Lecture 2.

INTEGRABILITY PROBLEM

$f: M \to M$ p.h $TM = \underbrace{E^s \oplus \overbrace{E^c}^{}}_{E^{cs}} \oplus E^u$, $E^{cu} = E^c \oplus E^u$

f is dynamically coherent

$\exists$ f-inv. W^{cs} & W^{cu} tangent to E^{cs} and E^{cu}.

E^s and E^u are uniquely integrable

<u>Prop</u> (Burago-Ivanov) γ curve tg to E^{cs} and transverse to E^s then $S = \bigcup_{y \in \gamma} W^s(y)$ is a surface tg to E^{cs}.

<u>Idea</u>: C^0-frobenius property.

Thm (Burago-Ivanov)

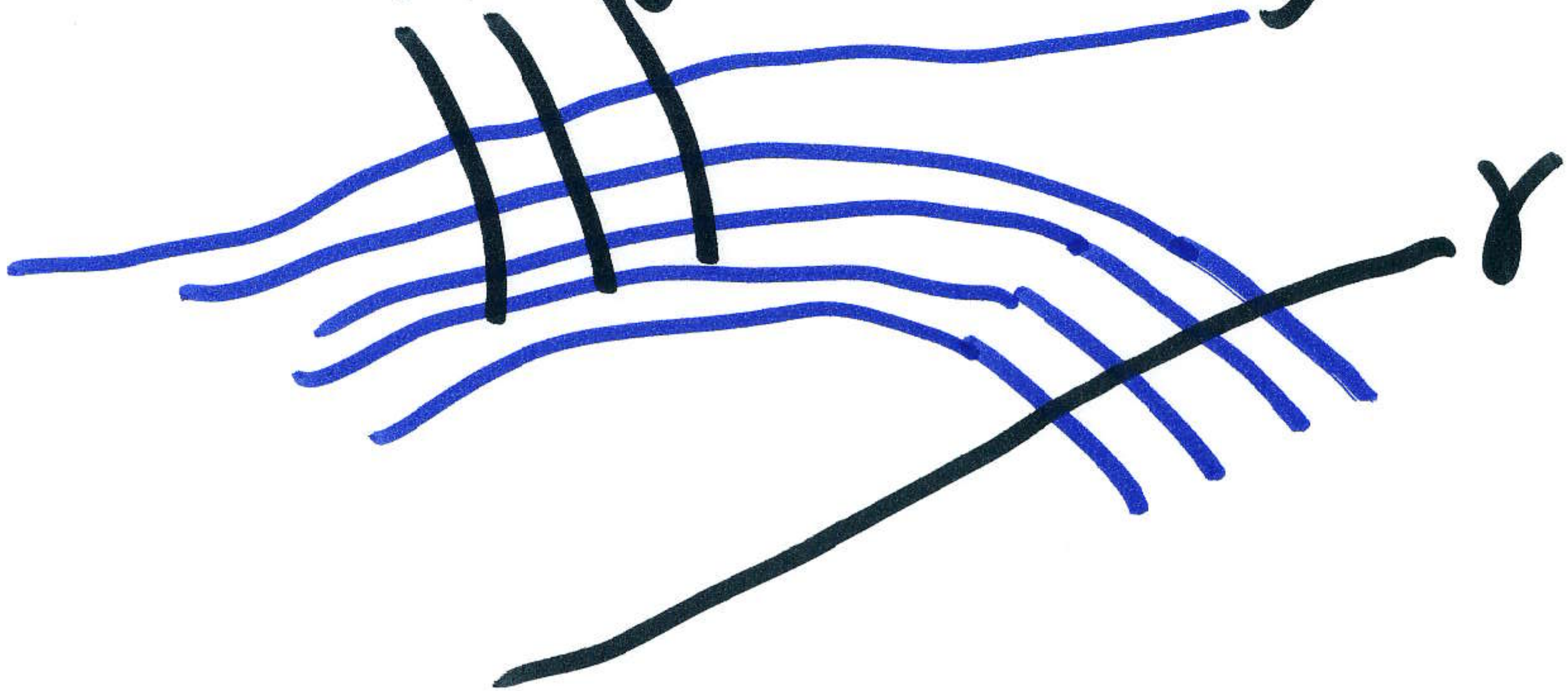
Assume E^{σ} are orientable, Df-pres.

$\exists\ \mathcal{F}^{cs}, \mathcal{F}^{cu}$ f-inv **branching foliations** and tangent to E^{cs} and E^{cu}.

Idea: Choose the "lowest" surfaces

→ f-inv.

→ no "topological crossings"



Branching Foliation:

$\mathcal{F}$ collection $\{L_\alpha\}$ of complete surfaces tangent to a continuous dist. E s.t.: $\rightarrow$ $\bigcup L_\alpha = M$

$\rightarrow$ no top. crossings.

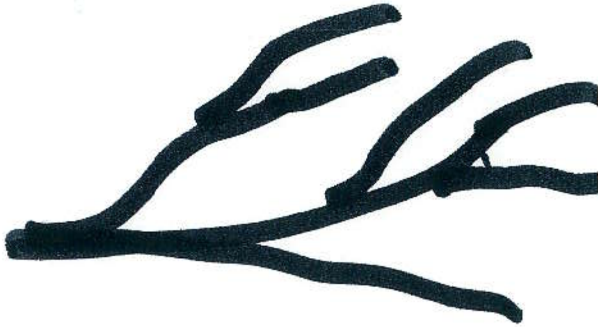
$\rightarrow$ $x_n \rightarrow x$, $x_n \in L_{\alpha_n}$

$L_{\alpha_n} \rightarrow L_\alpha$

Remark/Def (Bonatti-Wilkinson)

A ***foliation*** is a branching foliation without branching.

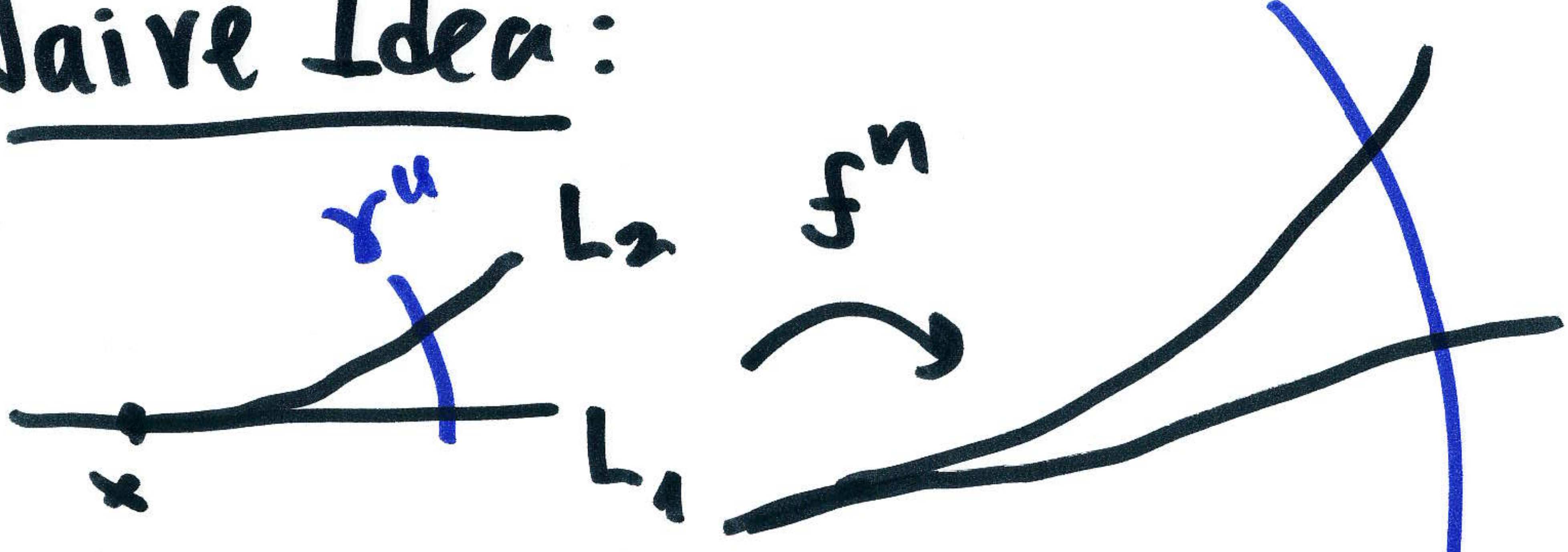
If $\exists W^{cs}$ then S^3 cannot admit p.h. diffeos.



[**Thm** (BI) It is possible to "blow up" the branching foliation]

Corollary S^3 does not admit ph diffeos

Naive Idea:



Under 2 assumptions this works:

Brin

- → Absolute partial hyperbolicity
- → quasi-isometry of $\mathcal{W}^u$.

Thm (BBI) $f : \mathbb{T}^3 \to \mathbb{T}^3$ absolute partially hyp. diffeo $\Rightarrow f$ is dynamically coherent

Proof: Showing $\mathcal{W}^s, \mathcal{W}^u$ are quasi-isometric.

$$d_{\tilde{\mathcal{W}}}(x,y) \leq a\, d(x,y) + b$$

Thm (J, F. Rodriguez Hertz, Ures)

$\exists$ open sets of p.h. in $\mathbb{T}^3$ which are not d.c.

Conj (RH, RH, U) If $\nexists$ T f-periodic tangent to E^{cs} or E^{cu} $\Longrightarrow$ f is d.c.

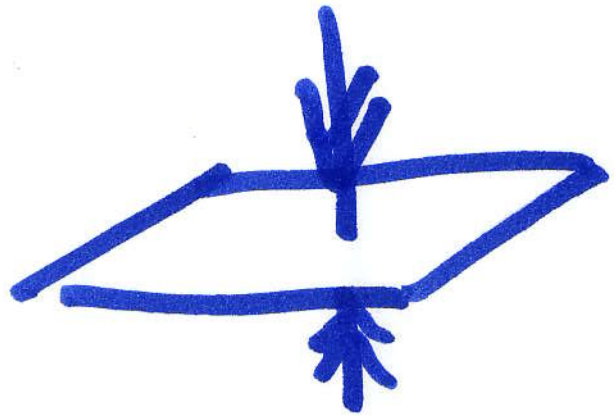
Example:

$A : \mathbb{T}^2 \circlearrowleft$ linear anosov difeo

eigenvalues $\lambda < 1$ and λ^{-1}

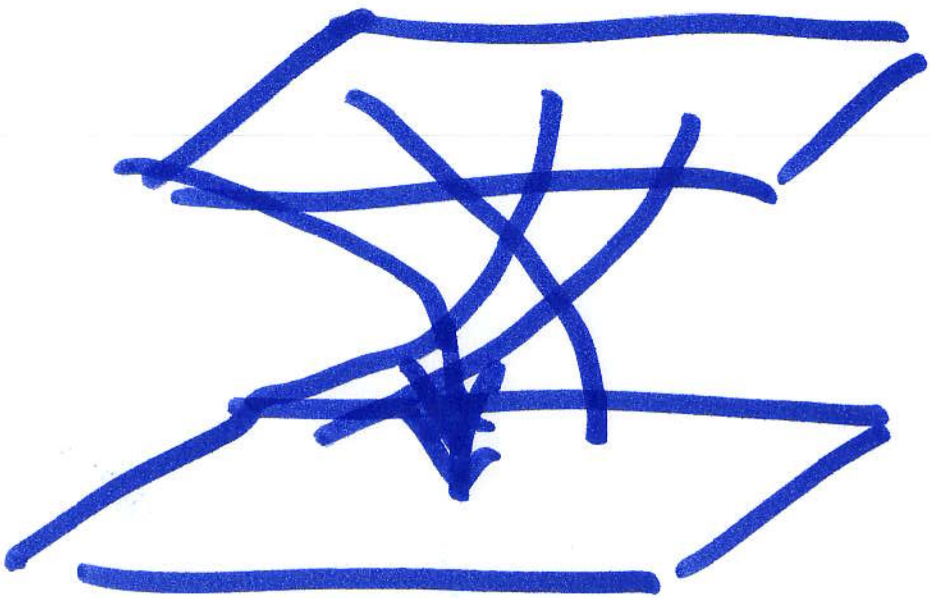
$f_1 : \mathbb{T}^2 \times [-1,1] \circlearrowleft$ $\quad f_1(x,t) = (Ax, \mu t)$

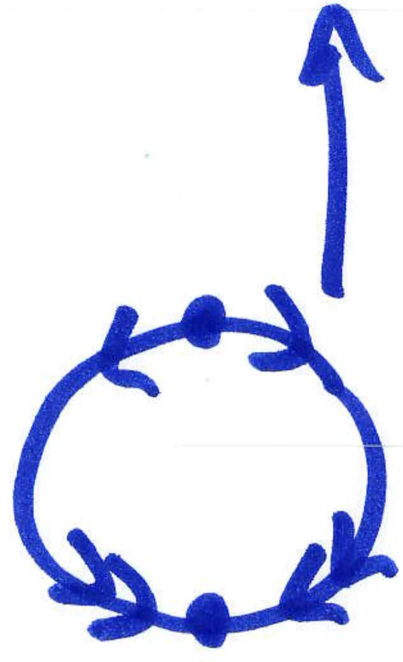
$\mu < \lambda$

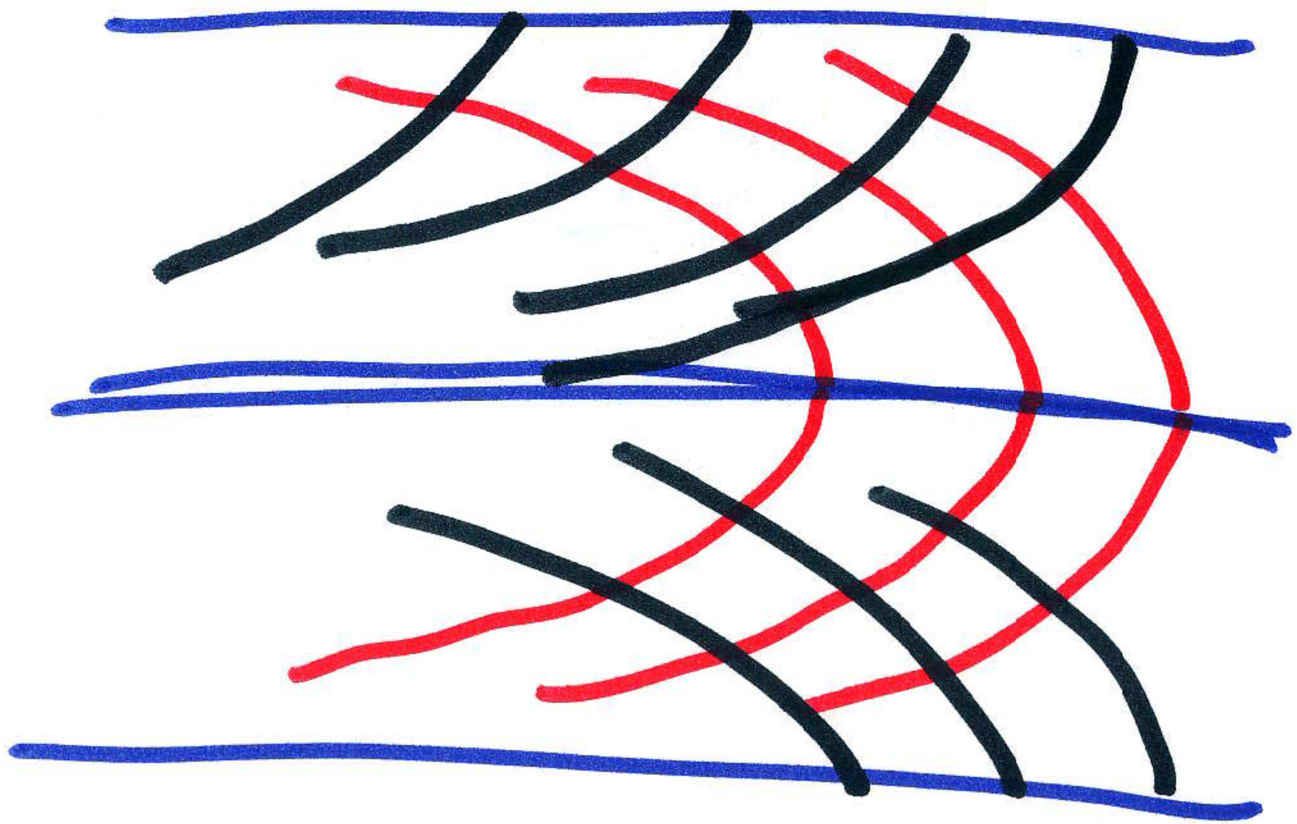


$f_2 : \mathbb{T}^2 \times [-1, 1]$ "$\circlearrowleft$" $\quad f_2(x, t) = (Ax, \theta t,$

$f : \mathbb{T}^2 \times S^1 \quad f(x, t) = (Ax + \varphi(t) v^s,$
$\psi(t))$






T^{ESu}
T^{CSu}

The example can be done in $M_A = \mathbb{T}^2 \times [0,1] / \sim$

Thm (HP) If $\nexists T$ cs or cu

$\Rightarrow f$ is dyn. coherent.

(if $\pi_1(M)$ is solvable)

$\rightarrow$ Transitivity, conservative

$\rightarrow$ absolutely p.h.

Prop If $\exists$ periodic cs or cu $T \Rightarrow f$ is not abs. p.h.

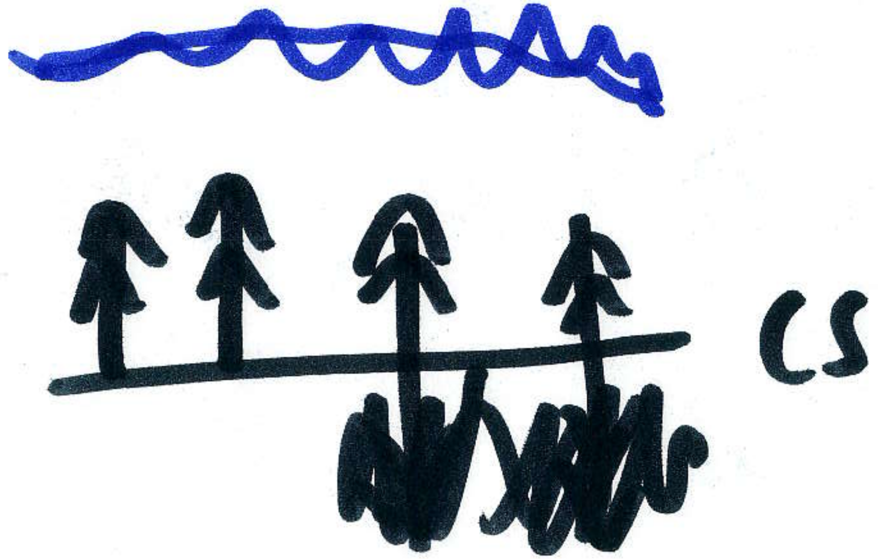
1) $f/_T$ where T is cs.

$$h_{top}(f/_T) = h_{top}(A)$$

$\left.\begin{array}{l}\text{Var. pple}\\ \text{Ruelle ineq}\end{array}\right\} \Rightarrow \exists \mu$

$$\lambda^c(\mu) > h_{top}(A) - \varepsilon$$

2)



W^u-saturated

Λ p.h. attractor

$f/_\Lambda$ semiconj to A

$$h_{top}(f/_\Lambda) = h_{top}(A)$$

$\exists\, \mu'$ such that $\lambda^u(\mu') \leq h_{top}(A)$