Does simultaneous solution matter for stellar evolution codes?*

Richard J. Stancliffe
Institute of Astronomy and Churchill College, Cambridge
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Abstract

A version of the STARS stellar evolution code has been developed that uses a non-simultaneous solution of the equations of stellar structure and evolution. In all other respects it is identical to the normal, fully simultaneous version. It is therefore possible to test the dependence of the solution on how the equations are solved. Two cases are investigated: a 5-$M_\odot$ and a 3-$M_\odot$ star, both of metallicity Z=0.02. Prior to the asymptotic giant branch, the models are almost identical. However once thermal pulses start, the two methods of solution yield diverging results with the non-simultaneous technique predicting longer interpulse periods. This is traced to difficulties associated with hydrogen burning caused by the use of a moving mesh. It is shown that, with careful control of the temporal resolution, the results of the simultaneous technique can be recovered.

Introduction

Calculations of TP-AGB evolution made with different stellar evolution codes show quite different results (Lugaro et al. 2003). The reason for this is not understood. Calculations could be effected by the choice of the method of solution of the equations of stellar structure and evolution. While it is thought that the method of solution will not affect the results obtained (Herwig 2005), this assumption has not been tested until now.

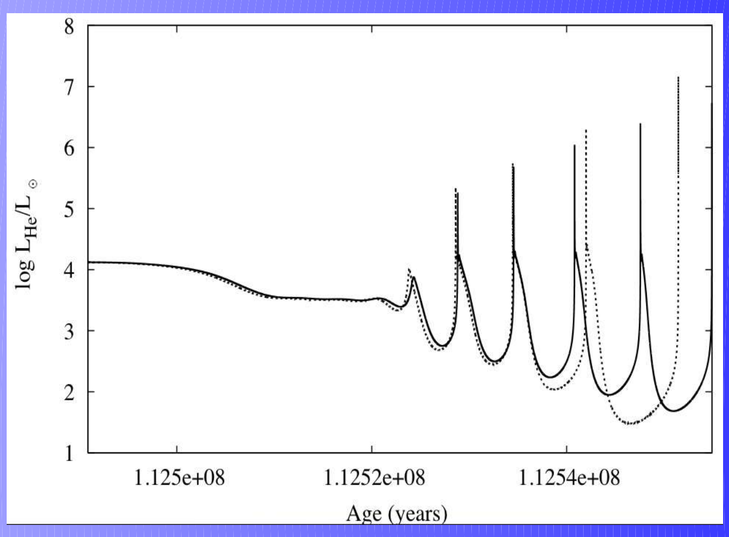

Fig 1. Evolution of the helium luminosity of the 5-$M_\odot$ model. The simultaneous solution (solid line) and non-simultaneous solution (dashed line). Despite starting from the same model, the two solutions rapidly diverge.

The Evolution Code

A version of the STARS stellar evolution code (Eggleton 1971) has been developed that employs a non-simultaneous solution of the equations of stellar structure and evolution. We can thus test the effect of the method of solution on the evolution obtained. By changing the method of solution employed it is possible to investigate the effect of this alone. We are not hampered by the differences in equation of state, reaction rates, timestep control, differencing scheme, etc. that would exist if a comparison were to be made between two different codes.

Models were evolved from the pre-main sequence to the TP-AGB with both methods of solution. Prior to the TP-AGB no discernable difference is seen in the evolution. The models begin the TP-AGB with core masses within 1% of each other – much less than the differences in core mass seen between different evolution codes. For example, the core masses of a 5-$M_\odot$ Z=0.02 model as calculated by Karakas et al. (2002) and Stancliffe et al. (2004) are different by about 7%. The core mass at which TDUP sets in is also the same to within a percent for both methods of solution. The method of solution does not account for the differences seen between different evolution codes.

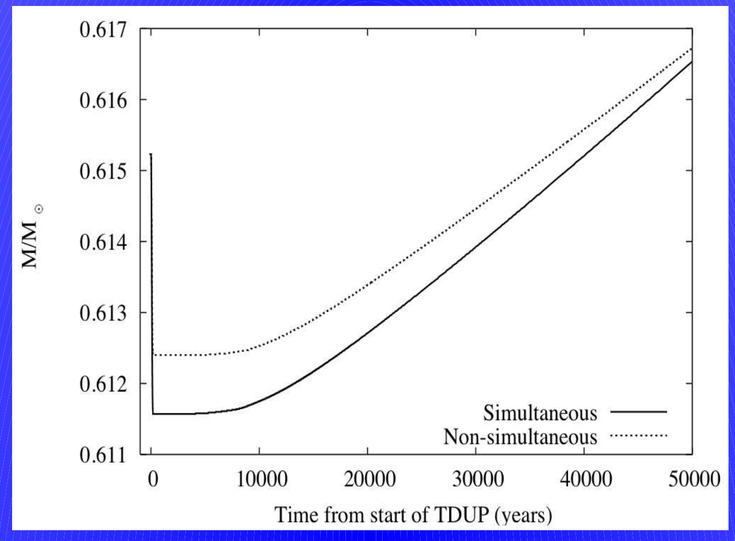

Fig 2. Evolution of the H-exhausted core mass after the 10th TP of a 3-$M_\odot$ model. Note that the dredge-up is shallower when the non-simultaneous method of solution is employed.

Results

Fig. 1 shows the evolution of a 5-$M_\odot$ star evolved from the same starting model along the TP-AGB with the two methods of solution. Within a few pulses the models are seen to diverge. Similar results are seen in with a 3-$M_\odot$ star.

The depth of TDUP is also effected. Fig. 3 shows the evolution of the H-exhausted core mass for the two methods of solution. These models were started from the same initial model, taken from after the 10th TP of a 3-$M_\odot$ model. The simultaneous solution gives noticeably deeper dredge-up.

It is found that the divergence is due to problems associated with H-burning with the non-simultaneous method of solution. Fig. 3 shows the evolution of the H-burning luminosity for a 1.5-$M_\odot$ model. If the timestep of the non-simultaneous solution is reduced by over an order of magnitude, the simultaneous solution is recovered.

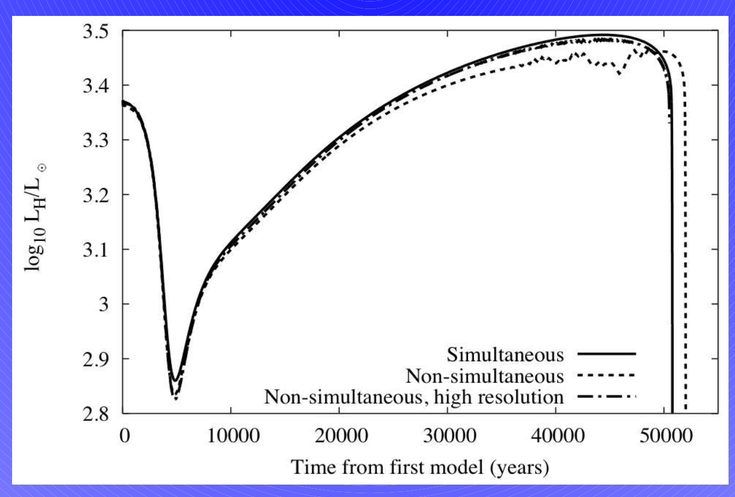

Fig 3. Evolution of the hydrogen-burning luminosity of a 1.5-$M_\odot$, Z=0.02 model. By using very small timesteps (high temporal resolution) with a non-simultaneous method of solution, the solution of the simultaneous method can be approached.

Numerical Diffusion

The H-burning problems are associated with the use of a non-Lagrangian mesh together with a non-simultaneous solution. When a sharp composition profile exists, movement of the mesh leads to numerical diffusion (as depicted in Fig. 4). This leads to extra fuel ending up where it should not physically be. The burning rate is enhanced and this feeds back to the structure.

In a Lagrangian mesh, this problem does not occur. With a non-Lagrangian mesh the problem can be dealt with if the timesteps are kept small as this limits the extent of the diffusion. This is why the simultaneous solution can be recovered when using a non-simultaneous solution.

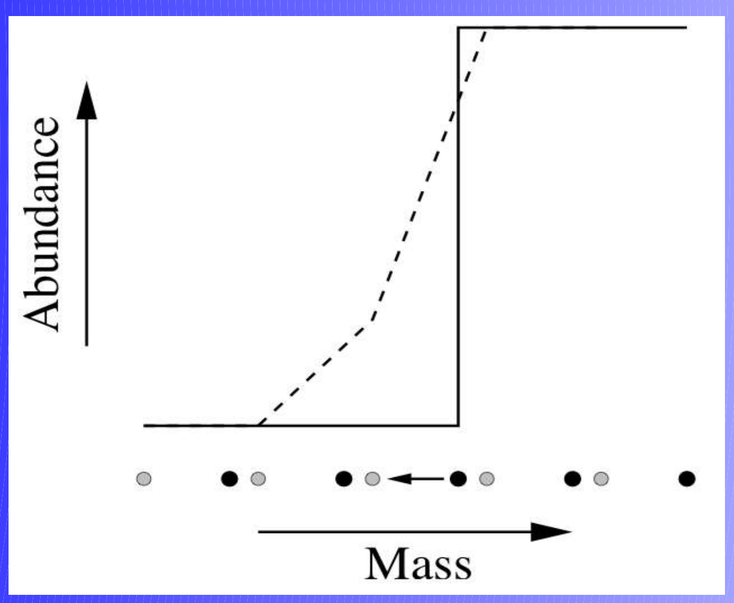

Fig 4. A schematic depiction of why numerical diffusion occurs. A sharp abundance profile (solid line) is set up over a few mesh points (black dots). If the mesh points then move in mass to the positions indicated by the grey circles, the abundance profile is modified to that indicated by the dashed line.

Future Work

The use of a non-Lagrangian mesh together with a non-simultaneous method of solution has been shown to be problematic. The next step would be to develop a version of the code that uses a Lagrangian mesh. It would then be possible to make a more detailed comparison of the effect of the method of solution and also of the type of mesh.

Acknowledgements
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