MODELLING PAPER TENSILE STRENGTH FROM THE STRESS DISTRIBUTION ALONG FIBRES IN A LOADED NETWORK

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This paper describes a new method for modelling paper strength and elastic properties by analysing the force distribution along single half fibres in the network. Paper mechanical properties are modelled from the point of view of single fibres of interest. Each half fibre of interest is in contact with i other crossing fibres, which connect to the network. A strain, ε, is applied to the network. The force that is developed at the j^{th} contact, F_j, is assumed to linearly proportional to the displacement of the contact relative to the network strain, δ_j. The stress transfer coefficient is β_j, and $F_j = \beta_j \delta_j$. This approach is similar to the Cox shear-lag model [1]. The problem is difficult to solve because the displacement at any contact is determined by the forces developed at all contacts, which in turn are directly proportional to the displacements at the contacts.

The solution used here was to express the displacement at each of the contacts as linear functions of the displacement of the last contact at the end of the fibre using the relationship between the displacements at the $n\text{-}1^{th}$ contact and the n^{th} contact, which is

$$\delta_{n-1} = \delta_n + \left(x_n - x_{n-1}\right)\left(\varepsilon - \frac{1}{EA}\sum_{j=n}^{j=i} \beta_j \delta_j\right)$$

where E is the fibre elastic modulus, A is the fibre cross-sectional area and x_n is the position of the n^{th} contact. The main advantage of this method of analysis over the shear-lag model is that fibre contacts are discrete and so it is possible to model variations in stress transfer from point to point as well as stochastically distributed crossing fibres. The main limitation is the assumption that the fibre is linearly elastic.

For the work presented here, the fibre elastic modulus was assumed to be 30GPa and a shear bond strength of 25 MPa [2] was used. The positions of the contact fibres were generated from Weibull distributions determined by fitting data measured [3] from sheets made of a never-dried radiata pine kraft with kappa number of 30. This included measurements on three sets of sheets where the fibre length, which had been reduced by wet cutting, ranged from a length weighted length of 3.14 to 2.10 mm. Sheets were manufactured and pressed at five different pressing levels. Sheet elastic modulus data showed only a small reduction in elastic modulus, as a function of sheet apparent density, as the fibre length was reduced.

Simulations were conducted to calculate the effective fibre modulus as a function of stress transfer coefficient, β. The effective fibre modulus was calculated by dividing the average stress along the fibre by the network strain. The effective fibre modulus is always less than the actual fibre elastic modulus because the force in the fibre builds from zero at the end of the fibre to a maximum in the middle. The more efficient the stress transfer in the network, the closer the effective fibre modulus will be to the true modulus. Figure 1 shows the results of the simulations. Each data point in Figure 1 is the result of 15 simulations. For each simulation, the intervals between the crossing fibres were generated using the corresponding Weibull functions for the fibre contact distributions. Thus the number and positioning of the set of contacting fibres is different from simulation to simulation.

The data show that at the lowest value of β=100, there is a 33% reduction in effective fibre modulus when comparing the sheets made with the longest and the shortest fibres. The data suggest that relatively high values of β are required to reduce the difference in effective fibre modulus to match the sheet elastic modulus data.

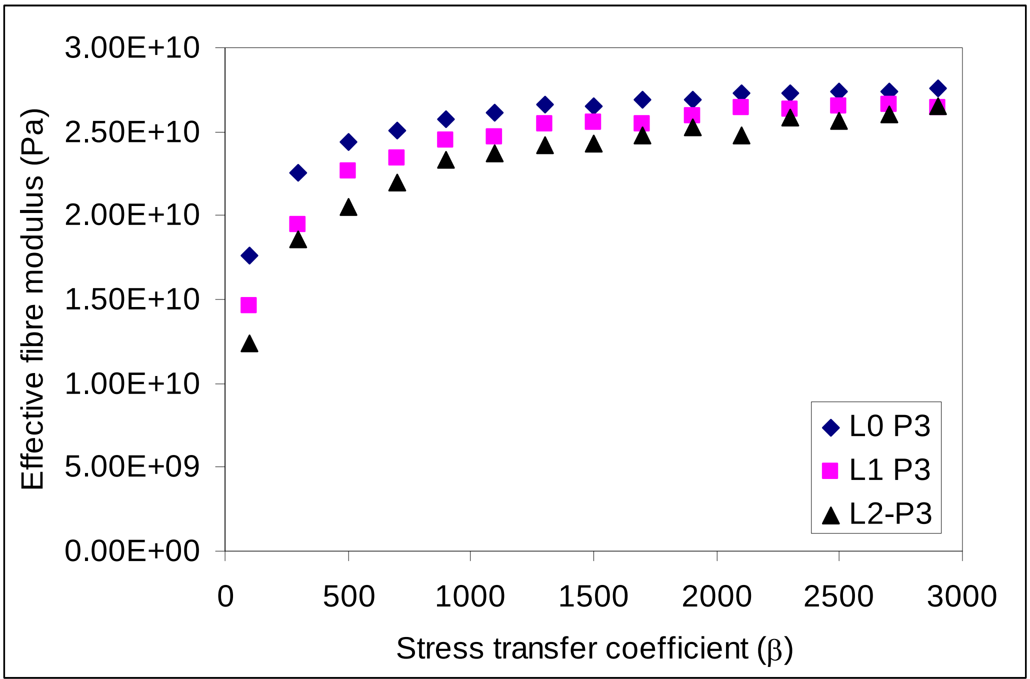

Figure 1. Effective fibre modulus as a function of stress-transfer coefficient with assumed fibre elastic modulus of 3.0E+10 Pa. Length weighted fibre lengths were 3.14, 2.53 and 2.10mm for LO, L1 and L2, respectively.

Accordingly, a value of $\beta = 2500$ was used to simulate the debonding and fracture process. The load-strain curves for the four fibres are not identical as the set of crossing fibres is different each time. For the simulations, the network strain was increased in steps of 0.002. After each increase, the forces at each contact point were calculated and compared with the breaking load of the bond. If the force on a bond exceeding the breaking load, then the bond was removed from the simulation and the forces at each bond recalculated. This procedure was repeated until the forces at each remaining bond was less than the breaking load of the bond, at which point the strain was then incremented by 0.002. The results are shown in Figure 2 and are consistent with the fracture process of paper, both in the average loads developed in the fibres and the network strains at which the fibre debonds from the network.

The simulation method seems a promising method to study paper mechanics.

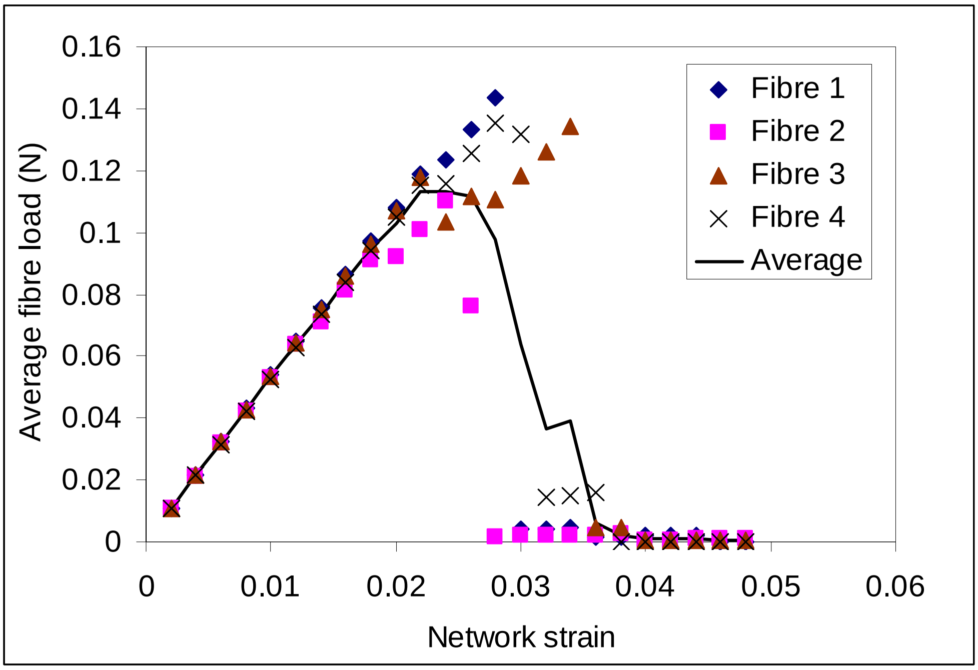

Figure 2 Simulation of the debonding of four fibres of the LO-P3 sample.

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