

Network modelling of underground mine layout: Two case studies

Doreen A. Thomas* Marcus Brazil* David H. Lee[†]
Nicholas C. Wormald[‡]

ARC Special Research Centre for Ultra-Broadband Information Networks (CUBIN)

*Department of Electrical and Electronic Engineering
The University of Melbourne, Victoria 3010 Australia
Email: {d.thomas, brazil}@ee.unimelb.edu.au*

*[†]Department of Mathematics and Statistics
The University of Melbourne, Victoria 3010 Australia
Email: dhlee@bigpond.net.au*

*[‡]Department of Combinatorics and Optimization
University of Waterloo, Waterloo ON N2L 3G1, Canada
Email: nwormald@uwaterloo.ca*

¹CUBIN is an affiliated program of National ICT Australia.

Abstract

The dominant working structure of an underground mine is a set of interconnected tunnels which provides access to ore zones and haulage of ore from the designated ore zones to the mill. This set of interconnected tunnels forms a network. We describe a mathematical network model and the modelling features of our two software tools for designing underground mine layouts that minimise associated costs. The application of these techniques is illustrated in two industry case studies. In the first, we apply the Underground Network Optimisation tool (UNOTM) to design an extension to an Australian gold mine where 15 new distinct ore bodies are located in a 3km long region, several hundred metres underground. In a second case study, we design a single decline for accessing a large orebody where a turning circle constraint is a significant factor. An efficient decline is found using the Decline Optimisation Tool (DOTTM).

1 Introduction

An underground mine can be viewed as a network of interconnected tunnels and shafts that provide access to designated ore zones and a means of transporting ore from these ore zones to the mill. There are several different basic forms for the infrastructure of an underground mine. Most underground mines use inclined tunnels, called ramps or declines, and horizontal tunnels, called drives (Figure 1). These need to be navigable by the large trucks that are used for haulage. In other long life or deeper mines a vertical shaft may also be used to haul mined material to the surface. A vertical shaft is an expensive piece of infrastructure, but it offers a lower unit cost per vertical meter for transporting ore and therefore has advantages over the more expensive truck haulage for large tonnage mines. Mines equipped with vertical haulage shafts often use orepasses - inclined chutes through which ore is dropped from the mining levels to the loading level of a shaft (Figure 2).

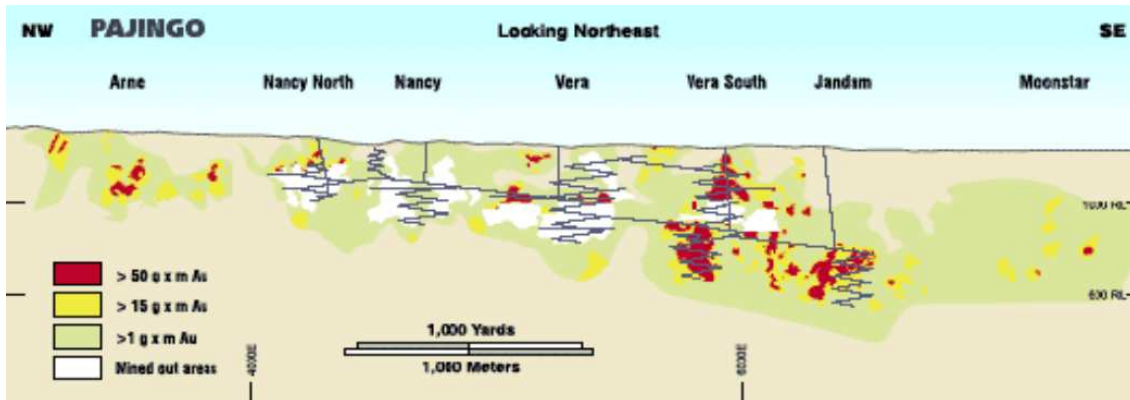


Figure 1: The network of ramps servicing Newmont’s Pajingo Gold mines in Australia. The near vertical lines indicate ventilation raises (not shafts).

Regardless of the particular infrastructure chosen for a mine, its layout should be designed so as to minimise both the construction costs as well as the haulage costs over the life of the mine. An efficient layout of the infrastructure is a key to achieving the low cost construction and haulage operations needed to make a mine profitable.

Current industry practice in underground mine development design involves no rigorous optimisation. The usual industry approach is to rely on the experience of mining engineers to find ‘good’ feasible design solutions. The Network Research Group based at The University of Melbourne have been developing techniques to find improved solutions to these design problems using new mathematical algorithms and software based on network optimisation. The value of this approach has been demonstrated in a number of real case studies, and this research is now receiving strong industry support.

In Section 2, we establish our network models for underground mine layouts and develop our approach for solving the associated network optimisation problems. In Section 3 we then describe the main features of the software tools we have developed

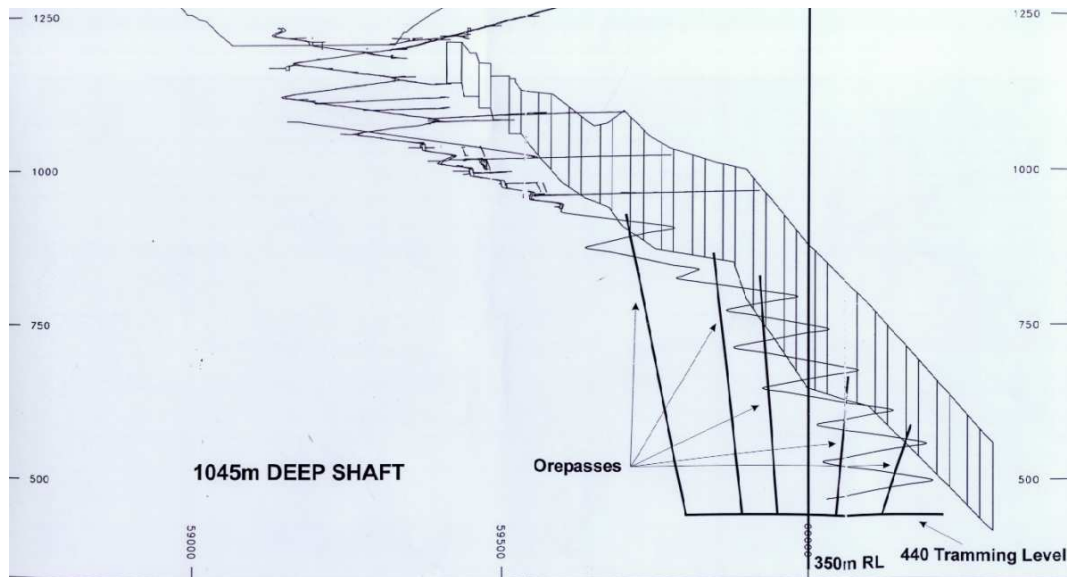


Figure 2: A proposed mine design incorporating a vertical shaft and orepasses

for minimising the cost of underground mine layouts, using this approach. Much of the material in these two sections has appeared before, but is summarised here to make this paper self-contained. The mathematics of gradient-constrained Steiner trees, which lies at the heart of Section 2, was introduced in [Brazil et al., 2001]. Early versions of the software packages in Section 3 were first described in [Brazil et al., 2000] (UNOTM) and [Brazil et al., 2003](DOTTM). In Section 4, the main section of this paper, we present two new industrial case studies that demonstrate the practical value of our approach. A summary of the first case study also appears in the paper [Brazil and Thomas, 2007].

2 Network models

In this section we discuss a network optimisation approach to the underground mine design problem and describe the mathematical features of the resulting network model. The basic situation considered here is that of a mine serviced only by declines for access and haulage. Our methods, however, can also be applied to more general mine layouts which may include shafts and orepasses [Brazil et al., 2005a].

The *network model* for an underground mine aims to capture the proposed layout and costs of a real mine. The network model is an example of a 3-dimensional mathematical network, which is a set of nodes interconnected by links, all embedded in 3-space. In the network model for an underground mine the set of nodes consists of terminals, whose positions are pre-determined before optimisation, and variable nodes. The terminals correspond to: the given surface portal of the mine, the given set of draw points (whence ore is drawn), and the given set of access points (points which must be accessed for operations such as drilling and blasting, but that are not draw points). The variable nodes (also known as *Steiner points*) correspond to junctions at which three or more ramps or drives meet. Links in the network model correspond to the centrelines of ramps and drives.

Ramps have large cross-sections - typically $4\text{m} \times 4\text{m}$ - and must be connected and navigable by mine equipment. Ramps are expensive to build with costs typically in the range of AU\$2,500 to AU\$3,500 per metre. A key navigability requirement is that the gradient of each ramp is constrained to be within a safe climbing limit for trucks, typically in the range 1:9 to 1:6.5. Hence, in the network model each link is gradient-constrained with a given maximum absolute slope m , and the optimal network can be considered as a gradient-constrained Steiner network, which we define below.

A Steiner network is a least cost network interconnecting a given set of nodes in a given metric space [Hwang et al., 1992]. The feature that distinguishes Steiner networks from minimum spanning networks is that a Steiner network may contain

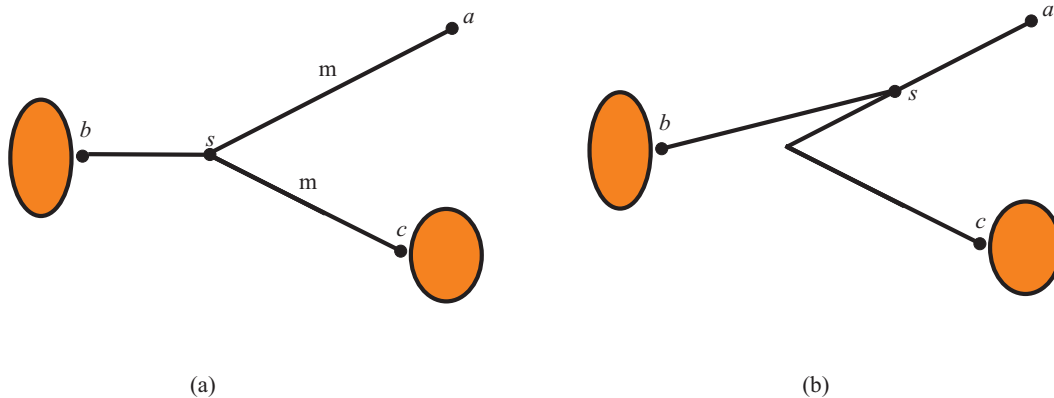


Figure 3: Minimum gradient constrained Steiner networks interconnecting three terminals a , b and c in a vertical plane. In (a) the length (modelling the infrastructure cost) is minimised. In (b) a cost function incorporating haulage costs is minimised.

extra nodes, referred to as Steiner points, in order to further reduce the total cost. The given nodes are generally referred to as terminals. This is illustrated by a simple example in Figure 3(a). Here the terminal a represents a surface portal, and terminals b and c represent draw points for two designated ore bodies. The length of the network is minimised by adding a single Steiner point s whose position ensures that links as and sc both have maximum gradient.

In general, the gradient constraint can most easily be modelled by assuming all links are straight line segments but varying the way we measure the length of a link, depending on whether or not the absolute gradient of the straight line segment is greater than m . This is done as follows.

Let x_p, y_p, z_p denote the Cartesian coordinates of a point p in three dimensional space. Assume that the z -axis is vertical. Given a link e in an underground mining network model, we can assume that one of the endpoints of e lies at the origin (by translating the coordinate axes if necessary). So, suppose $e = (o, p)$ where o is the

origin. Then, by the *gradient* of e , denoted $g(e)$, we mean the absolute value of the slope of the vector \vec{p} . That is,

$$g(e) = \frac{|z_p|}{\sqrt{x_p^2 + y_p^2}}.$$

If the link has the property that $g(e) \leq m$ then the embedding of the corresponding minimum centreline will be a straight line corresponding to the line segment op and its length is simply Euclidean length. If, however, $g(e) > m$ then the centreline of a minimum length ramp under the gradient constraint can be thought of as a zigzag or helical curve in space with constant gradient m at every differentiable point. Since the gradient is constant it follows that the length of this curve is a function only of $|z_p|$, and is independent of the shape of the curve. It is not difficult to see that the length of this curve is $|e|_v := (\sqrt{1 + m^{-2}})|z_p|$ which we define as the *vertical metric* for e .

We now define the *gradient metric*, which we use to measure the lengths of links in our network for a given maximum gradient m . This can be defined in terms of the Euclidean metric, $|\cdot|$, and vertical metric, $|\cdot|_v$. The length of a link $e = (o, p)$ in the gradient metric is defined to be

$$|e|_g = \begin{cases} |e| = \sqrt{x_p^2 + y_p^2 + z_p^2}, & \text{if } g(e) \leq m; \\ |e|_v = (\sqrt{1 + m^{-2}})|z_p|, & \text{if } g(e) \geq m. \end{cases}$$

It is easily checked that this defines a metric. Note that $|e|_g = \max\{|e|, |e|_v\}$. Since $|e|$ and $|e|_v$ are both convex functions it follows that the gradient metric is convex (though it is not strictly convex). The *gradient-constrained Steiner network problem* is the problem of finding the minimum cost Steiner network under the gradient metric, for a given m .

The simplest model of cost is that of construction (or *development*) only, where the cost is proportional to the total length of the links in the network. Even in this simple model the gradient-constrained Steiner network problem has been shown to be NP

hard. This is principally due to the large number of possible topologies (ie, patterns of adjacencies) that need to be considered [Brazil et al., 1998]. So, it is essential to have a very efficient method to find the least cost network for a given topology. However, the problem continues to be difficult even for a fixed topology; it has been shown that for the three terminal Steiner problem there are cases where there is no closed form for the coordinates of the Steiner point in the minimum network [Brazil et al., 2005b]. This means approximation or heuristic methods are required to effectively tackle this problem. Fortunately, the convexity properties of the metric have enabled us to construct good descent techniques for finding approximate solutions for a given topology. This is done by exploiting the geometric properties of minimum solutions as discussed in [Brazil et al., 2001].

A more realistic and useful model of the cost of a link depends both on the development costs and the associated haulage costs over the life of the mine. For each link the cost remains proportional to its length but each link has a different weighting (or constant of proportionality) depending on the given tonnage of ore to be hauled along that link, as well as the gradient of the link. This variable (that is, length-dependent) cost associated with a single link e of the network model is given by a function of the form:

$$w_e|e|_g := (D + HT + f(g_e)T)|e|_g$$

where:

D and H are given operational constants;

f is a given increasing function of g_e , the gradient of the embedded ramp or drive;

T is the total tonnage of ore to be transported along this ramp or drive over the life of the mine.

Here we can view the first term $D|e|_g$ as the development cost for this link, the second term $HT|e|_g$ as the haulage cost if we had assumed the ramp or drive was horizontal, and the final term $f(g_e)T|e|_g$ as the haulage penalty associated with the

true gradient of the link. Taking development and haulage into account, the cost of a single link, $w_e|e|_g$, is a monotonically increasing function of both length and gradient.

For real industry applications, we have to be able to solve the gradient-constrained Steiner network problem, where, for a given topology, the total cost function is $\sum w_e|e|_g$ where the sum is taken over all links e in the underground mining network. An example of the change to the physical design resulting from this cost function including haulage as compared to development only is illustrated in Figure 3(b). Here the incorporation of haulage (towards the surface portal at a) into the cost function results in the Steiner point s moving closer to terminal a . The effect of this change in position of s is to increase the infrastructure development costs of the link bs , but to reduce the total haulage costs between b and a . The new position of s gives an optimum network, which balances the combined increase in infrastructure costs with the savings in haulage costs. Note that if we consider the limiting case where the haulage costs are so great that the infrastructure costs are negligible in comparison, then the Steiner point s collapses into the terminal a .

Again an effective descent algorithm can be devised for converging to an optimal solution. The gradient penalty cost function $f(g_e)$ can generally be assumed to be linear, or at worst polynomial. It has been shown that under such an assumption the total cost function for the network continues to be convex for a fixed topology [Brazil et al., 2005a]. These principles have been incorporated into a software product called UNOTM - Underground Network Optimiser, which we describe in the next section.

3 Software Tools

UNOTM is a software tool for solving the gradient-constrained Steiner network problem, for a given set of terminals in three dimensional space, with a given cost function of the type described above. Essentially it is a heuristic method for finding approxi-

mate solutions to the following optimisation problem:

- **GIVEN:** A set of points N in Euclidean 3-space, and a gradient bound $m > 0$.
- **FIND:** A network T interconnecting N embedded in Euclidean 3-space, such that
 - the embedded links are piecewise smooth curves whose absolute gradient at each differentiable point is at most m ,
 - the total development plus haulage costs are minimised.

UNOTM aims to determine the topology of the network and the locations of the Steiner points, to minimise the total costs associated with the network over the life of the mine.

The input to UNOTM consists of the maximum gradient, the development and haulage costs mentioned in the previous section, the coordinates of the given terminals (draw points at orebodies to be reached by the network, as well as the surface portal, or breakout point from an existing decline, that all ore must be hauled to), and the tonnages of ore to be transported from each of the draw points. UNOTM constructs a tree interconnecting the terminals, and representing a network model of the mine, so that minimising the cost of the tree minimises development and haulage costs in the mine. Naturally, the amount of ore to be hauled through each section of the mine needs to be taken into account when costing the network model.

UNOTM finds heuristic solutions that are close to optimal for problems of the type and size discussed in this article. The algorithm uses simulated annealing and genetic algorithm methods to systematically search through the huge number of possible topologies. In this way the problem is reduced to one of efficiently minimising the cost for a single topology. As mentioned in the previous section, the convexity of the cost function with respect to the positions of the Steiner points allows us to apply a descent method. Such descent methods have been successfully developed

and implemented in the unweighted 3-dimensional Euclidean case [Smith, 1992]. The gradient-constrained problem to be solved by UNOTM is considerably more difficult, both because of the different weightings on individual links and the complicated geometry of the minimum configuration at each Steiner point [Brazil et al., 2001]. Despite this greater complexity, rapid descent methods have been developed for UNOTM which allow it to run very efficiently [Brazil et al., 2000].

The output is a set of coordinates for the junction points (Steiner points) as well as the topology of the best tree found. Once the endpoints of each link have been determined, constructing an embedding of the link corresponding to a minimum cost path is straightforward; the embedding of the link, however, is not unique in cases where the vertical metric alone is active - rather there is an infinite family of solutions. Finding a representative element of one of these families is easy; for example, we can take a straight line of maximum gradient joined to a helix for achieving the extra height. Because of the flexibility available in the possible embeddings of such edges, UNOTM can be thought of as providing an optimal design template. For large-scale design problems, other constraints on the physical design can often be incorporated after finding a UNOTM solution, with little or no increase in the cost function.

It follows that UNOTM tends to be very effective in large-scale mine design projects involving multiple ore bodies. An example of this is presented in Section 4.1. Suppose, however, there is a single ore-zone for a proposed new underground mine or an extension to an existing mine. We assume this ore zone is described by a triangulation of its boundary and either cross-cut entry or level access points on a sequence of levels. Two key design constraints, that are not handled by UNOTM, are that the declines should satisfy a pre-specified minimum turning circle constraint, in addition to satisfying the gradient constraint, and that the declines should avoid specified “no-go” regions. Typically the minimum turning circle will be in the range of 15m to 30m, depending on the equipment to be used in the mine. Here the topology of

the network is no longer an issue, since the decline forms a single path, but now the navigability constraint is likely to be a significant factor in the optimal solution and cannot be treated simply as a secondary optimisation objective.

To solve this problem we have developed a Decline Optimisation Tool, DOTTM, described in [Brazil et al., 2003]. The optimisation problem addressed by DOTTM can be formulated as follows:

- **GIVEN:** A set of points N in Euclidean 3-space with an ordering placed upon them, a gradient bound $m > 0$, a minimum radius of curvature bound, and a set of “no-go” regions.
- **FIND:** A network T interconnecting N embedded in Euclidean 3-space, such that a given cost function of T is minimised, where T satisfies the following constraints:
 - (a) T contains a smooth path (the decline) interconnecting the first and last terminals. The decline contains all Steiner points, and the terminals are connected to the decline in the given order via straight horizontal links perpendicular to the decline (corresponding to cross-cuts).
 - (b) Each link has gradient at most m .
 - (c) The decline satisfies the minimum radius of curvature bound.
 - (d) T avoids the specified “no-go” regions.

Note that DOTTM optimises a single decline; unlike UNOTM, it does not handle more complex networks where multiple declines join together.

We give a quick summary of the key features of DOTTM.

The mathematical model consists of a surface portal or breakout point and a decline, which is modelled as a concatenation of straight and curved ramps, with variable length crosscuts attached at points which we again call Steiner vertices.

We often assume that the crosscuts are perpendicular to the decline, although this condition can be varied as part of the input. Moreover the crosscuts can access the orebody at a variable or fixed point on a given level. This extra flexibility can produce considerable savings in tightly constrained designs. The cost functions associated with the different components of the network are of the same form as those used for UNOTM. The important constraints are curvature (turning circle) and gradient constraints.

Designing such a network so that it has optimal cost is an extremely difficult problem. In order to make the problem tractable, the algorithm focuses sequentially on each section between Steiner points where the initial and final directions of the path are determined in advance. Once a solution method has been developed for this modified problem, one can proceed with a dynamic programming methodology to solve the original problem, visiting the specified points and amalgamating the path entering a point and the one leaving it provided they have the same start and finish directions.

An abstract solution to the modified problem of finding minimal paths in three-dimensional space with given start and finish directions and a given minimal turning circle (but no gradient constraint) has been described in [Sussman, 1995]. This solution also has the unacceptable disadvantage of a continually varying gradient, which is an undesirable characteristic. It is an industry requirement that the gradient on each link is both bounded and unchanging. We have developed theory that shows that a shortest path satisfying these constraints consists of several helical and straight line segments joined together smoothly. This builds on the theory of minimum length paths with a curvature constraint in the plane developed by Dubins [Dubins, 1957].

The program DOTTM uses a heuristic algorithm based on this theory to produce near-optimal designs satisfying all the constraints. DOTTM generates a three-dimensional image of the decline's centerline and strings of co-ordinates which may

be loaded into standard mine graphics systems.

4 Two Case Studies

In this section we present two case studies, undertaken with industry support, and based on data for two large underground mines in Australia. These case studies demonstrate the applicability of our software to “real world” mine design problems. Our results for these case studies have been used by the mining companies to help determine the future development of these mines. For reasons of commercial confidentiality, some identifying details in these case studies have been withheld in this paper. All costs in these case studies are given in Australian dollars.

Disclaimer: please note that all cost figures quoted are indicative only for the purpose of reflecting the magnitude of improvement realised by the use of the UNOTM and DOTTM.

4.1 Case 1: Large scale extension of an existing mine

This case study illustrates an application of UNOTM to design the extension of a decline in an existing underground mine to service a new set of discrete ore bodies. Figure 4 is a representation of the newly discovered mineralisation with triangulated ore bodies shown relative to the existing decline. The existing decline in the figure shows some branching where it served old ore zones, however we were restricted to the lowest point of the existing decline for the breakout to the new extension.

The design brief required that a decline first be extended over the new ore zones to allow infill drilling of the mineralized zones. Given this development the aim was then to find the decline spanning the 15 orebodies which minimized the cost of infrastructure development to, and haulage from, these ore bodies. The maximum gradient parameter for this case study was 1:6.5. Figure 5 shows another view of the

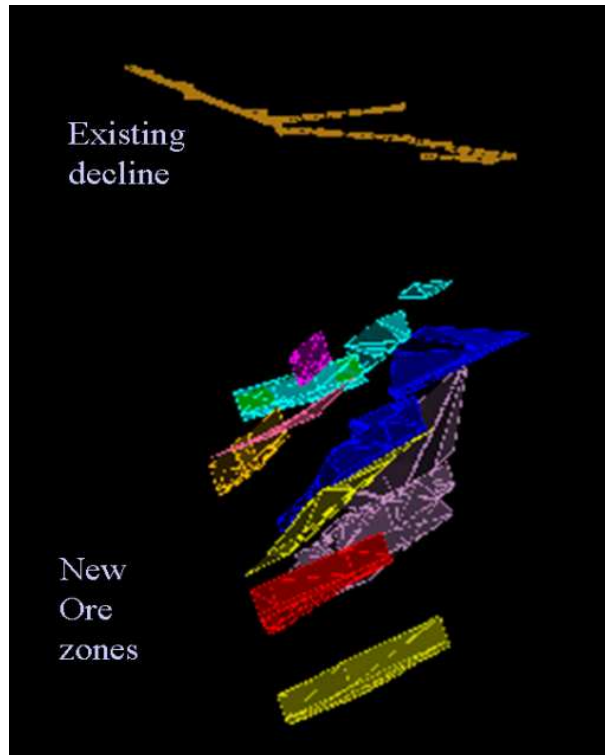


Figure 4: The existing decline and triangulated representations of the new ore bodies.

problem with the approximate path for the drilling decline, supplied by the client, shown as a dashed line.

The first stage in our modelling was to discretise the proposed path for the drilling design as a set of nodes spaced sufficiently close together so that UNOTM, in forming the minimum tree, would select this path as part of the design. This path then provides potential breakout points for branch declines to serve ore bodies below. Our model uses 14 points which must be serviced to represent the drill decline. The software permits alternative breakouts for branch declines to reach the 15 ore zone targets from any combination of points along the drill decline. It was agreed that centroids of the ore bodies provided an adequate definition of the ore zone targets

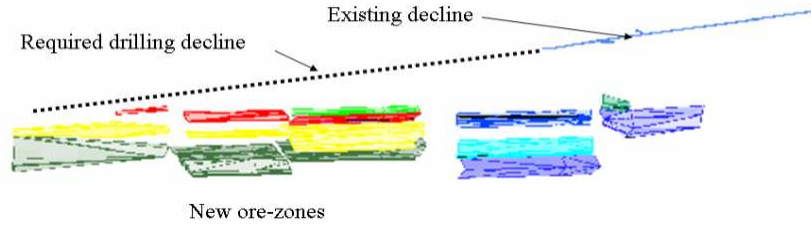


Figure 5: A view of the planned drilling decline and the new ore bodies.

for this global design stage; in other problems with larger orebodies we have used multiple points down the hanging or footwall wall side to better model the required access to the target ore zones.

UNOTM searched for a minimal network connecting all 29 points to the endpoint (breakout) of the existing decline. Here “minimal” means cheapest, and both development costs and haulage costs were included in the cost function. In this case there was no gradient penalty included in the haulage costs, so the gradient penalty function was set to 0. Since the points inserted to model the drill drive are close together, the minimal network joins them consecutively; these links are not new development design but replicate the required infill drilling ramp (if our solution did not join them consecutively, we could have inserted extra points to achieve this property.) In an UNOTM solution, the edges between the points on the drill drive may be bent slightly at a Steiner point to meet incoming drives from the orebodies, but this effect is generally negligible and can be compensated for at a later stage.

The best result found is shown in Figure 6, where the cost shown is in Australian dollars. Note that subtrees attach at a few different places (breakouts) on the drilling ramp (points 17 - 30 in Figure 6).

The second best result found has a significantly different topology as shown in

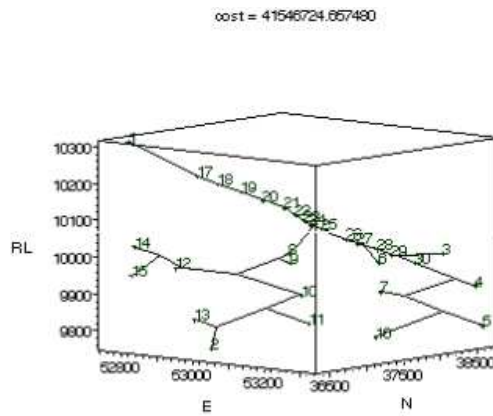


Figure 6: The best design found for the case study, computed using UNOTM.

Figure 7; while the best design is only marginally cheaper than this in percentage

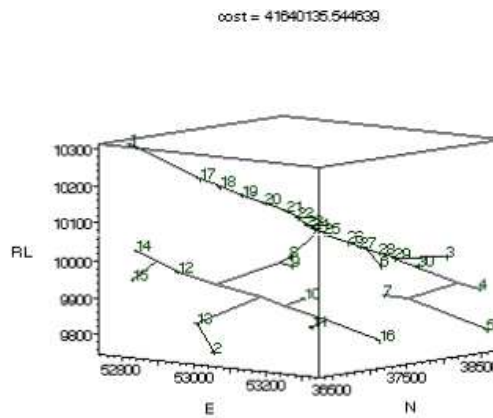


Figure 7: The second best UNOTM design found for the case study.

terms, it corresponds to a savings of \$100,000.

The software does not give a guaranteed minimum, but our conjecture that the best design found was close to optimal was confirmed by further runs with some of the clearly redundant points (labelled 17 to 22 in Figure 7) omitted. Virtually the same designs came up many times on different runs (which use different seeds in the random number generator for the genetic algorithm).

UNOTM has no in-built feature for planning the sequence of extractions to account for cash flow considerations. However, it allows management to explore ‘what if’ design alternatives by making simple adjustments to the input. For example, we can explore the solution obtained by solving for a “priority subset”. This can assist one to see what might be justified in a life of mine context. The solution for extracting ore from points labelled 6, 7, 8, 9, 10, 11, 16 in Figure 6 is shown in Figure 8, where in the rerun of UNOTM these points have now been labelled 2, 3, 4, 5, 6, 7, 8 respectively.

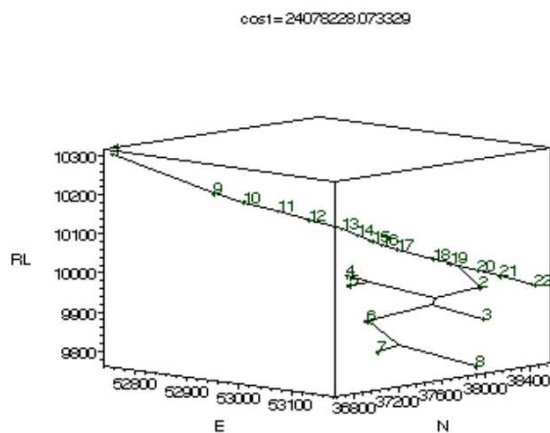


Figure 8: Best UNOTM design for a “priority subset”.

We note that originally the ramps in our designs were constrained to a maximum gradient of 1:7. Concerned at the project infrastructure cost for this gradient, the client sought designs with ramp gradients at 1:6.5 (about the limit for current mine trucks and machinery), which are the designs shown in this case study. These designs were achieved simply by rerunning UNOTM with a new maximum gradient parameter of 1:6.5.

While further work needs to be done to optimize tree designs taking into account turning circle constraints we can currently do this on a path by path basis as illustrated in Figure 9 which shows an optimal (or near-optimal) navigable implemen-

tation of the end subtree of Figure 6 serving orebodies 4,5,7 and 16 from breakout point 29.

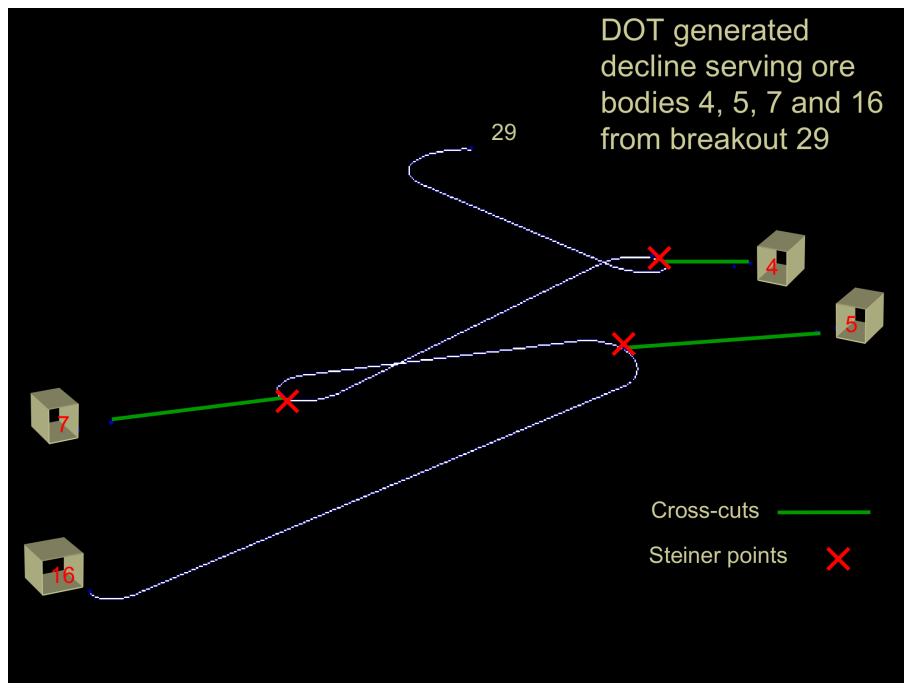


Figure 9: A navigable decline for part of the case study computed using DOTTM.

4.2 Case 2: Access decline design at Olympic Dam

A second case study involved the detailed design of an access decline serving a large stope at Olympic Dam Mine in South Australia. This project was well suited to the design and optimisation capabilities of DOTTM.

Here the brief was to design a new minimum-cost decline for accessing a well-defined stope from the existing infrastructure of a large underground mine. The decline was to break out from the existing mine from one of three possible breakout points above the stope (each with z coordinate more than 40m above the highest access points) and connect to access points lying on perimeter drives for the nominated

Breakout points	x	y	z	
Option 1	58524	33137	-380	
Option 2	58559	33245	-381	
Option 3	58833	33162	-365	
Access points to be serviced by decline	x	y	z	#
-405RL -41 Level	58725	32882	-405	0
	58803	32977	-405	1
	58929	33132	-405	2
	58987	33202	-405	3
-455RL -46 Level	58707	32906	-455	4
	58782	32995	-455	5
	58963	33093	-455	6
	59031	33202	-455	7
-505RL (and endpoint) -51 level	58659	32954	-505	8
	58747	33010	-505	9
	58853	33113	-505	10
	58920	33191	-505	11

Table 1: Breakout and access points for the decline. All coordinates are in metres.

stope via crosscuts (short horizontal drives). The 100m high stope is a designated ore-containing region that the mining company had decided could be profitably mined. It is to be accessed from three perimeter drives - horizontal tunnels along the length of the stope at three levels with vertical separations of fifty metres. These three perimeter drives are designated “-41 level”, “-46 level” and “-51 level”, in order of increasing depth. On each of the three perimeter drives on the southern wall of the stope there are four nominated access points whose coordinates were given. It was required that the decline attach to each perimeter drive at one of these perimeter access points. The data given is shown in Table 1.

An important constraint on the decline is that it must not enter the stope or indeed come closer to the stope than the perimeter drives. This was modelled as a DOTTM barrier by triangulating the set of perimeter access points as shown in

Figure 10. More precisely the triangulation referred to the access points shifted 20m south to allow for 20m crosscut drives to be used to join the decline to the perimeter drives. The other fundamental constraints on the decline were the maximum gradient which was 1:9 and the minimum turning radius which was 40m.

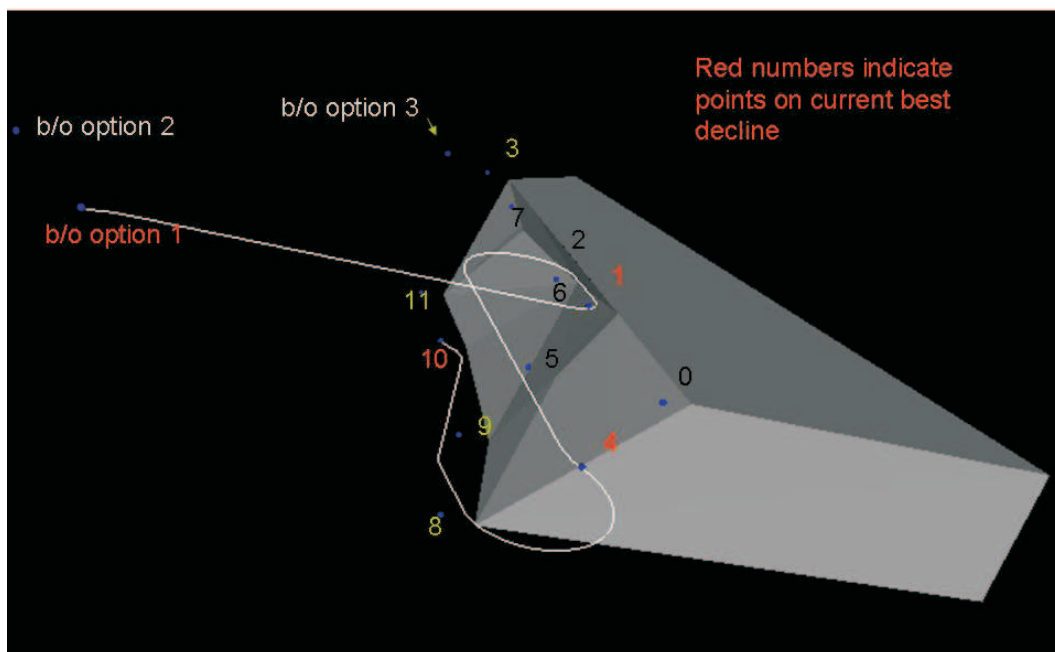


Figure 10: An initial strategic design for the access decline found using DOTTM.

In this case study the decline is to be used for access only and not truck haulage. The cost objective function is therefore development cost only:

$$\text{Development Cost} = D_{\text{decl}}a + D_{\text{xcut}} \sum_{i=1}^3 b_i$$

where

a = decline length;

b_i = length of i th crosscut (m);

D_{decl} = decline development cost rate (\$/m);

D_{xcut} = crosscut development cost rate (\$/m).

The specifications for the case study also included an additional set of design constraints: namely, that the decline path must avoid a number of mineralized zones in the intervening space between the breakout points and the perimeter drives. These constraints were introduced in a staged manner. Our preliminary “strategic” design allowed the decline a free path from the breakouts to perimeter accesses in order to find its preferred path in the absence of any of the additional constraints - a best case scenario. The design was then reviewed by inspecting where it had cut mineralized regions and, in consultation with the design engineer, barriers were inserted to prevent it from doing so in the next run of DOTTM. This step was repeated several times as more barriers were inserted until it was clear that the decline avoided all regions of interest.

The first “strategic” decline design (Figure 10), costing \$3.81M, breaks out at Option 1 and connects to point 1 at the -41 Level, point 4 at the -46 Level and point 10 at the -51 Level. The eastern end of the decline, however, extends behind the eastern boundary of the target stope into a mineralised region and was therefore unacceptable. As part of the iterative design process described above a barrier was added to the eastern end and, as a double check, access for the decline was denied to the eastern end access points 0, 4 and 8. Figure 11 shows the resultant DOTTM design. It is \$100,000 more expensive and extends beyond the western end of the stope.

The model was then extended to allow the decline to attach to the perimeter drive via a choice of either 20m or 40m crosscuts. This additional flexibility could potentially allow for a cheaper design (noting that crosscut lengths are included in the costing). A reasonable decline was found at a cost of \$3.905M but the mining

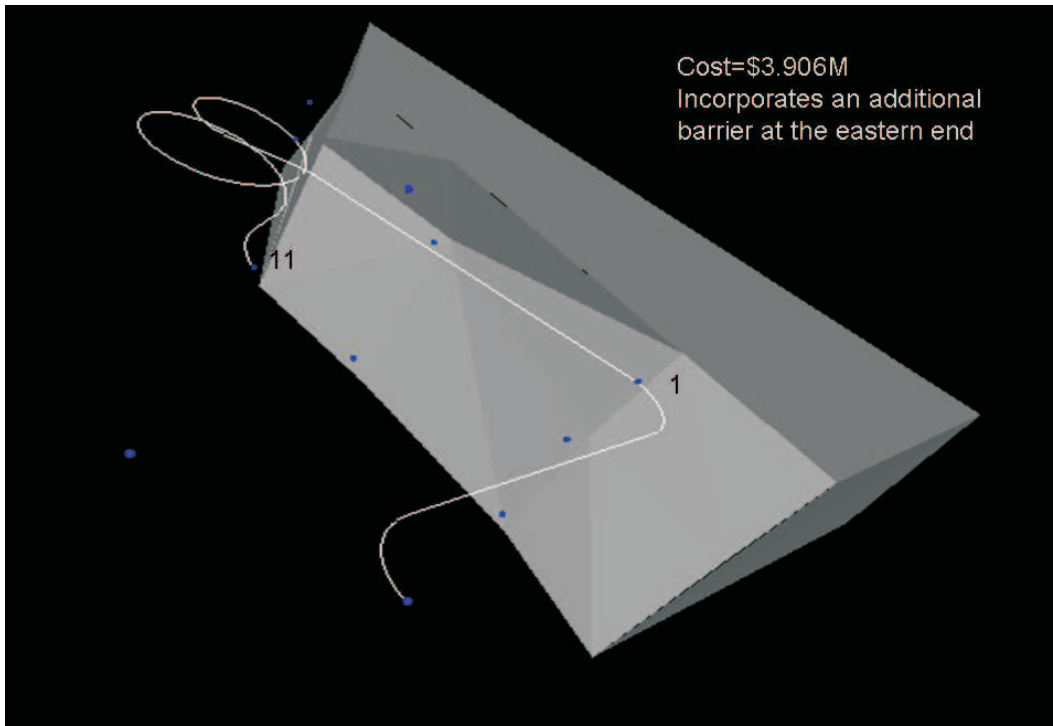


Figure 11: An improved design for the access decline, avoiding mineralised material at the eastern end of the stope.

engineer noted it transgressed some new zones of interest. Therefore new western and eastern barriers, nominated by the design engineer after inspecting the imported decline design in his block model of the ore body, were added to the model and it was run again. A new decline also costing \$3.905M was generated but it was in a very tight helix, making it less than ideal in terms of navigation, even though it satisfied the constraints.

Consequently, some “path straightening” - an option available in the version of DOTTM being used - was introduced, increasing the total cost to more than \$3.96M. Another apparently straighter and cheaper design found after path straightening ex-

hibited a flaw in the eastern barrier - it managed to slip through a small gap in the proposed barrier. The barrier was modified and a new design was generated costing just under \$3.96M. This is shown in Figure 12.

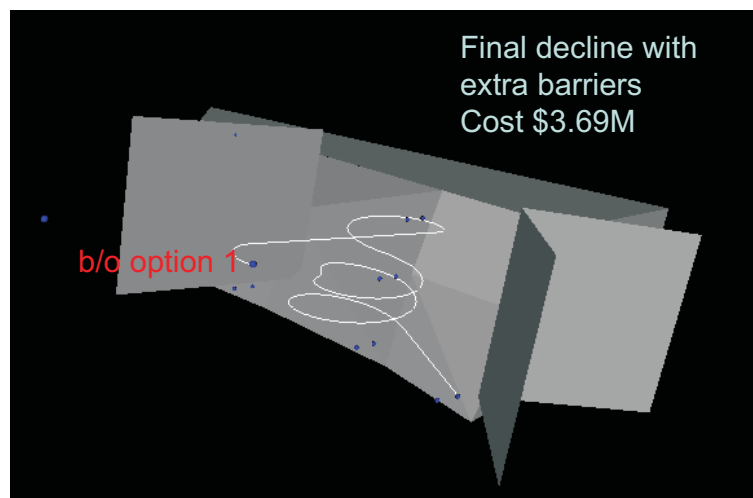


Figure 12: Final design for the access decline, avoiding all mineralised material and including some curve straightening.

The above design could theoretically have been completed in one pass by having a full set of barriers in place in the first run. However it is easier in practice to do the design progressively as described above when only a small subset of the set of all possible barriers need to be inserted in the design process.

Finally the design team noted that since this decline was to be used for access only and not truck haulage, there should be some scope for allowing a slightly steeper maximum gradient. It appeared that the decline could still serve its purpose even if the maximum gradient was increased to 1:8. An experimental 1:8 design showed a decline with a significantly lower cost of \$3.568M was possible. Given the significant savings for this change of parameter the mining company decided that this design would be seriously considered.

5 Conclusions

This paper describes an approach to the design of underground mines that aims to optimise the life-of-mine costs under a number of realistic constraints. This is a difficult problem that does not lend itself easily to standard optimisation techniques. The optimisation capability of the software tools we describe in this paper is important not only for producing better mining layout designs, but also for enabling good decision-making in underground mine planning. Many high-level planning decisions, such as whether a proposed mining project is economically viable, or whether haulage should take place via a shaft or decline, depend on being able to quickly and accurately model and optimise costs associated with competing designs. Mine planning that takes recent changes in mining technology into account may require a radical change in mindset (and substantial retraining) for mining engineers experienced in designing for traditional technologies; however, it would generally only require small parameter changes to the input when using the software tools described in this paper.

Acknowledgements. This research has been conducted with financial support from the Australian Research Council and Newmont Australia Limited via a Linkage Collaborative Research Grant. Nicholas Wormald is also supported by the Canada Research Chairs program. We acknowledge the collaboration of Hyam Rubinstein, Jia Weng and Peter Grossman in the development of the design tools described in this paper [Brazil et al., 2000], [Brazil et al., 2003]. We also acknowledge Richard Buckley of Maptek Australia Limited and Henry Adryszczak of BHP Billiton Limited for their assistance with the case studies.

References

[Brazil et al., 2003] Brazil, M., Lee, D.H., Leuven, M. Van, Rubinstein, J.H., Thomas, D.A., Wormald, N.C., 2003. Optimizing declines in underground mine. *Mining Tech-*

- nology: Trans. of the Institution of Mining and Metallurgy, Section A* 112, 164-170.
- [Brazil et al., 2000] Brazil, M., Rubinstein, J.H., Thomas, Lee, D.H., D.A., Weng, J.F., Wormald, N.C., 2000. Network optimization of underground mine design. *The Australasian Institute for Mining and Metallurgy Proc.* 305, 57-65.
- [Brazil et al., 2001] Brazil, M., Rubinstein, J.H., Thomas, D.A., Weng, J.F., Wormald, N.C., 2001. Gradient-constrained minimum networks, I: fundamentals. *J. of Global Optimization* 21, 139-155.
- [Brazil et al., 1998] Brazil, M., Thomas, D.A., Weng, J.F., 1998. Gradient constrained minimal Steiner trees. In: Pardalos, P.M., Du, D.-Z. (Eds.), *Network Design: Connectivity and Facilities Location (DIMACS Series in Discrete Mathematics and Theoretical Computer Science, Vol. 40)*. American Mathematical Society, Providence, pp. 23-38.
- [Brazil et al., 2005b] Brazil, M., Rubinstein, J.H., Thomas, D.A., Weng, J.F., 2005. Gradient-constrained minimum networks, II: labelled or locally minimal Steiner points. Preprint.
- [Brazil et al., 2005a] Brazil, M., Thomas, D.A., Weng, J.F., Rubinstein, J.H., Lee, D.H., 2005. Cost optimisation for underground mining networks. *Optimization and Engineering* 6, 241-256.
- [Brazil and Thomas, 2007] Brazil, M., Thomas, D.A., 2007. Network optimization for the design of underground mines. *Networks* 49, 40-50.
- [Dubins, 1957] Dubins, L E, 1957. On curves of minimal length with a constraint on average curvature and with prescribed initial and terminal positions and tangents. *Amer J Math* 79, 497-516.
- [Hwang et al., 1992] Hwang, F.K., Richards, D.S., Winter, P., 1992. The Steiner Tree Problem, *Annals of Discrete Mathematics* 53. Elsevier, Amsterdam.
- [Smith, 1992] Smith, W.D., 1992. How to find minimal trees in Euclidean d -space. *Algorithmica* 7, 137-177.

[Sussman, 1995] Sussman, H J, 1995. Shortest 3-dimensional paths with prescribed curvature bound. In: Proc. 34th Conf. on Decision and Control, IEEE: New Orleans, pp. 3306-3312.