MTH 41041 (Semester 1, 2017) Assignment 3

Important

(a) The assignment is due at 11:30 am on Friday, 24 March, 2017. You can either hand your assignment in at the start of class, hand it to me in person beforehand, or slip it under my office door.

(b) You can talk with the other students and with me about the problems, but you must write up and hand in your own work.

(c) We expect that you will put enough thought and effort into the presentation of your solutions so that they are neat, clear, and concise. Poorly presented assignments will be penalized.

Problem 1 Thinking of the n-sphere $S^n = \{ x \in \mathbb{R}^{n+1} \,|\, \|x\| = 1 \}$ as a hypersurface in $\mathbb{R}^{n+1}$, the tangent plane to a point $x \in S^n$ can be defined as

$$\mathcal{T}_x S^n = \{ v \in \mathbb{R}^{n+1} \,|\, \langle v|x\rangle = 0 \},$$

where $\langle \cdot|\cdot \rangle$ is the standard Euclidean inner product on $\mathbb{R}^{n+1}$. Show that $\mathcal{T}_p S^n$ is naturally isomorphic to the abstract definition of the tangent plane $T_p S^n$ from class. (*Hint:* Associate to each $v \in \mathcal{T}_x S^n$ a curve $\gamma_v(t) \in \mathbb{R}^{n+1}$ such that $\gamma_v(t) \in S^n$ for each t, $\frac{\partial}{\partial t}\big|_{t=0} \gamma_v(t) = v$, and $\gamma_v(0) = x$. Next, consider having v "act" on smooth functions $f \in C^\infty(S^n)$, which are just the smooth functions on $\mathbb{R}^{n+1}$ restricted to S^n, by defining $v(f) := \frac{\partial}{\partial t}\big|_{t=0} f(\gamma_v(t))$.)

Problem 2 Let $\mathbb{B}^n = \{ x \in \mathbb{R}^n \,|\, |x| < 1\}$ denote the open ball in $\mathbb{R}^n$. Show that $\mathbb{B}^n$ and $\mathbb{R}^n$ are diffeomorphic.