ASP2062 Introduction to Astrophysics

Star formation II — Daniel Price

Key revision points

1. Star formation is a competition between gravity and pressure
2. Timescale associated with gravity is the 'freefall time', involves only density
3. The Jeans length is the critical wavelength below which a small perturbation to gas is unstable to collapse under it's own gravity. Jeans mass gives a critical mass (mass within a sphere of radius the Jean's length).
4. Jeans length and mass increase with temperature but decrease with density
5. Jeans length decreasing with increasing density implies break-up of an isothermal cloud during collapse — '*fragmentation*'

1 The physics of star formation

1.1 How long does it take to form a star?

The main physical process involved in star formation is *gravity*. We can work out the typical timescale on which gravity acts by a simple dimensional analysis. Consider the gravitational constant G,

$$G = 6.673 \times 10^{-8}\mathrm{cm}^3\mathrm{s}^{-2}\mathrm{g}^{-1}. \tag{1}$$

This tells us the strength of the gravitational field *in some units*. An equivalent way of stating the strength of gravity is to ask "how long does it take gravity to pull something of length L and mass M to the origin?". Hence we want to phrase the gravitational constant in terms of a *timescale*, T. From (1) we see that the dimensions of G are

$$[G] = \frac{[L]^3}{[T]^2[M]}. \tag{2}$$

Rearranging this to obtain a timescale, we find

$$[T] = \sqrt{\frac{[L]^3}{G[M]}}, \tag{3}$$

and so we can estimate a timescale associated with gravity using

$$t_{\rm ff} \sim \sqrt{\frac{L^3}{GM}} \sim \sqrt{\frac{1}{G\rho}}, \tag{4}$$

where $\rho \equiv M/L^3$ is a density. This is known as the *free-fall timescale*, since it is the time it takes for material to collapse freely under it's own gravity.

A more rigorous definition of the free-fall time is *the time taken for material at a given radius in a uniform density, self-gravitating medium to collapse to the origin.* Using this definition (see the problem sheet) we arrive at the expression

$$t_{\rm ff} = \sqrt{\frac{3\pi}{32G\rho}}. \tag{5}$$

Notice that the dependence on density and the gravitational constant is the same as in our dimensional analysis, the only thing that has changed is that we know the factor out the front. Our estimate (4) was just a dimensional guess, but correct to within a factor of two from the precise definition — such precision is pointless anyway because molecular clouds are not uniform density spheres. Dimensional analysis is a powerful way to get a rough handle on the timescales involved.

A typical molecular cloud has a mean density of $10^3 - 10^4$ molecules per cm^{-3}, equivalent to a mass density $\rho \sim 10^{-21}$ to 10^{-20}g/cm^3 assuming the mass is mostly molecular hydrogen (H_2). Hence we can evaluate the free fall time for a typical molecular cloud as

$$t_{\rm ff} \sim \sqrt{\frac{1}{6.673 \times 10^{-8} \times 10^{-20}}} = 3.9 \times 10^{13}{\rm s} \approx 10^6{\rm yr}. \tag{6}$$

This is the *fastest* timescale on which star formation could possibly take place, since it is the timescale on which the cloud would collapse if there was nothing to resist collapse against gravity. Hence when we talk about *fast* or *slow* star formation, we mean fast or slow with respect to the free-fall time.

> Notice that the free-fall time has nothing to do with the *size* of the cloud, only it's density. Why is this so, and what does it tell us about gravity? Which would collapse faster, a large cloud or a small cloud of the same density?

1.2 Jeans length and Jeans mass

The main way gas can resist gravitational collapse is with *pressure*, more specifically a *pressure gradient* (this is how stars hold themselves together, as you will learn in the stellar evolution lectures). Again, we can get a long way with dimensional analysis. Communication of pressure disturbances occurs at the sound speed, $c_{\rm s}$, so we can estimate the *sound crossing time*, $t_{\rm s}$, as the time for a sound wave to travel a particular distance, i.e.

$$t_{\rm s} \sim \frac{L}{c_{\rm s}} \tag{7}$$

Comparing this to the free-fall timescale we can define a critical *lengthscale* on which pressure and gravity are communicated equally fast. We find

$$L_J \sim \frac{c_s}{\sqrt{G\rho}}. \tag{8}$$

This lengthscale is known as the *Jeans length*, after Jeans (1902). Again, dimensional analysis gives us the right combination of variables but not the coefficient out the front. A more rigorous analysis involves a perturbation analysis of the equations of self-gravitating fluid dynamics, with the precise definition of the Jeans length being *the critical wavelength at which a small perturbation to a self-gravitating, uniform density medium becomes unstable to gravitational collapse*, giving

$$L_{\rm J} = \sqrt{\frac{\pi c_{\rm s}^2}{G\rho}}. \tag{9}$$

On length scales $L < L_J$ pressure can support the gas against collapse, but on scales $L > L_{\rm J}$, gravity wins. The Jeans length defines the minimum size 'chunk' that a cloud can break up into. We can also define a critical mass, the *Jeans mass*, as the mass contained within a volume bounded by $L_{\rm J}$, i.e.

$$M_{\rm J} \sim L_{\rm J}^3\rho = G^{-3/2}c_{\rm s}^{3/2}\rho^{-1/2}. \tag{10}$$

Equivalently, we could define this as the mass contained within a sphere of radius L_J, i.e.

$$M_J = \frac{4}{3}\pi\left(\frac{L_J}{2}\right)^3\rho, \tag{11}$$

1.3 The sound speed in molecular clouds

For an ideal gas the equation of state relating pressure (P) to density (ρ) and temperature (T) is given by

$$P = \frac{\rho k_{\rm B} T}{\mu m_{\rm H}}, \tag{12}$$

where $k_{\rm B} = 1.38 \times 10^{-16}$ erg/K is the Boltzmann constant, $m_{\rm H} = 1.67 \times 10^{-24}$g is the mass of a Hydrogen atom and μ is the mean molecular weight. If the temperature is constant then the sound speed is given by

$$c_{\rm s}^2 \equiv \frac{\partial P}{\partial \rho} = \frac{k_{\rm B} T}{\mu m_{\rm H}}. \tag{13}$$

Here's the interesting bit: Molecular clouds cool very efficiently (molecules are good at jiggling!) — so efficiently that the temperature is constant at $\approx$ 10K to a very good approximation *across a huge range in density*. This has profound implications for star formation, as we shall see.

To get the sound speed from the temperature we need to know the composition. Molecular clouds are mostly molecular hydrogen with a sprinkling of heavier molecules like water and carbon monoxide (these are important coolants but not important by mass), so the mean molecular weight is close to that of molecular hydrogen ($\mu = 2$), or $\mu \approx 2.4$. From (13) the sound speed is

$$c_{\rm s} = \sqrt{\frac{k_{\rm B} T}{\mu m_{\rm H}}} = 0.2{\rm km/s}. \tag{14}$$

This is similar to the sound speed in air (330 m/s), but for a very different reason — the temperature in air is higher by a factor of ~ 30, but so is the mean molecular weight.

Mean Jeans length and Jeans mass in a molecular cloud

Molecular clouds are *not* uniform density. Nevertheless, evaluating the mean Jeans length and Jeans mass is interesting. Assuming typical sound speed (from 14) and mean density (10^{-20} g/cm^3) we have

$$L_{\rm J} \sim \frac{2 \times 10^4 {\rm cm/s}}{\sqrt{6.673 \times 10^{-8}{\rm cm^3 s^{-2} g^{-1}} \times 10^{-20}{\rm g cm^{-3}}}} \approx 7.7 \times 10^{17}{\rm cm} \sim 0.25{\rm pc}, \quad (15)$$

and

$$M_{\rm J} \sim L_{\rm J}^3 \rho \sim (7.7 \times 10^{17}{\rm cm})^3 \times 10^{-20}{\rm g/cm^3} \approx 4.6 \times 10^{33}{\rm g} = 2.3 M_\odot. \quad (16)$$

It is interesting that this is 'close to' the typical mass of stars ($\sim 0.5 M_\odot$), but it is not terribly meaningful since the assumption of uniform density is very poor. The precise value is also very sensitive to the assumed density, for example what is the mean Jeans mass if you use 10^{-21} g/cm^3 instead?

1.4 Fragmentation

More interesting is how the Jeans length and Jeans mass depend on density and temperature. We have

$$L_{\rm J} \propto \frac{c_{\rm s}}{\sqrt{\rho}} \propto T^{1/2} \rho^{-1/2}. \quad (17)$$

So the Jeans length *increases with temperature* but *decreases with density*. Similarly, for the Jeans mass we have

$$M_{\rm J} \propto T^{3/2} \rho^{-1/2}, \quad (18)$$

so if the temperature goes up, so does the minimum mass and size that can resist the pull of gravity (obvious, since there is more pressure), leading to bigger, more massive stars. As the density goes up, the critical mass and size both decrease, leading to smaller, less massive stars.

Now what happens inside a real molecular cloud? In molecular clouds $T \approx 10$K *over a huge range of density* (they cool very efficiently!), implying $P \propto \rho$. So as gravity starts to collapse a region of the cloud, what will happen to the Jeans length and Jeans mass if T=const?

The fact that the Jeans length and Jeans mass become ever *smaller* as the density increases leads to the idea of *fragmentation*, first put forward by Hoyle (1953), in other words the idea that a molecular cloud will 'fragment' into ever smaller pieces.

In practice, fragmentation occurs because molecular clouds are *turbulent*, with motions that are highly supersonic (observed motions are $\sim$1–2 km/s, or 5–10 times the sound speed). The effect of this turbulence is to produce a spectrum of density fluctuations on which the Jeans instability can act.

When does fragmentation stop?

Fragmentation cannot go on for ever, otherwise one would end up breaking up the cloud into infinitely small pieces, giving infinitely small and infinitely low mass stars. So before next lecture, try to answer this: What physical change causes fragmentation to stop?

References

Hoyle, F.: 1953, 'On the Fragmentation of Gas Clouds Into Galaxies and Stars.'. ApJ **118**, 513.

Jeans, J. H.: 1902, 'The Stability of a Spherical Nebula'. *Phil. Trans. Roy. Soc. Lon. A* **199**, 1–53.