Modelling Paper Tensile Strength from the Stress Distribution along Fibres in a Loaded Network

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Tensile strength of paper

- Important to predict
- Approaches
  - Analytical models
  - Finite Element simulations of low density sheets
Problems with strength modelling

- Qualitative not quantitative predictions
- Many variables difficult to measure
- Unverified assumptions about fracture process
- Simplified models with average fibre properties
New approach presented here

- Model paper behaviour with single fibre stress development
  - Usually fibres in stress direction
- Micro-Macro approach
  - Single fibre of interest
  - Discrete contacts transferring stress from solid Matrix
  - Apply strain to matrix
  - Cox type shear lag assumption of stress transfer at each contact
\( x = 0 \)
The Problem!

- Consider steps: ONE contact with $F \propto \delta$
  - Strain Matrix
  - Displaces contact
  - Produces Force in fibre
  - Force reduces contact displacement
  - Reduces force etc etc etc

- Equilibrium: force produces displacement required to generate force

- How do find displacements for multiple contacts??
Solution

Express $\delta$ at each contact in terms of displacement at last contact

Solve to obtain all displacements

\[
\delta_{n-1} = \delta_n - \frac{(x_n - x_{n-1})}{E_n A_n} \left( \sum_{j=n}^{j=i} \beta_n \delta_n - \varepsilon \right)
\]

$E_n$ : Elastic Modulus of $n^{th}$ segment

$A$ : Cross-sectional area of $n^{th}$ segment

$\varepsilon$ : Matrix strain
Equation to solve for three contacts

- Complexity increases as power to the number of contacts, $i$.
- More than four contacts - very tedious to write
- Instead solve symbolically using Matlab

\[
0 = x_1 \beta' + \left( \delta_3 + (x_3 - x_2)(\epsilon + \beta' \delta_3) \right) + \\
\left( x_2 - x_1 \right) \left( \epsilon + \left( \beta' \delta_3 + \beta_2 \left( \delta_3 + (x_3 - x_2)(\epsilon + \beta_3 \delta_3) \right) \right) \right] \\
+ x_2 \beta_2 \left( \delta_3 + (x_3 - x_2)(\epsilon + \beta_3 \delta_3) \right) + x_3 \beta_3 \delta_3 - \epsilon x_3 - \delta_3 \\
\beta' = \beta / EA
\]
An example: fixed contact positions, randomly varying $\beta'$ (0-10,000 fibres 1-3, 0-1000 fibres 4-6)
Example: stress development in a fibre. Randomly placed contacts. Contacts removed when load exceeds bond strength of bond
Advantages of method
- Random contact positions
- $\beta$ can vary contact to contact
- Elastic modulus, X-section area can vary segment to segment

Disadvantages of method
- Linear elastic only

Data needed:
- $\beta$, $E$, $A$
- Bond strength
- Fibre contact positions
Comparison Experimental Data

- Radiata pine
- Never dried
- Unbleached
- Kappa number 30
- Fibre dimension variation: cutting and fractionation
# Samples

<table>
<thead>
<tr>
<th>Label</th>
<th>Length weighted fibre length (mm)</th>
<th>Fibre wall area (µm²)</th>
<th>Pressing Levels (Contacts)</th>
</tr>
</thead>
<tbody>
<tr>
<td>L0</td>
<td>3.14</td>
<td>203</td>
<td>Middle (P3)</td>
</tr>
<tr>
<td>L1</td>
<td>2.53</td>
<td>204</td>
<td>Middle (P3)</td>
</tr>
<tr>
<td>L2</td>
<td>2.12</td>
<td>196</td>
<td>Middle (P3)</td>
</tr>
<tr>
<td>L3</td>
<td>1.79</td>
<td>196</td>
<td>Middle (P3)</td>
</tr>
<tr>
<td>Accepts</td>
<td>3.00</td>
<td>220</td>
<td>P1, P3, P5</td>
</tr>
<tr>
<td>Rejects</td>
<td>3.34</td>
<td>193</td>
<td>P1, P3, P5</td>
</tr>
</tbody>
</table>

Hydrocyclone fractionation
Measurements

- Sheet density, elastic modulus, tensile strength
- Fibre shape (fill factor), cross-sectional area, length
- Fibre contacts:
  - distances between contacts,
  - Weibull distributions of contacts
  - Full / partial contacts
Cross-section image before (A) and after (B) thresholding and binarisation. Fibre 2 and 3 in (B) make two full contacts, fibre 1 makes a partial contact, and fibre 4 is not in contact with the fibre of interest.
Distance between contact frequency distribution

\[ f(g) = \frac{c}{b} \left(\frac{g}{b}\right)^{c-1} \exp\left(-\left(\frac{g}{b}\right)^c\right) \]

- \( b \) is the scale parameter
- \( c \) is the shape parameter
Average elastic modulus along fibre

- Average: 30 simulations per point
- Fibre elastic mod: assumed 30 Gpa
- Effective elastic modulus: average load along fibre/X-section area/matrix strain
- Effective elastic mod=30 Gpa
  - Perfect stress transfer
Average elastic modulus along fibre

![Graph showing the relationship between stress transfer coefficient and effective fibre modulus, with data points for L0 P3, L1 P3, and L2-P3]
Elastic Stiffness Index for $\beta'$

~5% reduction in stiffness with fibre length implies $\beta' \sim 2500$
Strength modelling

- $\beta' = 2500$
- Bond strength $= 25$ MPa (IPPC 2007)
- Average 30 simulations with measured fibre contact statistics
- Fibre elastic modulus $= 30$ GPa
- Contacts
  - Assumed both full, partial contacts contribute
  - But bond breaking load scaled to contact area
- Fibre fracture not considered
- Model fibres in stress direction only
Six simulations for L0 P3

Matrix strain

Average Load (N)

Series 1
Series 2
Series 3
Series 4
Series 5

Retention (N)
Six simulations for RR P1

![Graph showing data for six simulations with different series, each represented by distinct markers and colors. The x-axis is labeled 'Axis Title' and the y-axis is labeled 'Average fibre load (N)'. The series are labeled Series1, Series2, Series3, Series4, Series5, and Series6.]
30 Simulations Averages

Network strain vs. Average force along fibre (N)

Network strain

Average force along fibre (N)
Max. Calculated force v measured tensile index
Where to next?

- Method to treat partial contacts
  - Relation to segment activation?
- Relationship between stress transfer and crossing angle?
- Inclusion of fibre fracture
Acknowledgements

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