Lecture 6

Thm $f : M_A \to M_A$ p.h. s.t.
$\nexists$ f-periodic $\mathbb{T}$ tang to E^{cs} or E^{cu}
$\Rightarrow$ f is dynamically coherent
and (an iterate) is leaf conj to
the susp. of an Anosov.

$A \in SL(2,\mathbb{Z})$
hyp.

$$M_A = \mathbb{T}^2 \times [0,1] \big/ (x,1) \sim (Ax,0)$$

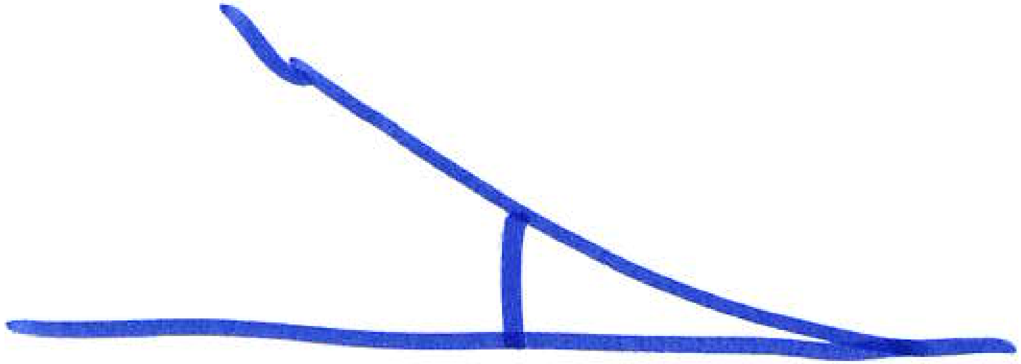
Anosov flows:

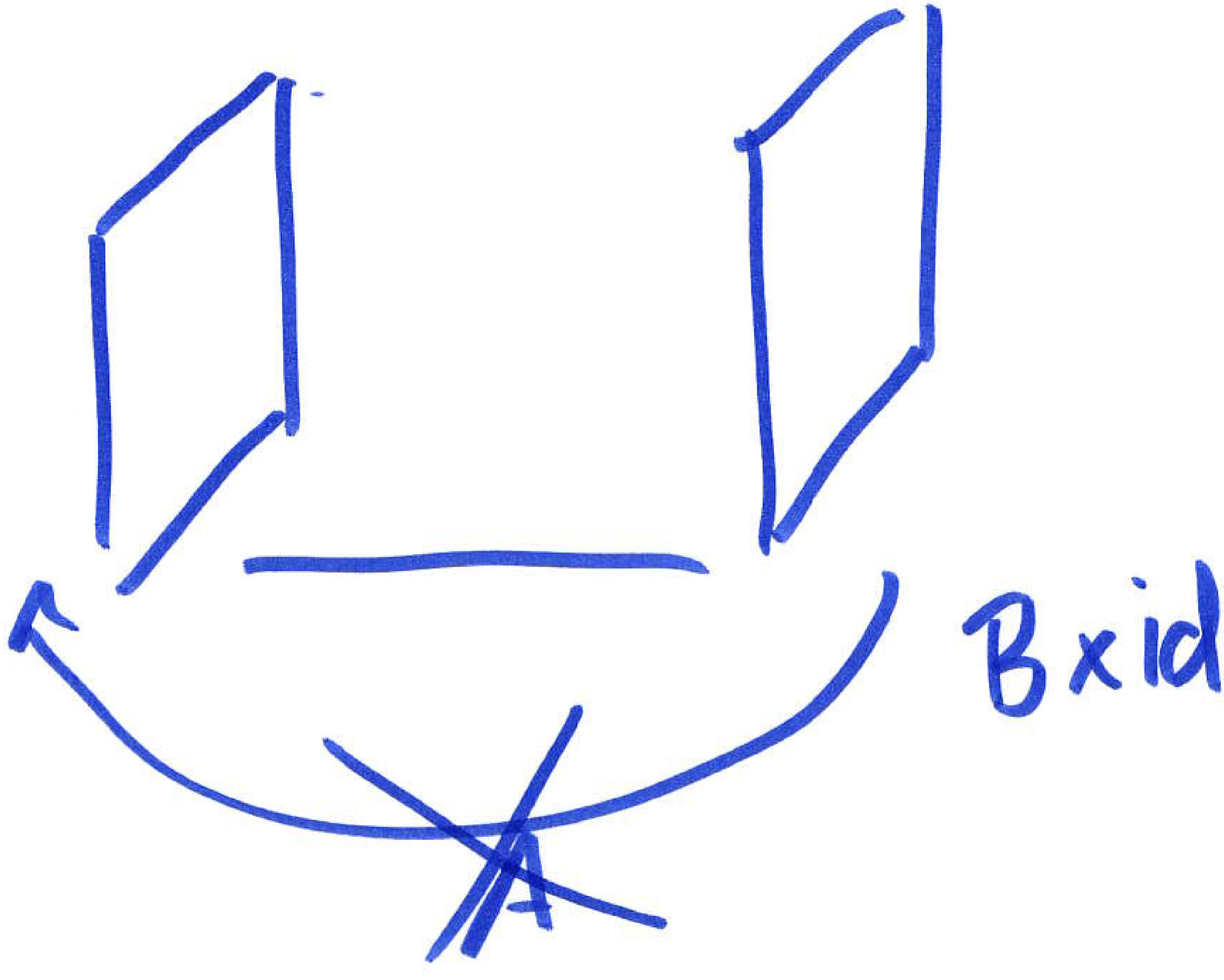
→ center leaves must be fixed

→ no periodic points in the universal cover.

Tools: 1) Classify foliations without tori (leaves separate)

2) Mapping class group in M_A is finite

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$$\tilde{M}_A = \mathbb{R}^2 \times \mathbb{R}$$

$$\gamma_1(x,t) = (x+(1,0),t)$$

$$\gamma_2(x,t) = (x+(0,1),t)$$

$$\gamma_3(x,t) = (Ax,t-1)$$

$$\Phi_s(x,t) = (x, t+s)$$

$$\mathcal{A}^{cs} = E^s_A \times \mathbb{R}$$

$$\mathcal{A}^{cu} = E^u_A \times \mathbb{R}$$

Prop: Every $\mathcal{F}$ foliation w.o. tori of M_A is Almost parallel to either $\mathcal{A}^{cs}$ or $\mathcal{A}^{cu}$.

Th (BI) $\exists$ $\mathcal{F}^{cs}$ and $\mathcal{F}^{cu}$ f-inv branching foliations which are almost parallel to true foliations

$\mathcal{F}^{cs}$ is AP either $\mathcal{A}^{cs}$ or $\mathcal{A}^{cu}$

$\mathcal{F}^{cu}$ is AP " $\mathcal{A}^{cs}$ or $\mathcal{A}^{cu}$

<u>Assumption</u>: $\mathcal{F}^{cu}$ is $\underset{\substack{\uparrow \\ R}}{\text{AP}}$ to $\mathcal{A}^{cu}$

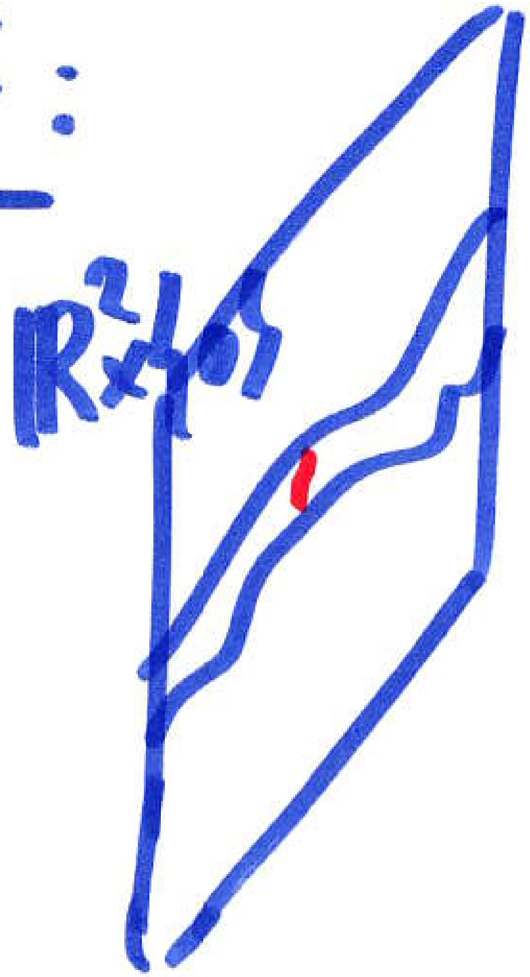
1st GOAL: Show $\mathcal{F}^{cs}$ is AP to $\mathcal{A}^{cs}$

Choose $\tilde{f}$ a lift $d(\tilde{f}, id) < K_0$

For a leaf $\mathcal{L}$ of $\mathcal{A}^{cu}$ consider $\Gamma \subseteq \tilde{\mathcal{F}}^{cu}$ the set of leaves at dist. $< R$ from $\mathcal{L}$.

Lemma Γ is a unique leaf.
(no Denjoy)

Pf:



A^{cu} is fixed by a deck tr.

$(x,t) \mapsto (x+n,t)$

Slice $\times\, [P(n), p(n)]$

$\blacksquare$

Corollary: Every surface of $\tilde{\mathcal{F}}^{cu}$ is fixed by $\tilde{f}$.

$$\tilde{\mathcal{F}}^{cu} \;\updownarrow\; \tilde{\mathcal{F}}^{cs}$$

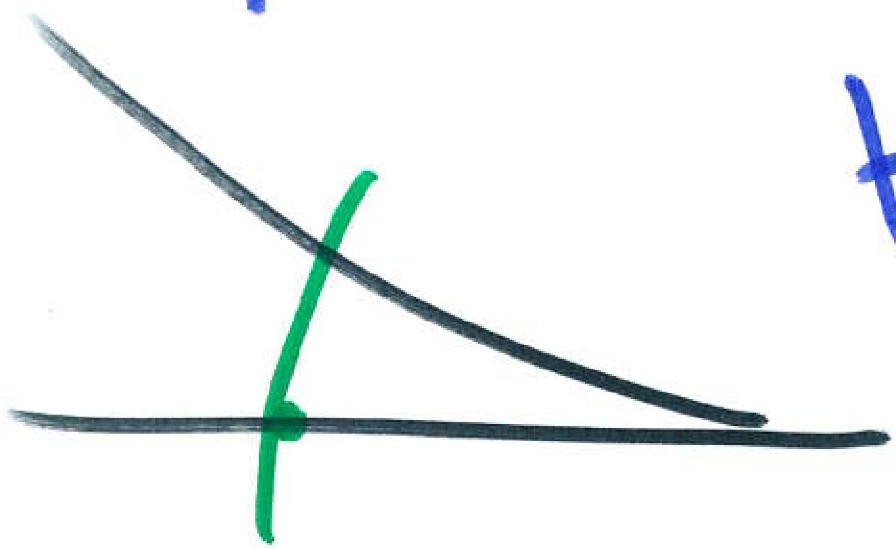
$$p : \mathbb{R}^2 \times \mathbb{R} \to \mathbb{R}$$

$$(x, t) \longmapsto t$$

<u>Crucial lemma</u> $\exists K_1$ such that $\forall x \in \mathbb{R}^3$ we have that

$$p(\tilde{f}^n(x)) - p(x) > -K_1$$

$$\forall n \geq 0$$



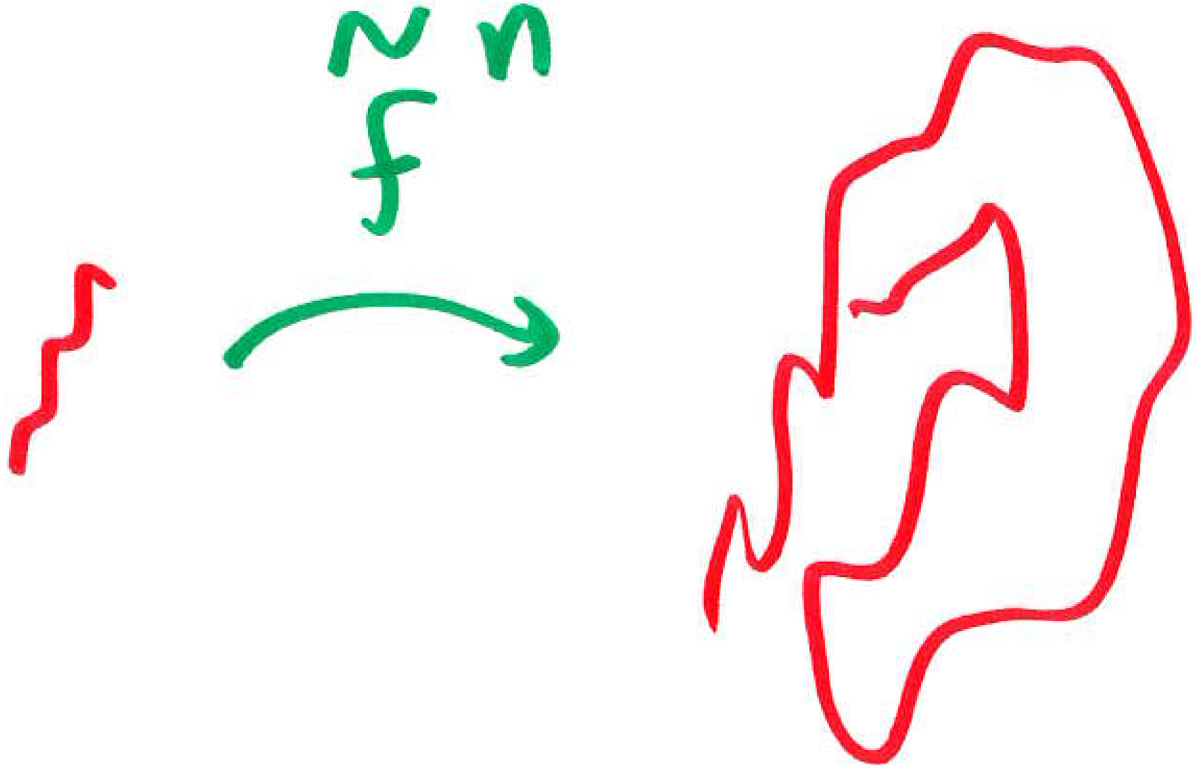
Prop: $\tilde{\mathcal{F}}^{cs}$ is AP to $\mathcal{A}^{cs}$.

Pf: Assume $\mathcal{F}^{cs}$ AP to $\mathcal{A}^{cu}$

$$\forall x \in \mathbb{R}^3 \quad p(\tilde{f}^{-n}(x)) - p(x) > -K_1$$
$$\forall n \geq 0$$

$$\Rightarrow \forall x \in \mathbb{R}^3 \quad \forall n \in \mathbb{Z}$$

$$-K_1 < p(\tilde{f}^{n}(x)) - p(x) < K_1$$

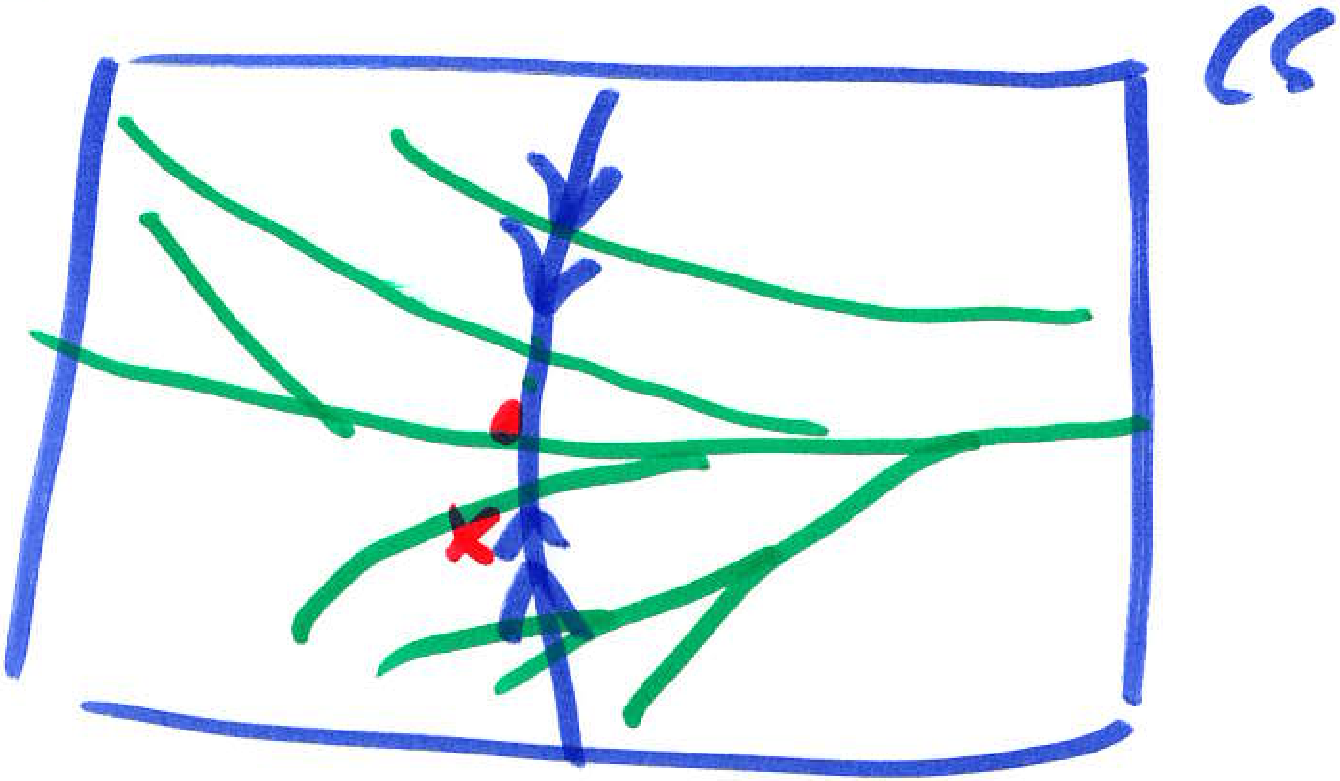


Novikov Thm.

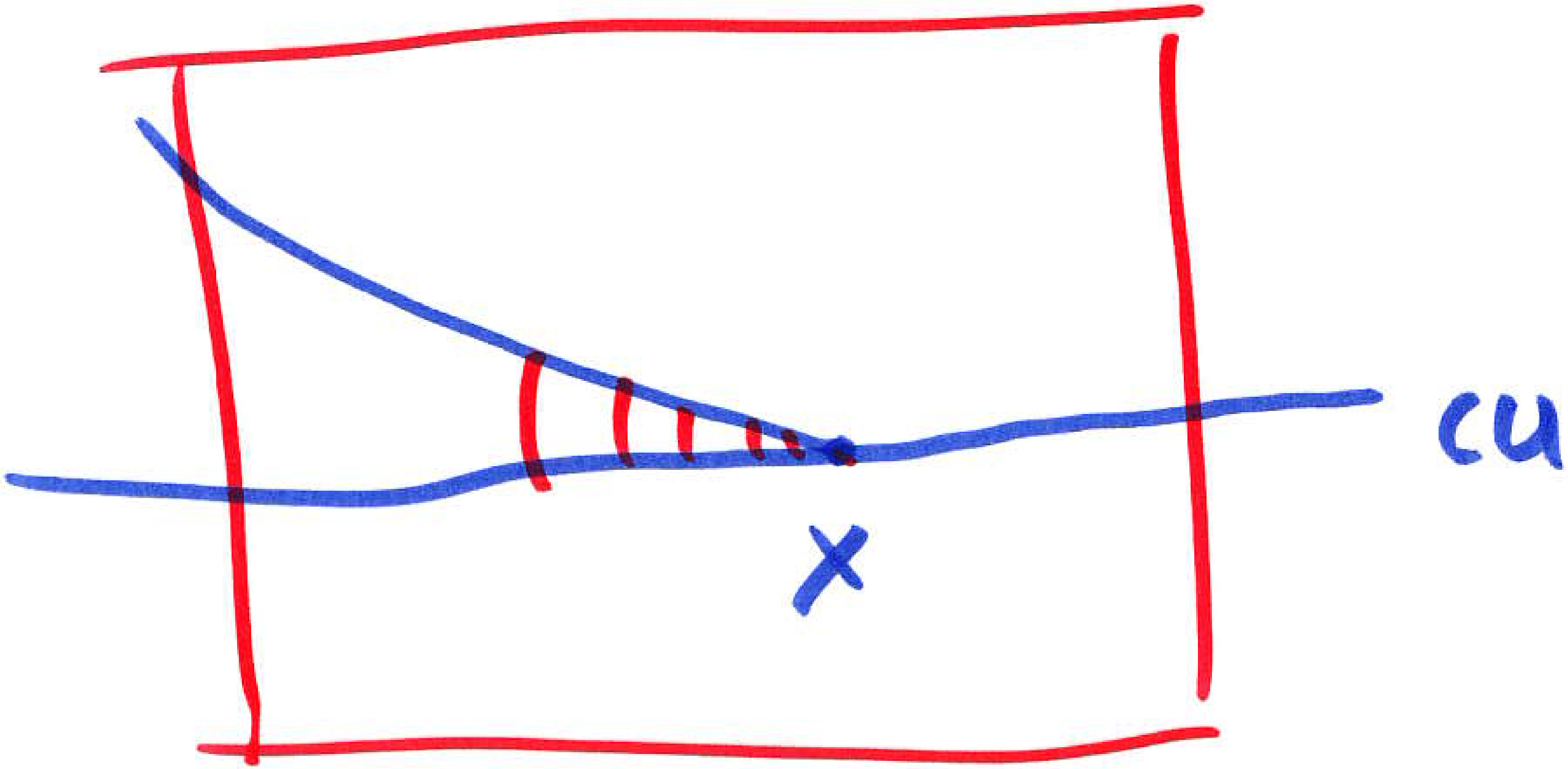
Consequence (modulo iterate)

Every intersection between cs-surfaces and cu-surfaces is fixed by $\tilde{f}$.

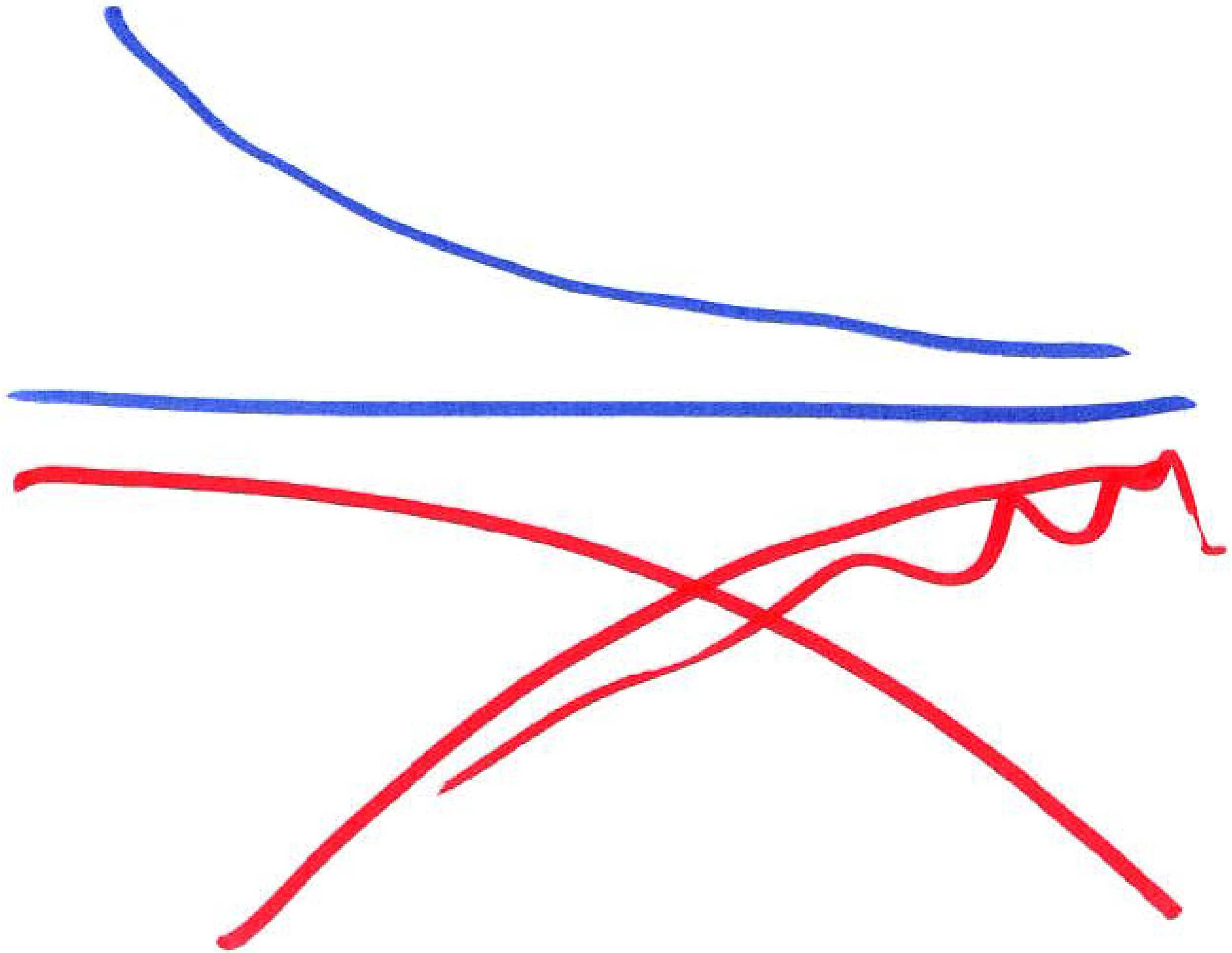
BW argument : No periodic points in the universal cover



Dynamical coherence:



$\Longrightarrow$ OK + uniform advance in flow direction



Conj: Every p.h. $\left\{ \begin{array}{l} \text{transitive} \\ \text{dyn.coh.} + \end{array} \right\}$ $\dim M = 3$

leaf. is

→ Anosov

→ Skew product

→ Anosov flow.