Determination of Load-Bearing Element Length in Paper using Zero/Short Span Tensile Testing

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Presented at 1999 Paper Physics Conference San Diego
Introduction

- Paper strength
- Effect of drying treatment
- Defects in fibres
- Load-bearing element
Load-bearing element

- NOT a fibre
  - Fibres can be made up of many elements
  - Joined by kinks etc
- Properties:
  - Length, \( l \)
  - Cross sectional area, \( C \)
  - Young’s modulus, \( E \)
Test and Sample Dimensions

\[ A = L_{\text{sheet}} W_{\text{sheet}} \]
Probabilities

- Single load-bearing element in sheet, length, $l$, angle $\theta$
- $P_1$: probability that element is gripped by Jaw 1
- $P_2$: probability of element gripped by Jaw 2 if also gripped by Jaw 1
- Assumption: $W_{\text{sheet}}, L_{\text{sheet}}, W_j$ much greater than $l$.

\[
P_1 = \frac{l \cos \theta}{L_{\text{sheet}}} \cdot \frac{W_j}{W_{\text{sheet}}}
\]

\[
P_2 = 1 - \cos \theta \cdot \frac{G}{l}
\]
Overall strain: \( \varepsilon = \frac{\Delta G}{G} \)
Load on single fibre

At strain, $\varepsilon$, force on fibre spanning both jaws is

$$F_f = EC\varepsilon \cos^2 \theta$$

and component in loading direction is

$$F_f = EC\varepsilon \cos^3 \theta$$

Average contribution by randomly located, oriented fibre

is $F_f P_1 P_2$ or

$$F_{av} = (1 - \overline{f_c})EC\varepsilon \frac{lW_j}{A} \frac{2}{\pi} \int_0^{\cos^{-1}(G/l)} \left( \cos^4 \theta - \frac{G}{l} \cos^3 \theta \right) d\theta$$

where $\overline{f_c}$ is fraction of fibres not bearing load due to out of plane curl
Integratin g get

\[ F_{av} = (1 - f_c)EC \varepsilon \frac{lW_j}{A} \frac{2}{\pi} \left[ -\frac{1}{12} \left( \frac{G}{l} \right)^3 \sqrt{1 - \left( \frac{G}{l} \right)^2} \right. \\
\left. + \frac{3}{8} \arccos \frac{G}{l} - \frac{7}{24} \frac{G}{l} \sqrt{1 - \left( \frac{G}{l} \right)^2} \right] \]

(1)

Use Taylor series expansion obtain :

\[ F_{av} = (1 - f_c)EC \varepsilon \frac{W_j}{A} \frac{3}{8} \left[ l - \frac{32}{9\pi} G \right] \]

(2)

Assumption s

\[ G < 0.7l \]  Random orientatio n  No fibre - fibre bonding

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Comparison between equations 1 and 2

Approximate limit for accuracy of Taylor’s series approximation
**Total force**

- For gap, $G$, if have $I(G)$ load bearing elements with $l > 32G / 9\pi$
- then the total force, $F$, is

\[
F(\varepsilon,G) = I(G)(1-f_c)\varepsilon \frac{W_j}{A} \left[ \frac{3}{8} E(G)C(G)l(G) - \frac{32}{9\pi} \frac{E(G)C(G)}{G} \right]
\]

- where $E(G)C(G)l(G)$ is the average of the $I(G)$ elements
\[ l(0) = - \frac{32}{9\pi} \frac{F(\varepsilon_{frac}, 0)}{dF(\varepsilon_{frac}, G)} \bigg|_{G=0} \]

\[ = -\frac{32}{9\pi} \frac{\frac{dF(\varepsilon_{frac}, G)}{dG}}{dF(\varepsilon_{frac}, G)} \bigg|_{G=0} \]

\[ = -\frac{32}{9\pi} \frac{dF(\varepsilon_{frac}, G)}{dG} \bigg|_{G=0} \]

\[ \frac{dF(\varepsilon_{frac}, G)}{dG} \bigg|_{G=0} \]

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**Assumptions:**

- \( E(0)C(0)l(0) \approx E(0)C(0)l(0) \)
- \( E(G)C(G), \ l(G) \) independent of \( G \) if \( G << l \)
Validity of Assumptions

- Major assumption: \( \frac{E(0)C(0)l(0)}{E(0)C(0)} \approx \frac{E(0)C(0) l(0)}{l(0)} \)
- Test: artificial distributions of fibre properties
- Result: \( K < 1 \) (always) if longer fibres are stiffer

\( \frac{E(0)C(0)l(0)}{E(0)C(0)} \approx K \frac{E(0)C(0) l(0)}{l(0)} \)

<table>
<thead>
<tr>
<th>Furnish</th>
<th>Relative number of fibres</th>
<th>EC</th>
<th>L</th>
<th>( E(0)C(0)l(0) )</th>
<th>( E(0)C(0) l(0) )</th>
<th>K</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hardwood</td>
<td>0.5</td>
<td>1.0</td>
<td>0.6</td>
<td>0.9</td>
<td>0.875</td>
<td>0.972</td>
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<tr>
<td></td>
<td>0.5</td>
<td>1.5</td>
<td>0.8</td>
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</tr>
<tr>
<td>Hardwood &amp; softwood</td>
<td>0.8</td>
<td>1.0</td>
<td>0.5</td>
<td>1.2</td>
<td>0.96</td>
<td>0.8</td>
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<tr>
<td></td>
<td>0.2</td>
<td>2.0</td>
<td>2.0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Experimental Method

◆ Five samples
  ◆ Unbleached Eucalypt NSSC pulp*
  ◆ *E. globulus* kraft pulp- Laboratory pulped, oxygen bleached to kappa no. 17.9*
  ◆ Unbleached brown mixed waste pulp for packaging grades*
  ◆ Unbleached *P. Radiata kraft pulp #1*
  ◆ Unbleached *P. Radiata kraft pulp #2, 45 kappa**

* Refrigerated for up to 1 year before making handsheets.
**Collected from pulp mill brown-stock washer. Handsheets made immediately.
Experiments

- Handsheet preparation
  - British Standard Handsheet machine
  - Not refined in PFI mill
- Zero/short span tests
  - Pulmac zero span tester
  - Span: 0.0, 0.05, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6 mm
- Fibre length measurements
  - Kajaani FS 200
- Sheet tensile strength
3 Drying Conditions

1. Make handsheets; air-dry under restraint
   - Labelled never/air dried

2. Make handsheets; air dry under restraint; reslush; make handsheets; air dry under restraint
   - Labelled air/air dried

3. Make handsheets; oven dry without restraint; reslush; make handsheets; air dry under restraint
   - Labelled oven/air dried
Results

- Zero/short span measurements
  - Plotted with residual span of 0.2 mm
- Tensile strength
- Fits of data to obtain load-bearing element length
- Comparison with measured fibre length
Zero/short span results: NSSC Eucalypt

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Zero/short span results: E. globulus kraft pulp

![Graph showing the relationship between True Span (mm) and Tensile index (Nm/g) for different drying conditions: Never/Air Dried, Air/Air Dried, and Oven/Air Dried.](image-url)
Zero/short span results: waste paper

![Graph showing the relationship between true span (mm) and tensile index (Nm/g) for different drying methods.](image)

- Never/Air Dried
- Air/Air Dried
- Oven/Air Dried
Zero/short span results: P. radiata kraft pulp #1

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Zero/short span results: P. radiata kraft pulp #2

![Graph showing tensile index vs. true span for different drying conditions.](image-url)
Sheet tensile strength under different drying conditions

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Fits to zero/short span data

- Quadratic
- Linear (full data set)
- Linear (restricted data set)
- No residual span
NSSC Eucalypt - Air/Air dried

Pine kraft #1 - Air/Air dried

\( \bar{l}(0) = 0.74 \text{ mm} \)

\( \bar{l}(0) = 2.02 \text{ mm} \)

\( \bar{l}(0) = 1.02 \text{ mm} \)

\( \bar{l}(0) = 1.32 \text{ mm} \)
<table>
<thead>
<tr>
<th></th>
<th>Never/Air Dried</th>
<th></th>
<th>Air/Air Dried</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Quadratic</td>
<td>Linear (0-0.4mm)</td>
<td>Linear</td>
<td>Quadratic</td>
</tr>
<tr>
<td>Euc NSSC</td>
<td>0.61</td>
<td>0.82</td>
<td>0.97</td>
<td>0.74</td>
</tr>
<tr>
<td>Euc kraft</td>
<td>0.67</td>
<td>0.92</td>
<td>1.04</td>
<td>0.79</td>
</tr>
<tr>
<td>Waste</td>
<td>0.63</td>
<td>0.82</td>
<td>1.26</td>
<td>0.73</td>
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<tr>
<td>Pine #1</td>
<td>1.00</td>
<td>1.19</td>
<td>1.97</td>
<td>1.33</td>
</tr>
<tr>
<td>Pine #2</td>
<td>0.89</td>
<td>1.29</td>
<td>1.69</td>
<td>1.26</td>
</tr>
</tbody>
</table>

Load-bearing element lengths (mm) determined by different fitting methods
**Effect of residual span**

Average load-bearing element lengths (mm) from fitting zero/short span data using different residual spans

<table>
<thead>
<tr>
<th></th>
<th>Never/Air Dried</th>
<th>Air/Air Dried</th>
<th>Air/Oven Dried</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Residual 0.2 mm</td>
<td>No Residual</td>
<td>Residual 0.2 mm</td>
</tr>
<tr>
<td>Euc NSSC</td>
<td>0.61</td>
<td>0.57</td>
<td>0.74</td>
</tr>
<tr>
<td>Euc Kraft</td>
<td>0.67</td>
<td>0.57</td>
<td>0.79</td>
</tr>
<tr>
<td>Waste</td>
<td>0.63</td>
<td>0.57</td>
<td>0.73</td>
</tr>
<tr>
<td>Pine #1</td>
<td>1.19</td>
<td>0.97</td>
<td>1.32</td>
</tr>
<tr>
<td>Pine #2</td>
<td>1.29</td>
<td>1.07</td>
<td>1.21</td>
</tr>
</tbody>
</table>

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<table>
<thead>
<tr>
<th></th>
<th>Never/Air Dried</th>
<th>Air/Air Dried</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Arithmetic (FS 200)</td>
<td>Load-bearing element length (fit)</td>
</tr>
<tr>
<td>Euc NSSC</td>
<td>0.63</td>
<td>0.61</td>
</tr>
<tr>
<td>Euc kraft</td>
<td>0.57</td>
<td>0.67</td>
</tr>
<tr>
<td>Waste</td>
<td>0.61</td>
<td>0.63</td>
</tr>
<tr>
<td>Pine #1</td>
<td>1.23</td>
<td>1.19</td>
</tr>
<tr>
<td>Pine #2</td>
<td>1.56</td>
<td>1.29</td>
</tr>
</tbody>
</table>

**Comparison of load-bearing element length (mm) with arithmetic and length weighted fibre lengths (mm)**
Conclusions

- Load-bearing element length approximately the same as arithmetic fibre length for these pulps
- Drying treatment- no effect on average load-bearing element length
Acknowledgements

◆ Daniel Ouellet, Paprican and University of British Columbia Pulp and Paper Centre
◆ Russell Allan, Amcor Research and Technology Centre
◆ Glenda Spencer, Michael Briggs and Geoff Ennis for assistance with the experiments
◆ Funding from the Commonwealth Government of Australia through the Cooperative Research Centre for Hardwood Fibre and Paper Science