MTH 41041 (Semester 1, 2017) Assignment 5

Important

(a) The assignment is due at 11:30 am on Friday, 7 April, 2017. You can either hand them in at the start of class, hand them to me in person beforehand, or slip it under my office door.

(b) You can talk with the other students and with me about the problems, but you must write up and hand in your own work.

(c) We expect that you will put enough thought and effort into the presentation of your solutions so that they are neat, clear, and concise. Poorly presented assignments will be penalized.

Questions.

(1) Let ∇ be a connection on a manifold M, let X and Y be vector fields on M and let U be a neighborhood of a point p. Show that $\nabla_X Y(p)$ depends only on the values that X and Y take on U.

Note that after proving this question, you may without loss of generality reduce a pointwise problem about a connection on a manifold M to a problem about a connection on an open subset of $\mathbb{R}^m$.

(2) Let ∇ be a connection on a manifold M and let X, Y and Z be vector fields on M. Show that if $X(p) = Y(p)$ at a point $p \in M$, then $\nabla_X Z(p) = \nabla_Y Z(p)$.

(3) Let ∇ be an arbitrary connection on $\mathbb{R}^m$. (That is, ∇ is not necessarily the Euclidean connection.) Let $\gamma : \mathbb{R} \to \mathbb{R}^m$ be the embedding $\gamma(t) = (t, 0, 0, \cdots, 0)$. Show that if X and Y are vector fields on $\mathbb{R}^m$ such that $X(p) = \partial_1$ and $Y(p) = 0$ for all $p \in \gamma(\mathbb{R})$, then $\nabla_X Y(p) = 0$ for all $p \in \gamma(\mathbb{R})$.

Here, $\partial_1 = \frac{\partial}{\partial x^1}$ is the vector field pointing in the direction of the first coordinate of $\mathbb{R}^m$.

(4) Using the last question, show that for an immersed curve $\gamma : I \to M$ and a vector field $v \in \mathscr{X}(\gamma)$ over the curve that the covariant derivative of v is well defined.