ERRATUM TO ERGODIC COMPONENTS OF PARTIALLY HYPERBOLIC SYSTEMS

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This erratum addresses two issues with the proofs in the paper [Ham17]. The first issue is that proposition (6.4) as stated is not correct.[1] For instance, the automorphism $\mathbb{Z}^2 \to \mathbb{Z}^2$, $(x, y) \mapsto (5x+2y, 2x+y)$ gives a counterexample as it fixes a coset of $\mathbb{Z} \times 2\mathbb{Z}$. The flaw in the proof is that it confuses invertibility in $GL(n,\mathbb{Z})$ with invertibility in $GL(n,\mathbb{R})$ and the notions are not equivalent. In fact, the proposition holds in the following revised version.

Proposition 1. *Let G be a torsion-free, finitely-generated, nilpotent group and suppose $\phi \in \mathrm{Aut}(G)$ is such that $\phi(g) \ne g$ for all non-trivial $g \in G$. If H is a normal, ϕ-invariant subgroup, then ϕ fixes at most* ***finitely many*** *cosets of H.*

We prove this revised version below. The original proposition (6.4) is used in only two places in the proofs of theorem (4.3) and lemma (6.5) and we show below how to use the revised version of the proposition to recover the proofs of those two results.

The other issue to address in the original paper comes at the start of section 8 which deals with AB-systems. That section states that hfh^{-1} is homotopic to f_{AB} and uses this to lift hfh^{-1} a map on $N \times \mathbb{R}$. In fact, the two functions are not homotopic in general. For instance, for the linear partially hyperbolic maps on the 3-torus $\mathbb{T}^3 = \mathbb{R}^3/\mathbb{Z}^3$ given by the matrices

$$\begin{pmatrix} 5 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 5 & 2 & 0 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

both have vertical center foliations and the identity map is a leaf conjugacy between the two systems. The two systems are not homotopic to each other and attempting to lift the two systems to AI-system on $\mathbb{T}^2 \times \mathbb{R}$ as in section 8 will not work. To fix this, we amend the definition of an AB-system to add the homotopy as an assumption. That is, a partially hyperbolic diffeomorphism f is an *AB-system* if

(1) it preserves the orientation of the center bundle E^c,
(2) there is a leaf conjugacy h between f and an AB-prototype f_{AB}, and
(3) hfh^{-1} is homotopic to f_{AB}.

[1] Note that the numbering of sections in some preprint versions may differ from the published version.

This additional assumption can always be achieved by lifting f and f_{AB} to finite covers:

Proposition 2. *If a partially hyperbolic diffeomorphism f satisfies conditions (1) and (2) above, then a lift of f to a finite cover satisfies all of (1), (2), and (3).*

The proof of this is given in the final section of this erratum.

For those readers interested only in the case where the nilmanifold N is a torus $\mathbb{T}^d$, we have structured the proofs below so that most of the details specific to the non-toral case may be skipped over.

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Karel Dekimpe was also very helpful and suggested an alternative method to establish proposition 1. Instead of proving the proposition directly, one can instead show the following fact, from which the proposition follows as a corollary:

> Let G be a finitely generated nilpotent torsion free nilpotent group and $\varphi \in \mathrm{Aut}(G)$ be fixed point free. Assume that H is a φ invariant subgroup of G such that G/H is torsion free. Then it follows that the induced automorphism on G/H is also fixed point free.

PROOF OF PROPOSITION 1

This section gives a proof of proposition 1. We first prove this in the abelian case and then use induction on the nilpotency class to handle the non-abelian case.

Lemma 3. *Let G be isomorphic to $\mathbb{Z}^d$ and suppose $\phi \in \mathrm{Aut}(G)$ is such that $\phi(g) \neq g$ for all non-trivial $g \in G$. If H is a normal, ϕ-invariant subgroup, then ϕ fixes at most finitely many cosets of H.*

Proof. Assume $G = \mathbb{Z}^d$ and define a linear map $A : \mathbb{Q}^d \to \mathbb{Q}^d$ such that $Az = \phi(z)$ for all $z \in \mathbb{Z}^d$. If A had an eigenvalue of 1, the corresponding eigenspace would intersect $\mathbb{Z}^d$ in a non-trivial fixed point $\phi(z) = z \in \mathbb{Z}^d$. Hence, 1 is not an eigenvalue of A.

Let $V \subset \mathbb{Q}^d$ be the set of all $\mathbb{Q}$-linear combinations of elements of H. We may assume H has infinite index in $\mathbb{Z}^d$, and so V is a proper A-invariant subspace of $\mathbb{Q}^d$. It induces a linear map $\bar{A}$ on the quotient space $\mathbb{Q}^d/V$. If $z \in \mathbb{Z}^d$ is such that $\phi(z+H) = z+H$, then $\bar{A}(z+V) = z+V$ and so $\bar{A}$ and therefore A has an eigenvalue of 1. $\square$

Lemma 4. *Let $\phi : G \to G$ be a group automorphism and let X be a normal ϕ-invariant subgroup. If $\phi|_X$ has at most finitely many fixed points and ϕ fixes at most finitely many cosets of X, then ϕ itself has finitely many fixed points.*

Proof. If $\phi(g) = g$ and $\phi(g') = g'$ are fixed points in the same coset $gX = g'X$, then $\phi(g'g^{-1}) = g'g^{-1}$ is a fixed point in X. Hence, each of the finitely many fixed cosets has finitely many fixed points. □

Corollary 5. *Suppose ϕ is an automorphism of a group G with center Z, and H is a ϕ-invariant normal subgroup of G. If the induced maps on $Z/(H \cap Z)$ and G/HZ have finitely many fixed points, then the induced map on G/H has finitely many fixed points.*

Proof. Apply the previous lemma to the quotient

$$0 \to Z/(H \cap Z) \to G/H \to G/HZ \to 0$$

□

Lemma 6. *Suppose G is a finitely generated torsion free nilpotent group and let $\phi : G \to G$ be an automorphism. Let Z denote the center of G. If there is a non-trivial fixed coset $\phi(gZ) = gZ$, then ϕ has a non-trivial fixed point.*

Proof. By the properties of such groups [Dek96], Z is isomorphic to $\mathbb{Z}^d$ and G/Z is torsion free. Suppose $\phi(gZ) = gZ$ is a non-trivial fixed coset. Let Y be the subgroup generated by g and Z. Then Y is isomorphic to $\mathbb{Z}^{d+1}$ and within Y, there are infinitely many fixed cosets: $\phi(g^k Z) = g^k Z$ for $k \in \mathbb{Z}$. Lemma 3 implies that $\phi|_Y$ has a non-trivial fixed point. □

Proof of proposition 1. We prove this by induction on the length of the upper central series of G. The abelian base case is given by lemma 3. Assume now that G is non-abelian with center Z and that proposition 1 is already known to hold for the quotient map $\Phi : G/Z \to G/Z$.

Since $\phi|_Z$ has no non-trivial fixed points, lemma 3 implies that $\phi|_Z$ fixes at most finitely many cosets of $H \cap Z$. Lemma 6 implies that Φ has no non-trivial fixed points. By the inductive hypothesis, Φ fixes at most finitely many cosets of HZ/Z. Then corollary 5 implies that ϕ (on all of G) fixes at most finitely many cosets of H. □

Revised proof of lemma (6.5)

The incorrect proposition (6.4) is used in the proof of (6.5) only to establish $\lambda \neq 1$. Recall in that proof that there is $F \in \mathrm{Aut}(G)$ with no non-trivial fixed points and a non-zero homomorphism $\tau : G \to \mathbb{R}$ such that $\tau F = \lambda \tau$. Define $H \subset G$ to be the kernel of τ. Note that the cosets of H are exactly the level sets of τ. If $\lambda = 1$, then every level set of τ is fixed by F. Since τ is non-zero, there are infinitely many such level sets and proposition 1 above gives a contradiction.

Circle bundles over nilmanifolds

Before revising the proof of (4.3), we first prove the following.

Proposition 7. *Suppose M is a circle bundle with oriented fibers over a nilmanifold N. If M has a compact horizontal submanifold Σ, then M is a trivial bundle.*

Remark. We consider everything in the C^0 setting here. The circle bundle is defined by a C^0 map $p : M \to N$ and a compact horizontal submanifold Σ is a codimension one C^0 submanifold such that $p|_\Sigma : \Sigma \to N$ is a covering map of finite degree. To show that M is trivial, it enough to find another horizontal submanifold Σ_1 such that $p|_{\Sigma_1} : \Sigma_1 \to N$ is a homeomorphism. To simplify the proof, we assume that the circle fibers are tangent to a C^0 vector field as is the case for the center leaves of a partially hyperbolic skew product.

Proof. Assume Σ intersects each fiber in exactly k points. We may define a metric on each fiber such that the length of every fiber is exactly one and that its points of intersection with Σ are equally spaced; that is, the distance between one point of intersection and the next is exactly $\frac{1}{k}$. We may choose these metrics to vary continuously along M.

Let $\pi : \tilde{M} \to M$ be the universal covering map. We may assume that $\tilde{M} = \tilde{N} \times \mathbb{R}$ where $\tilde{N}$ is the nilpotent Lie group covering N and such that the fibers of M lift to lines of the form $v \times \mathbb{R}$ with $v \in \tilde{N}$. We further assume that the metric on a fiber of M lifts to a metric on $v \times \mathbb{R}$ which is equal to the standard Euclidean metric given by $\mathbb{R}$. In particular, $\pi^{-1}(\Sigma)$ intersects each fiber $v \times \mathbb{R}$ in a set of points of the form

$$\{\, (v, \sigma(v) + t) \;:\; t \in \tfrac{1}{k}\mathbb{Z} \,\}$$

for some $\sigma(v)$ depending on v. We may assume $\sigma : \tilde{N} \to \mathbb{R}$ is continuous. To see this, choose a connected component $\tilde{\Sigma}$ of $\pi^{-1}(\Sigma)$ and define $\sigma(v)$ to be the unique intersection of $v \times \mathbb{R}$ with $\tilde{\Sigma}$.

Write $G = \pi_1(M)$, and $H = \pi_1(N)$. The bundle projection $p : M \to N$ induces a surjective homomorphism $p_* : G \to N$. We now use $\tilde{\Sigma}$ to define a homomorphism $\tau : G \to \frac{1}{k}\mathbb{Z}$. Without loss of generality, assume $\sigma(e) = 0$ where e is the identity element of $\tilde{N}$. For a deck transformation $g \in G$, let $\tau(g)$ be such that $(e, \tau(g))$ is the unique intersection of $g(\tilde{\Sigma})$ with $e \times \mathbb{R}$. Similar to lemma (7.6) in the original paper, one may show that $\tau : G \to \frac{1}{k}\mathbb{Z}$ is a homomorphism. We claim the following.

> **Claim.** There is a (not necessarily unique) homomorphism
> $\psi : H \to \frac{1}{k}\mathbb{Z}$ such that $\psi p_*(g) - \tau(g) \in \mathbb{Z}$ for all $g \in G$.

We leave the proof of this to the end and first show that this gives the desired result. By the properties of nilmanifolds [Mal51], ψ determines a Lie group homomorphism $\psi : \tilde{N} \to \mathbb{R}$ where if we regard H as a discrete subgroup of $\tilde{N}$ then this is an extension of ψ from H to all of $\tilde{N}$. Define a submanifold $\tilde{\Sigma}_1$ as the graph of $\sigma - \psi$; that is, $(v, t) \in \tilde{\Sigma}_1$ if and only if $t = \sigma(v) - \psi(v)$. By the above claim, for all deck transformations $g \in G$, the intersection of $g(\tilde{\Sigma}_1)$ with $e \times \mathbb{R}$ lies inside $e \times \mathbb{Z}$. Hence $\tilde{\Sigma}_1$ quotients down to a compact horizontal submanifold $\Sigma_1 \subset M$ which intersects each fiber exactly once and therefore shows that the circle bundle is trivial

It remains to prove the claim. We first consider the abelian case where H is isomorphic to $\mathbb{Z}^d$. Let $\{h_1, \dots h_d\}$ be a generating set for H and choose elements $g_i \in G$ such that $p_*(g_i) = h_i$. Let $z \in G$ be the deck transformation $(v, t) \mapsto (v, t + 1)$ corresponding to going once around a fiber of the circle fibering. Note that $\tau(z) = 1$. As explained in the original proof of (4.3), $\langle z \rangle$ is the kernel of p_*, and so $\{z, g_1, \dots g_d\}$ is a generating set for G. Define $\psi : H \to \frac{1}{k}\mathbb{Z}$ by $\psi(h_i) = \tau(g_i)$. Then $\psi p_*(z) - \tau(z) = -1$ and $\psi p_*(g_i) - \tau(g_i) = 0$. As $\psi p_* - \tau$ takes integer values on a generating set for G, it must take integer values on all of G.

We now extend this argument to the non-abelian case. Note that both M and N are nilmanifolds. Consider the root set G_1 of the commutator subgroup of G. That is $g \in G_1$ if and only if $g^k \in [G, G]$ for some $k \ge 1$. Such sets are discussed in detail in Chapter 1 of [Dek96] (where the notation there is $\sqrt[G]{\gamma_2(G)}$ instead of G_1). In particular, G_1 is a normal subgroup and any homomorphism from G to a torsion-free abelian group R is identically zero on G_1 and so factors through $G \to G/G_1 \to R$ We can therefore define a homomorphism $\tau_1 : G/G_1 \to \frac{1}{k}\mathbb{Z}$ as the quotient of τ.

Similarly write H_1 for the root set of $[H, H]$. Then H/H_1 is a torsion-free abelian group homomorphic to $\mathbb{Z}^d$ for some d [Dek96], and $p_* : G \to H$ descends to a map $p_1 : G/G_1 \to H/H_1$. Adapting the argument above, we may define a map $\psi_1 : H/H_1 \to \frac{1}{k}\mathbb{Z}$ such that $\psi_1 p_1 - \tau_1$ takes integer values on all of G/G_1. Then ψ_1 determines a map $\psi : H \to \frac{1}{k}\mathbb{Z}$ as desired. $\square$

REVISED PROOF OF THEOREM (4.3)

This section revises the proof of theorem (4.3) to use proposition 1 above in place of the incorrect proposition (6.4) of the original paper. By virtue of proposition 7 above, we need only show that the partially hyperbolic system has a compact us-leaf.

The proof of (4.3) is unchanged up to the definition of $\hat{\tau} : G \to \mathbb{R}/\mathbb{Z}$ and the first use of (6.4). Using instead proposition 1 above, the most we can say is that $\hat{\tau}$ has a finite image. In other words, there is an integer $k \ge 1$ such that $\tau(G) = \frac{1}{k}\mathbb{Z}$.

The existence of τ is given by (6.1) and (6.2). From the proofs of those results, we can see that then there is a measure μ on $\tilde{S}$ invariant under the action of G and such that $\tau(g) = \mu[x, g(x))$ for any $x \in \Lambda$ and $g \in G$. Here, Λ is the intersection of the non-open accessibility classes Γ with $\tilde{S}$. Choose some point $x_0 \in \Lambda$ and for each $t \in \frac{1}{k}\mathbb{Z}$, define a set $X_t \subset \Lambda$ by

$$X_t = \{x \in \Lambda \ : \ \mu[x_0, x) = t\}.$$

The sets X_t are disjoint and the action of $g \in G$ on Λ takes X_t to $X_{t+\tau(g)}$. Define $y_t = \sup X_t$ where we are identifying $\tilde{S}$ with $\mathbb{R}$ in order to define the supremum. Then $\{y_t : t \in \frac{1}{k}\mathbb{Z}\}$ is a discrete subset of Λ which is invariant under the action of G. This implies that for any point y_t, its accessibility class $AC(y_t) \subset \tilde{M}$ quotients down to a compact us-leaf on M.

PROOF OF PROPOSITION 2

We now prove proposition 2. Assume $f : M \to M$ is a partially hyperbolic diffeomorphism which preserves the orientation of E^c and $h : M \to M_B$ is a leaf conjugacy to $f_{AB} : M_B \to M_B$. We want to show that after lifting f and f_{AB} to maps $\hat{f}$ and $f_{\hat{A}\hat{B}}$ on finite covers $\hat{M}$ of M and $M_{\hat{B}}$ of M_B that there is a leaf conjugacy $\hat{h} : \hat{M} \to M_{\hat{B}}$ such that $\hat{h}\hat{f}\hat{h}^{-1}$ and $f_{\hat{A}\hat{B}}$ are homotopic. As all of the manifolds involved are Eilenberg-MacLane spaces of type $K(\pi,1)$, the existence of such a homotopy is purely a question involving the actions of the functions on the fundamental groups of the manifolds. (See, for instance, Propositions 1.33 and 1B.9 of [Hat02].) In particular, we do not need to use the smoothness of f in any way. Therefore, we may replace f by hfh^{-1} and assume without loss of generality that $M = M_B$ and that h is the identity map.

In this section, we write G for the simply connected nilpotent Lie group and Γ for the cocompact lattice such that $N = \Gamma \backslash G$ is the nilmanifold. Then quotienting G by $[G,G]$ yields an abelian Lie group isomorphic to $\mathbb{R}^d$ for some d. This defines a projection from G to $\mathbb{R}^d$, and for $x \in G$, we write $\bar{x} \in \mathbb{R}^d$ for its image under the projection. This projection may further be chosen such that Γ is mapped to $\mathbb{Z}^d$. (If the nilmanifold is a torus $N = \mathbb{Z}^d \backslash \mathbb{R}^d$, then the projection $G \to \mathbb{R}^d$ is the identity map and all of the overlines in what follows may be safely ignored.)

Let A, $B : G \to G$ be the commuting Lie group automorphisms defining the AB-prototype. These induce linear automorphisms $\bar{A}$ and $\bar{B}$ on $\mathbb{R}^d$ with the property that $\overline{A(x)} = \bar{A}(\bar{x})$ and $\overline{B(x)} = \bar{B}(\bar{x})$.

The universal cover of M_B is $G \times \mathbb{R}$. Define $\beta(x,t) = (B(x), t-1)$. For $\gamma \in \Gamma$, define $\tau_\gamma(x,t) = (\gamma \cdot x, t)$. Note that $\beta\tau_\gamma = \tau_{B(\gamma)}\beta$ and that every deck transformation may be written in the form $\tau_\gamma \beta^n$ for $\gamma \in \Gamma$ and $n \in \mathbb{Z}$.

Lift f to a diffeomorphism $\tilde{f} : G \times \mathbb{R} \to G \times \mathbb{R}$ such that $\tilde{f}(0 \times \mathbb{R}) = 0 \times \mathbb{R}$ where 0 is the identity element of G. Such a lift exists because of the leaf conjugacy. This lift then determines an automorphism f_* of the fundamental group $\pi_1(M_B)$ defined by the property $f_*(\tau) \circ \tilde{f} = \tilde{f} \circ \tau$ for all deck transformations τ. Since $0 \times \mathbb{R}$ projects to an f-invariant circle in M_B, one can show that $f_*(\beta) = \beta$. By the leaf conjugacy, $\tilde{f}(x \times \mathbb{R}) = A(x) \times \mathbb{R}$ for all $x \in G$, and so for each $\gamma \in \Gamma$, there is an integer $L(\gamma)$ such that $f_*(\tau_\gamma) = \tau_{A(\gamma)}\beta^{L(\gamma)}$. Using that f_* is a group homomorphism, one can show that $L(\gamma_1 \cdot \gamma_2) = L(\gamma_1) + L(\gamma_2)$ and $A(\gamma_1 \cdot \gamma_2) = A(\gamma_1)B^{L(\gamma_1)}\,\mathrm{A}(\gamma_2)$ for all $\gamma_1, \gamma_2 \in \Gamma$. This implies that $L : \Gamma \to \mathbb{Z}$ is a group homomorphism and that there is $k \ge 0$ such that $L(\Gamma) = k\mathbb{Z}$ and B^k is the identity map on G. If $k = 0$, then f induces the same action on $\pi_1(M_B)$ as the AB-prototype f_{AB} and this would imply the desired result. Therefore, we assume in what follows that $k \ge 1$.

By the properties of nilmanifolds [Mal51], L extends to a Lie group homomorphism $L : G \to \mathbb{R}$. Since $\mathbb{R}$ is abelian, $L|_{[G,G]} \equiv 0$ and there is a linear map $\bar{L} : \mathbb{R}^d \to \mathbb{R}$ such that $\bar{L}(\bar{x}) = L(x)$ for all $x \in G$. Let I denote the identity map on $\mathbb{R}^d$. As $\bar{A}$ is hyperbolic, $\bar{A} - I$ is invertible. Define $\bar{S} : \mathbb{R}^d \to \mathbb{R}$ by $\bar{S} = \bar{L}(\bar{A} - I)^{-1}$ and

$S : G \to \mathbb{R}$ by $S(x) = \bar{S}(\bar{x})$. By Cramer's rule, $\bar{S}(m\mathbb{Z}^d) \subset k\mathbb{Z}$ where $m = \det(\bar{A} - I)$. Using $f_*(\beta\tau_\gamma) = f_*(\tau_{B(\gamma)}\beta)$, one can show $LB(\gamma) = L(\gamma)$ for all $\gamma \in \Gamma$. Hence, $LB = L$ as functions on G and one may use this to show $\bar{L}\bar{B} = \bar{L}$, $\bar{S}\bar{B} = \bar{S}$, and $SB = S$.

Define $\Gamma_0 \subset \Gamma$ by $\gamma \in \Gamma_0$ if and only if $\bar{\gamma} \in m\mathbb{Z}^d$. Since $\bar{A}(m\mathbb{Z}^d) = m\mathbb{Z}^d$ and $\bar{B}(m\mathbb{Z}^d) = m\mathbb{Z}^d$, it follows that $A(\Gamma_0) = \Gamma_0$ and $B(\Gamma_0) = \Gamma_0$. Hence, A and B define commuting automorphisms $\hat{A}$ and $\hat{B}$ of a nilmanifold $\hat{N} = \Gamma_0 \backslash G$ that finitely covers N. Using this we define a new AB-prototype $f_{\hat{A}\hat{B}}$ on a new suspension manifold $M_{\hat{B}}$ which finitely covers the original. Further, $\tilde{f}$ quotients to a function $\hat{f} : M_{\hat{B}} \to M_{\hat{B}}$ which is a lift of the original f.

Define $\tilde{h} : G \times \mathbb{R} \to G \times \mathbb{R}$ by $\tilde{h}(x,t) = (x, t + S(x))$. If $\gamma \in \Gamma_0$, then $S(\gamma) \in k\mathbb{Z}$ and since B^k is the identity, it follows that $\beta^{S(\gamma)}(x,t) = (x, t - S(\gamma))$ which may be used to show that $\tilde{h}\tau_\gamma = \tau_\gamma \beta^{-S(\gamma)} \tilde{h}$. This implies that $\tilde{h}$ quotients to a diffeomorphism $\hat{h}$ on $M_{\hat{B}}$ and that induced action on $\pi_1(M_{\hat{B}})$ satisfies $\hat{h}_*(\beta) = \beta$ and $\hat{h}_*(\tau_\gamma) = \tau_\gamma \beta^{-S(\gamma)}$ for all $\gamma \in \Gamma_0$. Further note that $\hat{h}$ is a leaf conjugacy between $\hat{f}$ and $f_{\hat{A}\hat{B}}$. Since

$$\hat{h}_* \hat{f}_* \hat{h}_*^{-1}(\tau_\gamma) = \hat{h}_* f_* \hat{h}_*^{-1}(\tau_\gamma) = \tau_{A(\gamma)} \beta^{-SA(\gamma)} \beta^{L(\gamma)} \beta^{S(\gamma)} = \tau_{A(\gamma)},$$

it follows that $\hat{h}\hat{f}\hat{h}^{-1}$ and $f_{\hat{A}\hat{B}}$ have the same action on $\pi_1(M_{\hat{B}})$ and so are homotopic.

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