - \varepsilon\right)$$

E_n : Elastic Modulus of nth segment

A: Cross-sectional area of nth segment

ε: Matrix strain

- Express δ at each contact in terms of displacement at last contact
- Solve to obtain all displacements

Equation to solve for three contacts

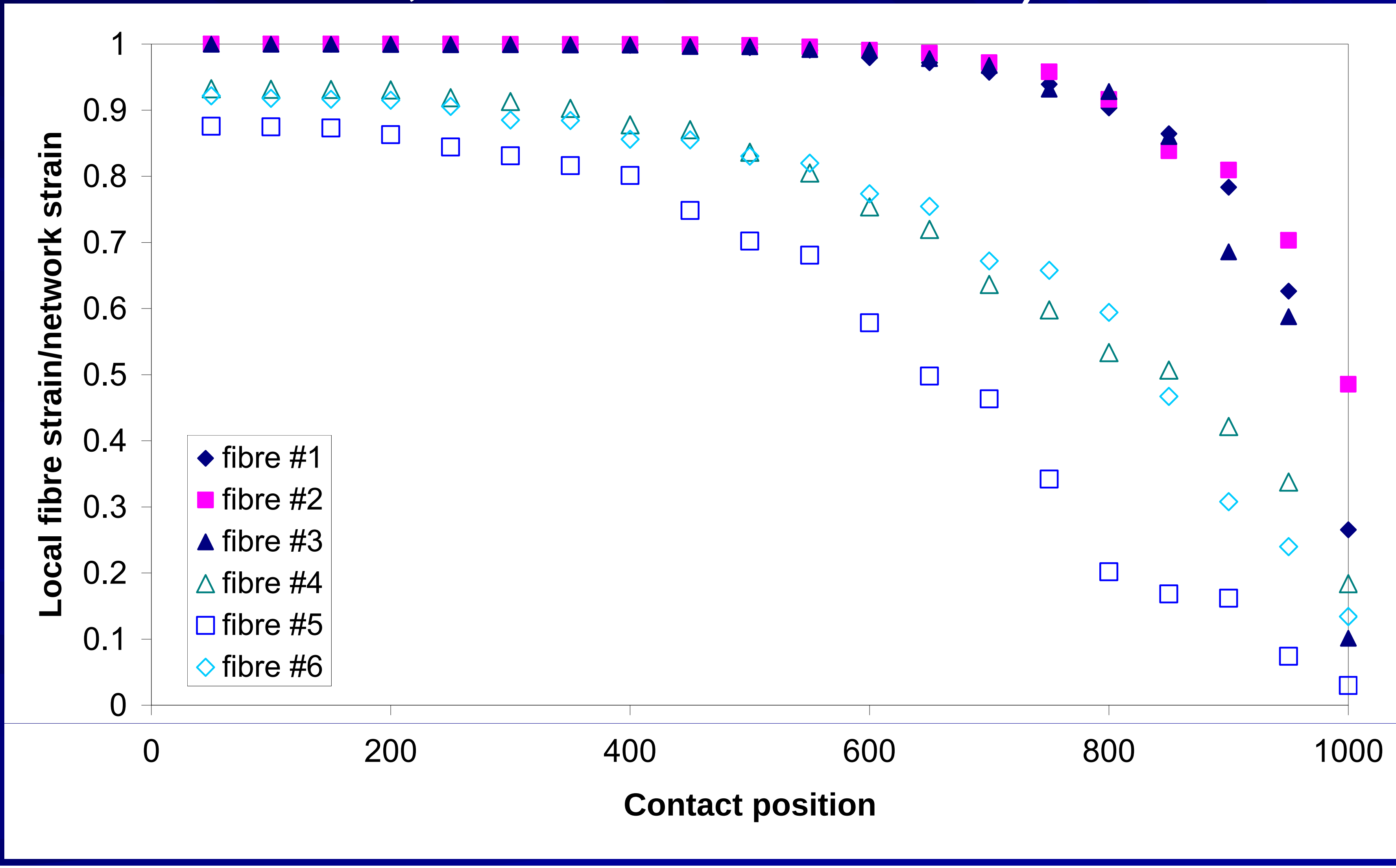
- Complexity increases as power to the number of contacts, *i*.
- More than four contacts- very tedious to write
- Instead solve symbolically using Matlab

$$0 = x_1\beta_1'\left(\begin{array}{l}\delta_3 + (x_3 - x_2)(\varepsilon + \beta_3'\delta_3) + \\ (x_2 - x_1)\left(\varepsilon + \left(\beta_3'\delta_3 + \beta_2'\left(\delta_3 + (x_3 - x_2)(\varepsilon + \beta_3'\delta_3)\right)\right)\right)\end{array}\right)$$

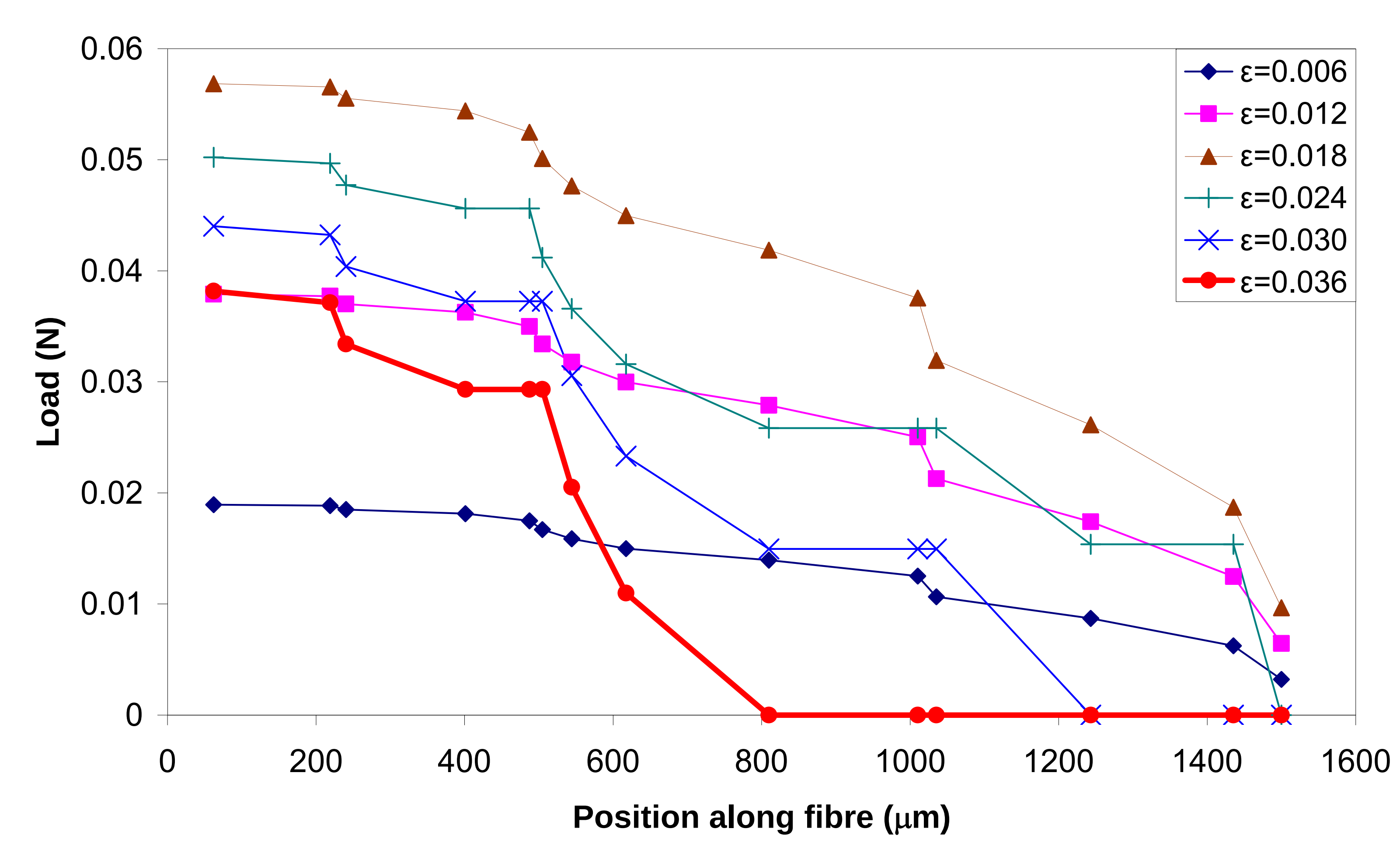
$$+x_2\beta_2'\left(\delta_3 + (x_3 - x_2)(\varepsilon + \beta_3'\delta_3)\right) + x_3\beta_3'\delta_3 - \varepsilon x_3 - \delta_3$$

$$\beta' = \beta / EA$$

An example: fixed contact positions, randomly varying $\beta^{/}$ (0-10,000 fibres 1-3, 0-1000 fibres 4-6)



Example: stress development in a fibre. Randomly placed contacts. Contacts removed when load exceeds bond strength of bond



- Advantages of method
  - Random contact positions
  - *β* can vary contact to contact
  - Elastic modulus, X-section area can vary segment to segment
- Disadvantages of method
  - Linear elastic only
- Data needed:
  - *β, E, A*
  - Bond strength
  - Fibre contact positions

Comparison Experimental Data

- Radiata pine
- Never dried
- Unbleached
- Kappa number 30
- Fibre dimension variation: cutting and fractionation

Samples

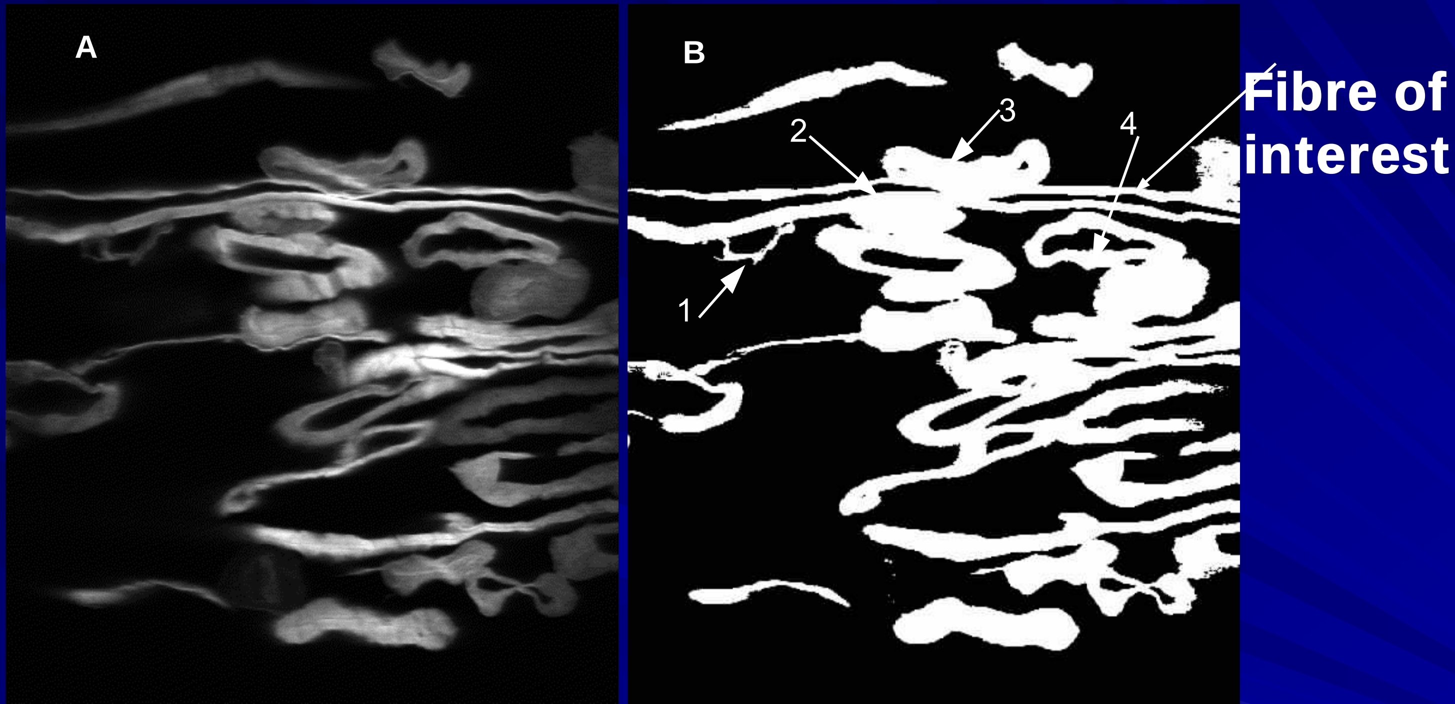
Label	Length weighted fibre length (mm)	Fibre wall area (μm^2)	Pressing Levels (Contacts)
L0	3.14	203	Middle (P3)
L1	2.53	204	Middle (P3)
L2	2.12	196	Middle (P3)
L3	1.79	196	Middle (P3)
Accepts	3.00	220	P1, P3, P5
Rejects	3.34	193	P1, P3, P5

→ Hydrocyclone fractionation

Measurements

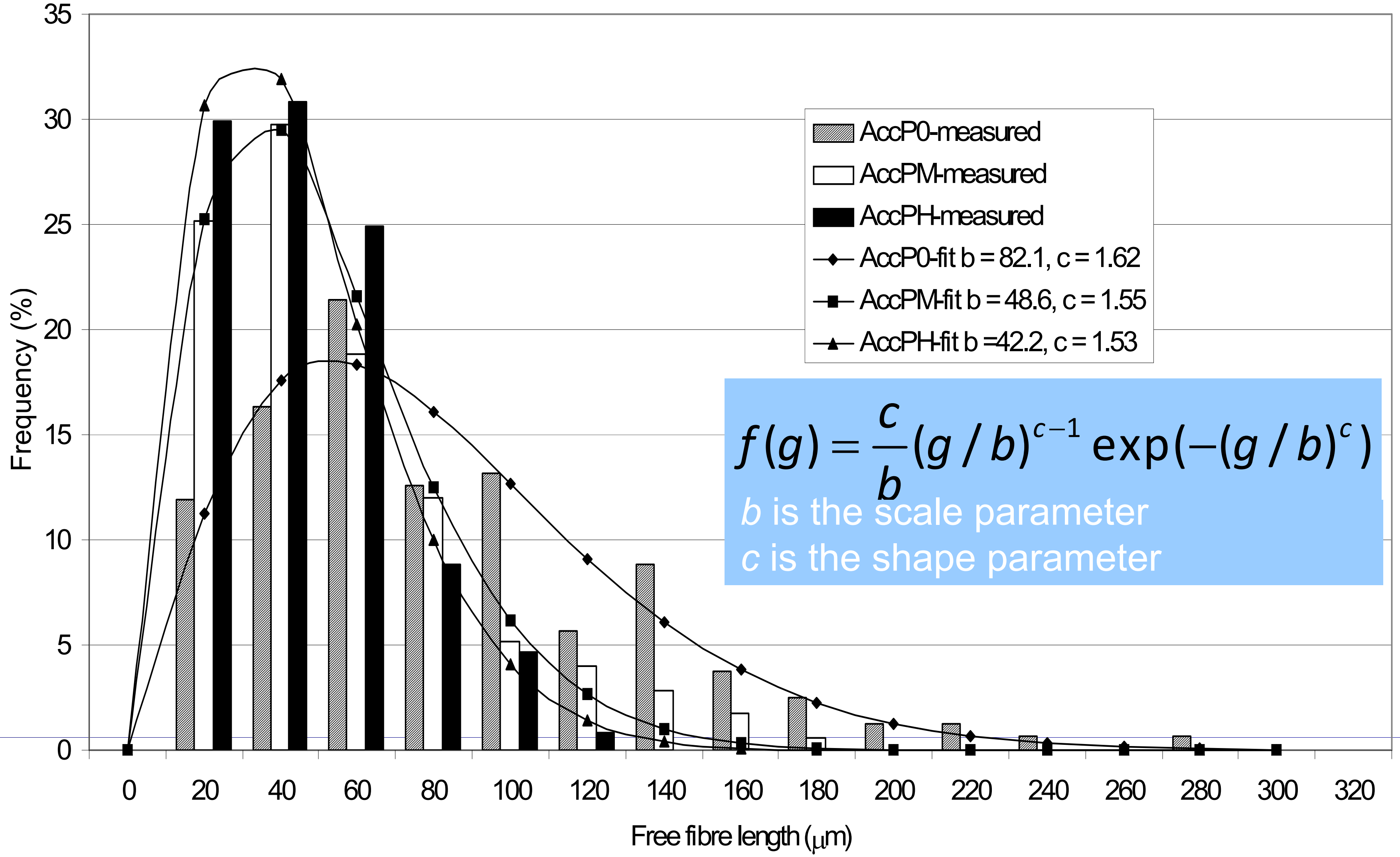
- Sheet density, elastic modulus, tensile strength
- Fibre shape (fill factor), cross-sectional area, length
- Fibre contacts:
  – distances between contacts ,
  – Weibull distributions of contacts
  – Full / partial contacts

Contact measurements



Cross-section image before (A) and after (B) thresholding and binarisation. Fibre 2 and 3 in (B) make two full contacts, fibre 1 makes a partial contact, and fibre 4 is not in contact with the fibre of interest.

Distance between contact frequency distribution



$$f(g) = \frac{c}{b}(g/b)^{c-1}\exp(-(g/b)^{c})$$

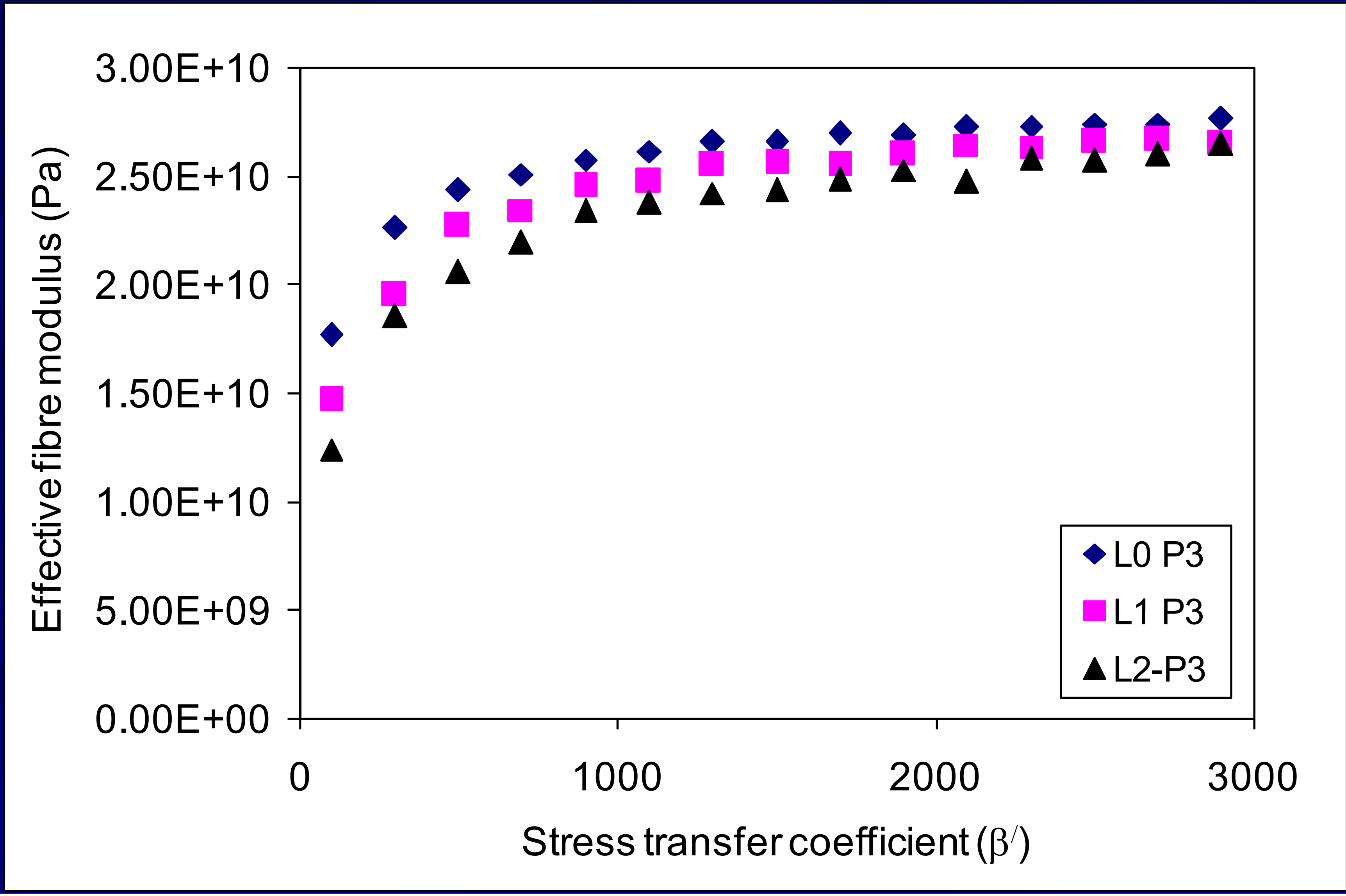
b is the scale parameter

c is the shape parameter

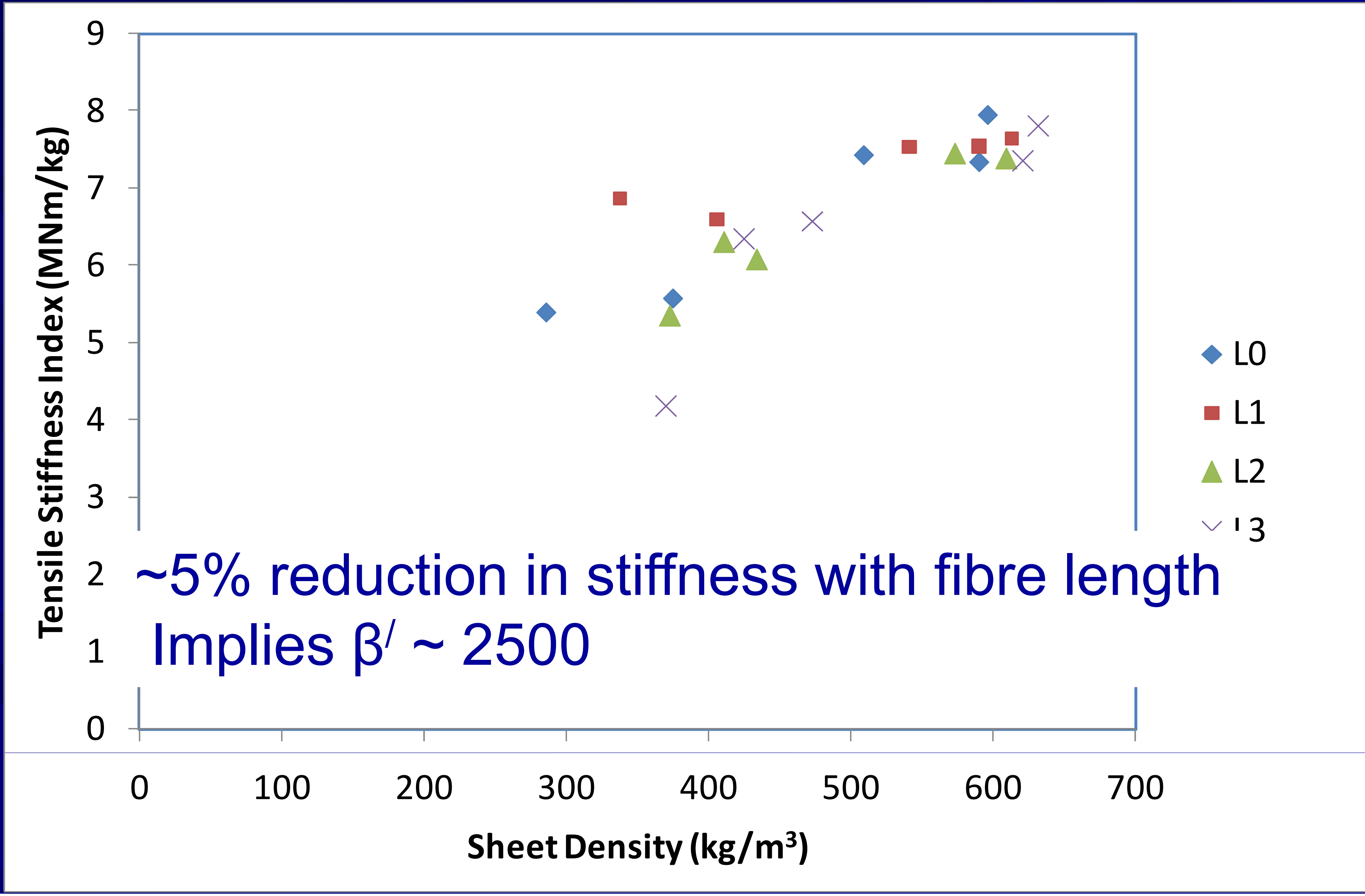
Average elastic modulus along fibre

- Average: 30 simulations per point
- Fibre elastic mod: assumed 30 Gpa
- Effective elastic modulus: average load along fibre/X-section area/matrix strain
- Effective elastic mod=30 Gpa
  - – Perfect stress transfer

Average elastic modulus along fibre



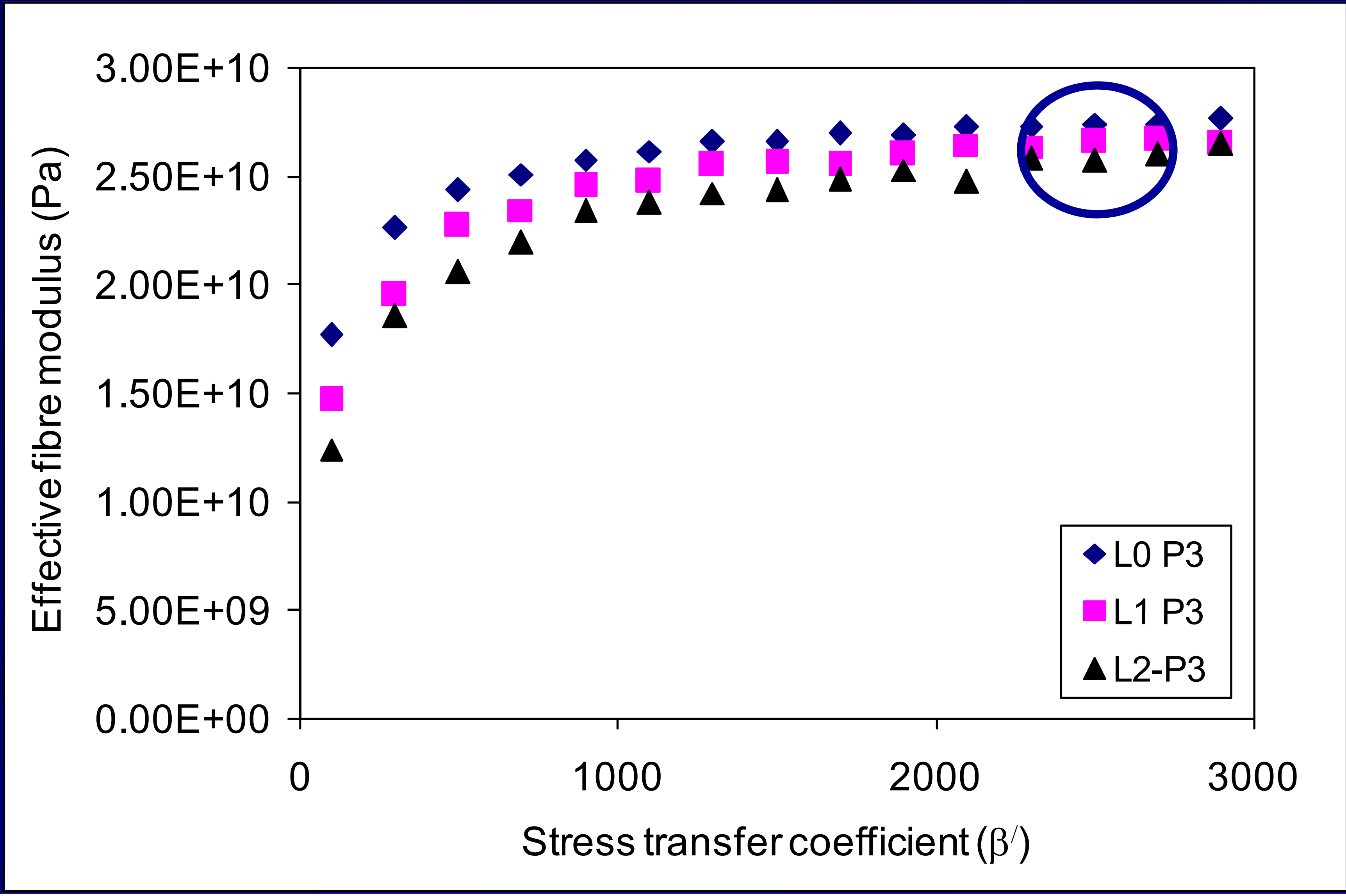
Elastic Stiffness Index for $\beta^{/}$



~5% reduction in stiffness with fibre length

Implies $\beta^{/} \sim 2500$

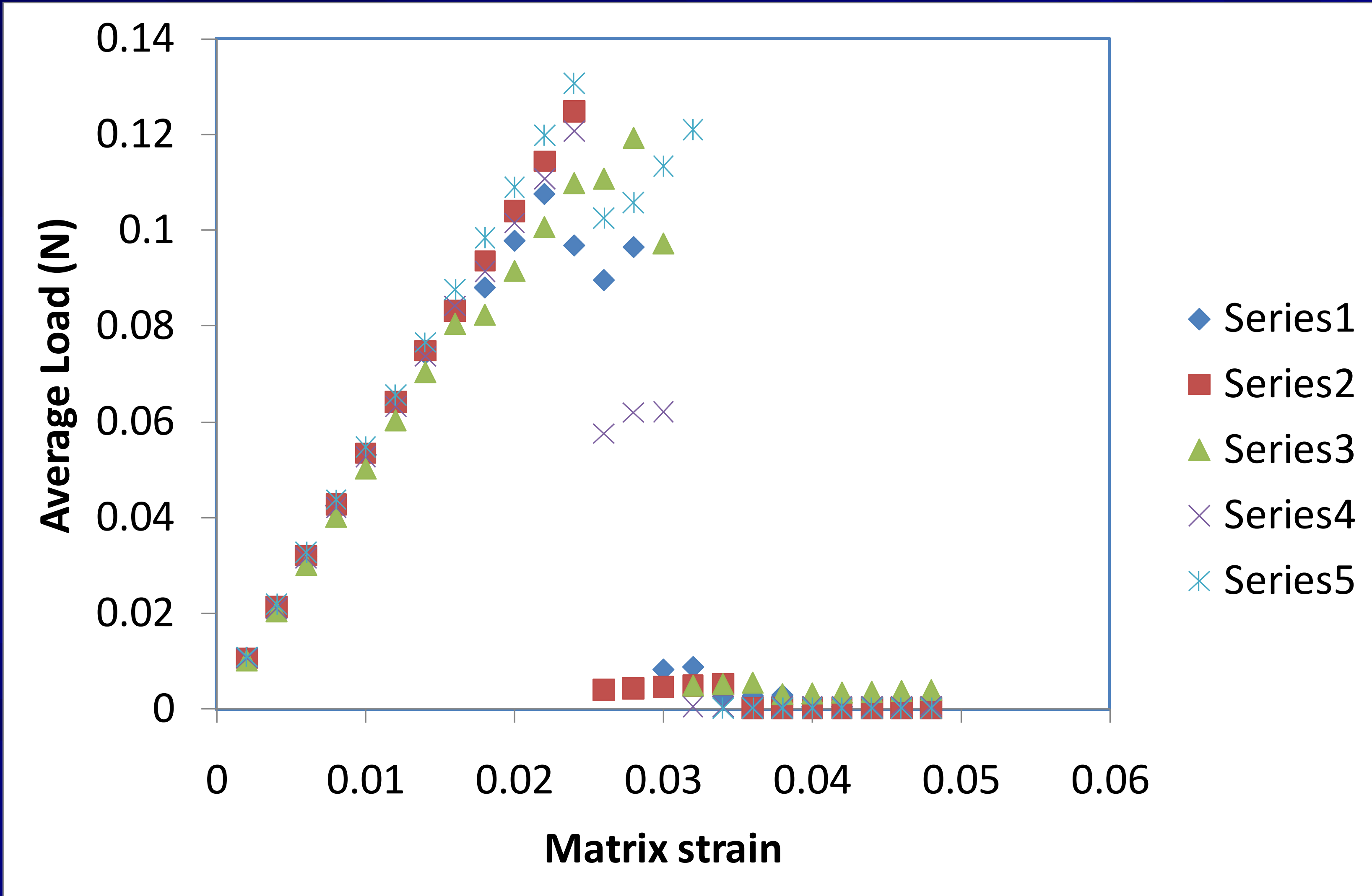
Average elastic modulus along fibre



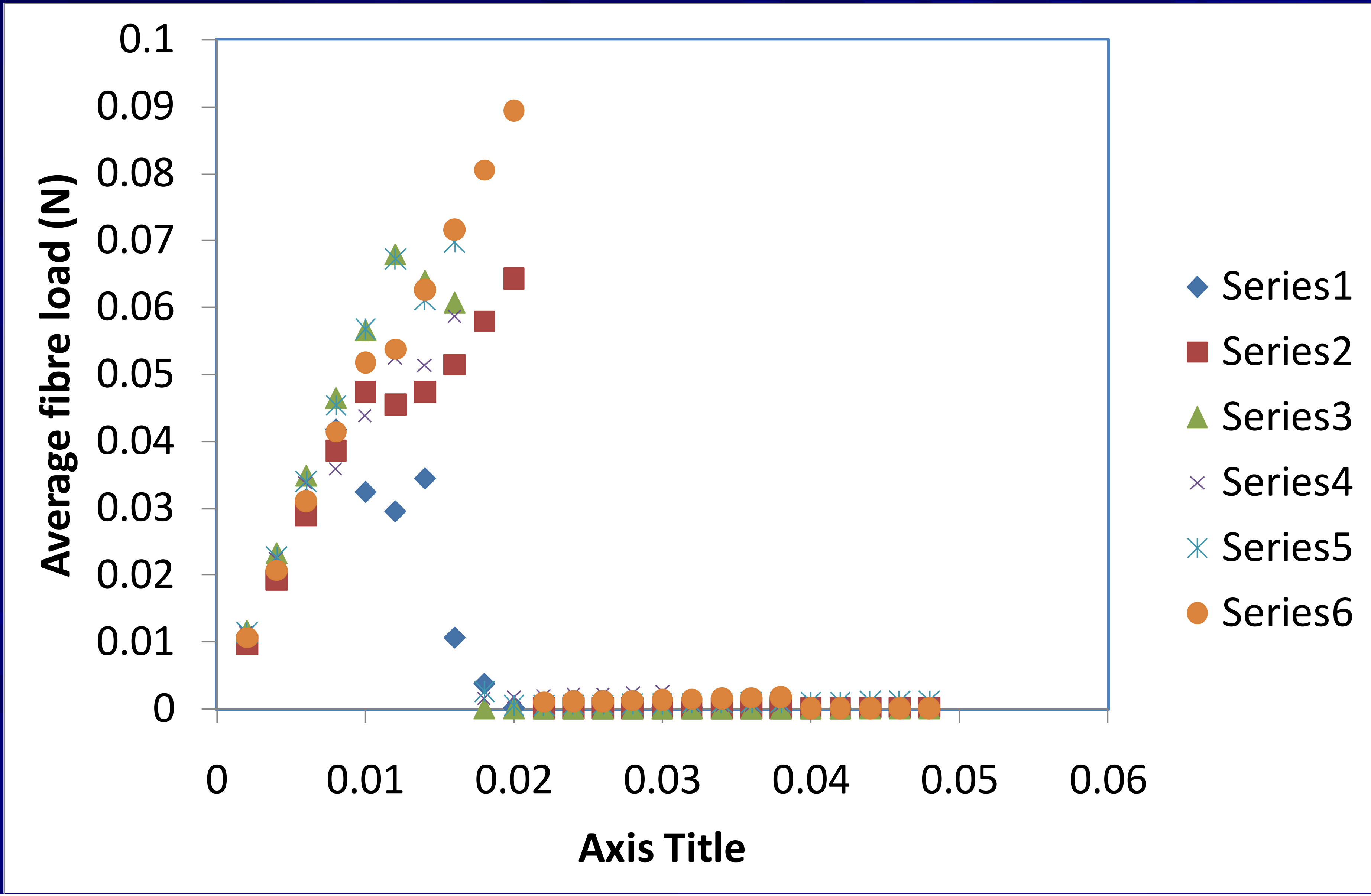
Strength modelling

- $\beta' = 2500$
- Bond strength= 25 MPa (IPPC 2007)
- Average 30 simulations with measured fibre contact statistics
- Fibre elastic modulus= 30 GPa
- Contacts
  - Assumed both full, partial contacts contribute
  - But bond breaking load scaled to contact area
- Fibre fracture not considered
- Model fibres in stress direction only

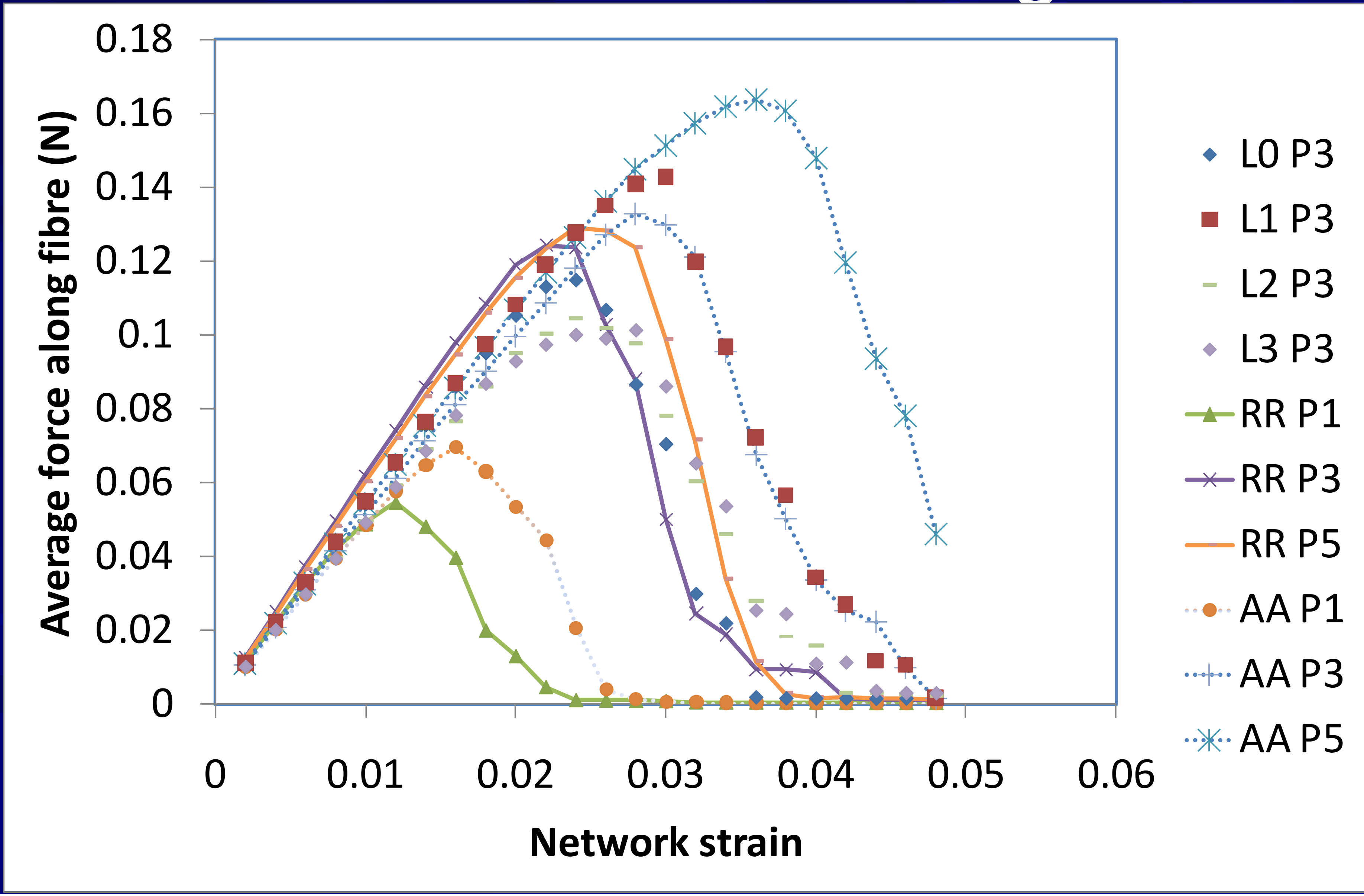
Six simulations for L0 P3



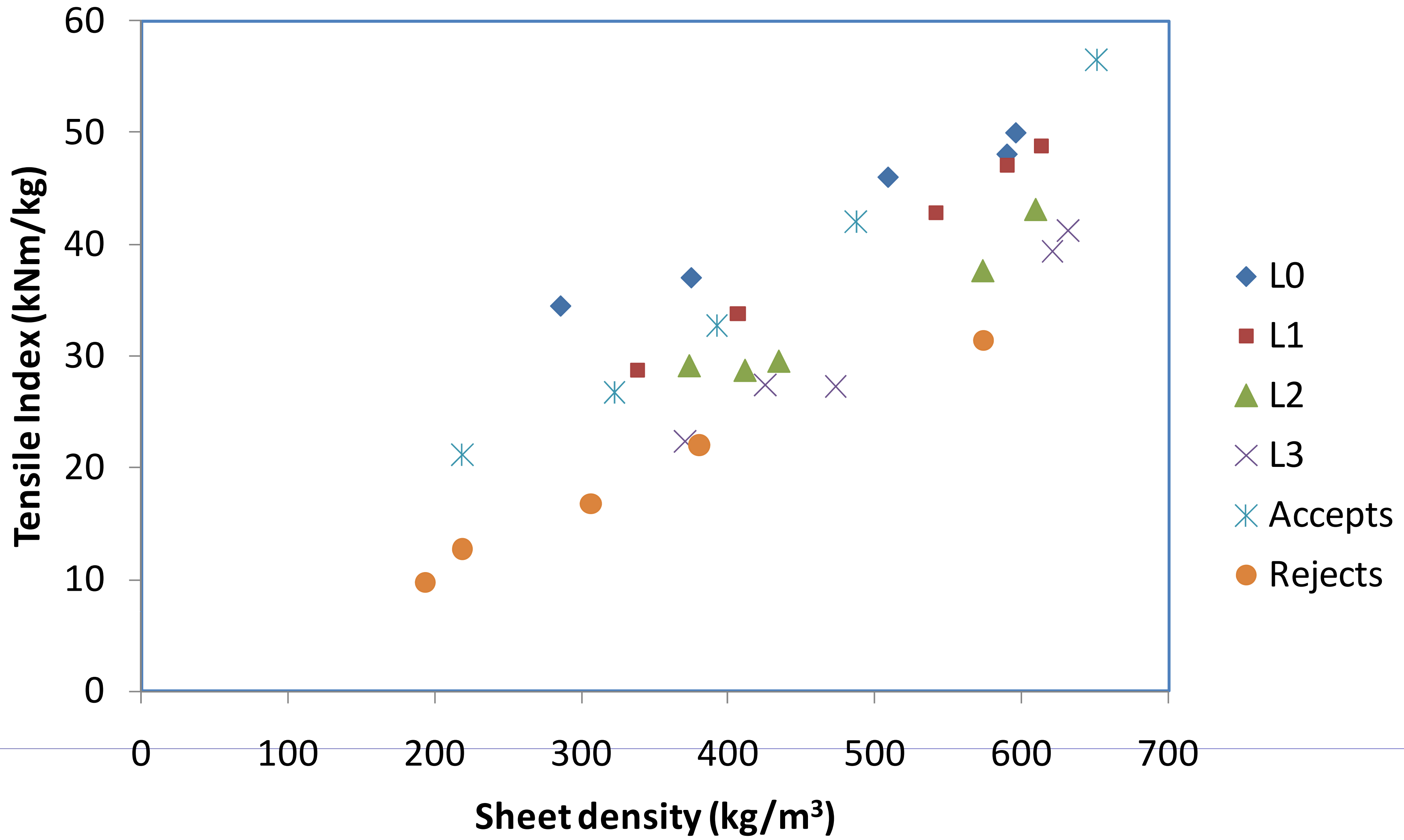
Six simulations for RR P1



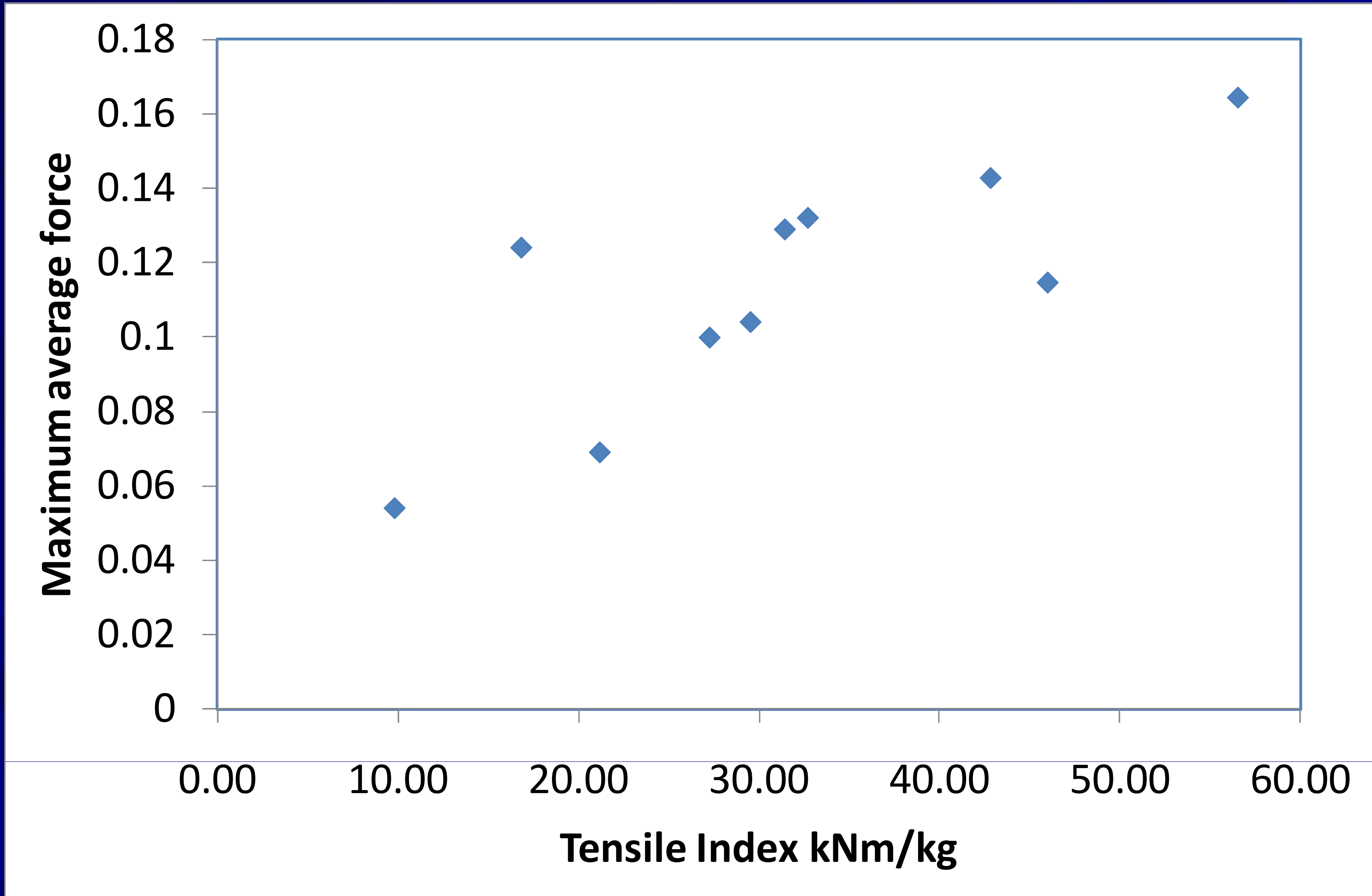
30 Simulations Averages



Tensile Index data



Max. Calculated force v measured tensile index



Where to next?

- Method to treat partial contacts
  - Relation to segment activation?
- Relationship between stress transfer and crossing angle?
- Inclusion of fibre fracture

Acknowledgements

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