MODELLING DUST

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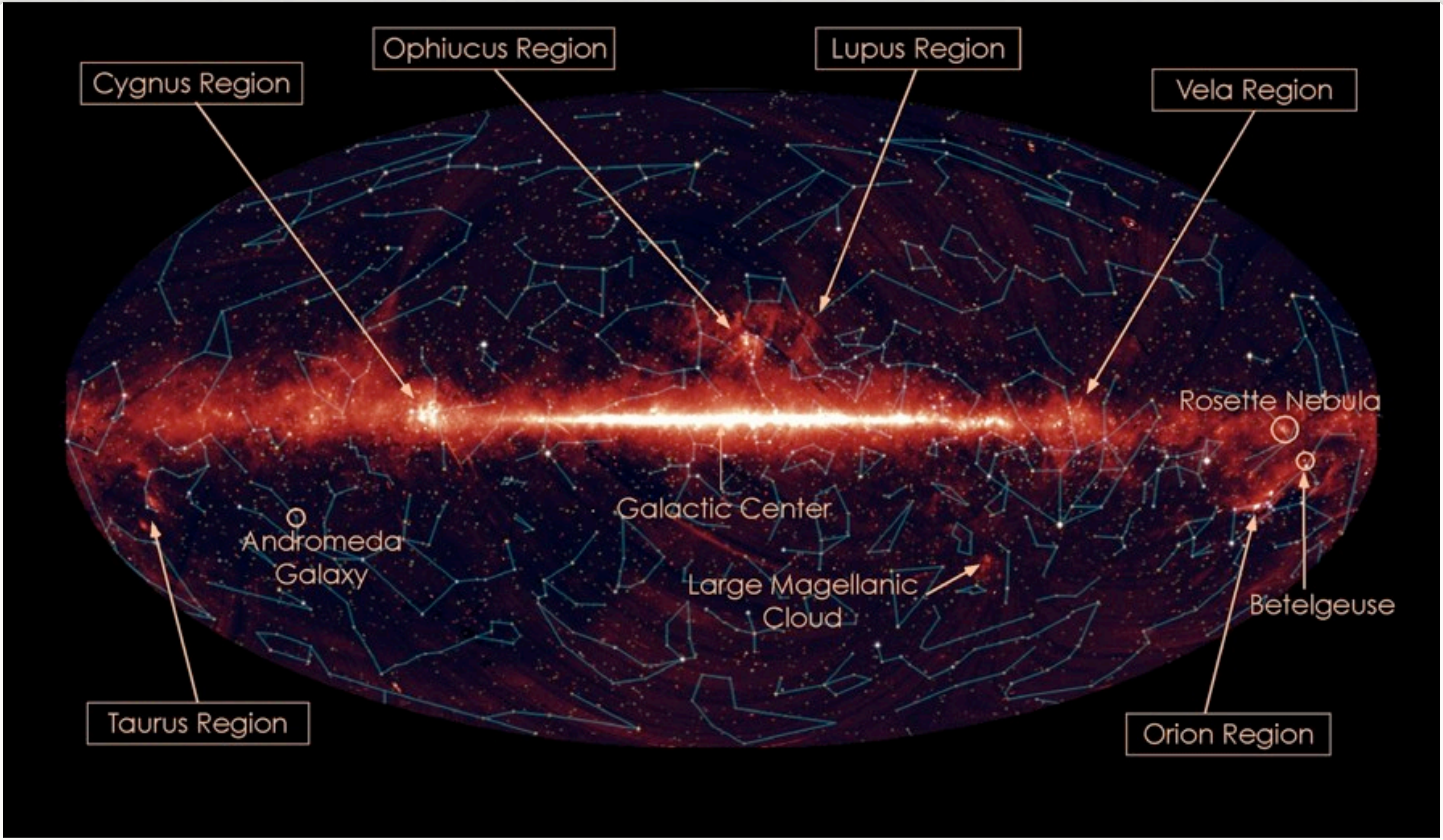
Mind the Gap, Cambridge, 8th-12th July 2013

MoCA
Monash Centre for Astrophysics

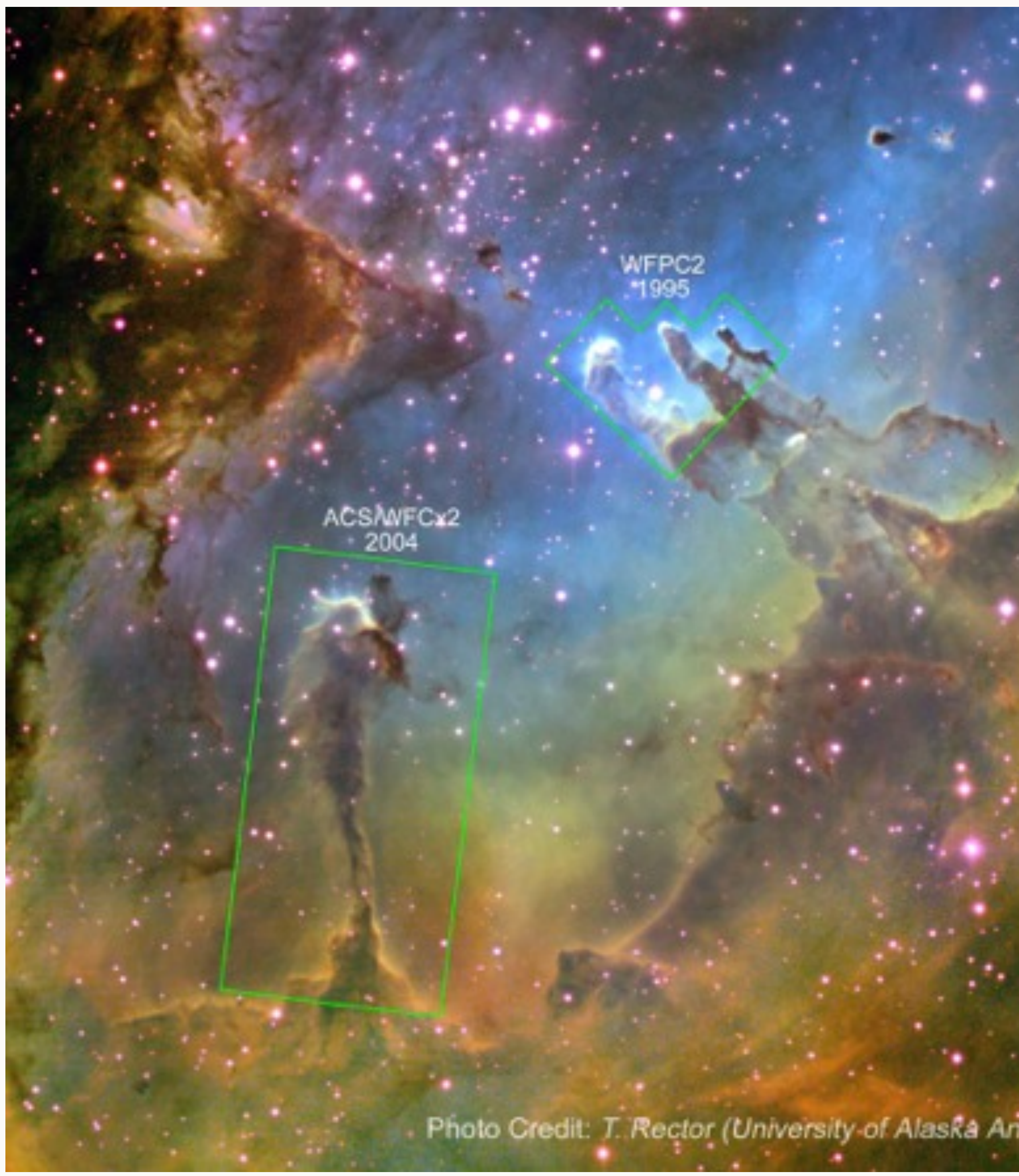
DUST BLOCKS

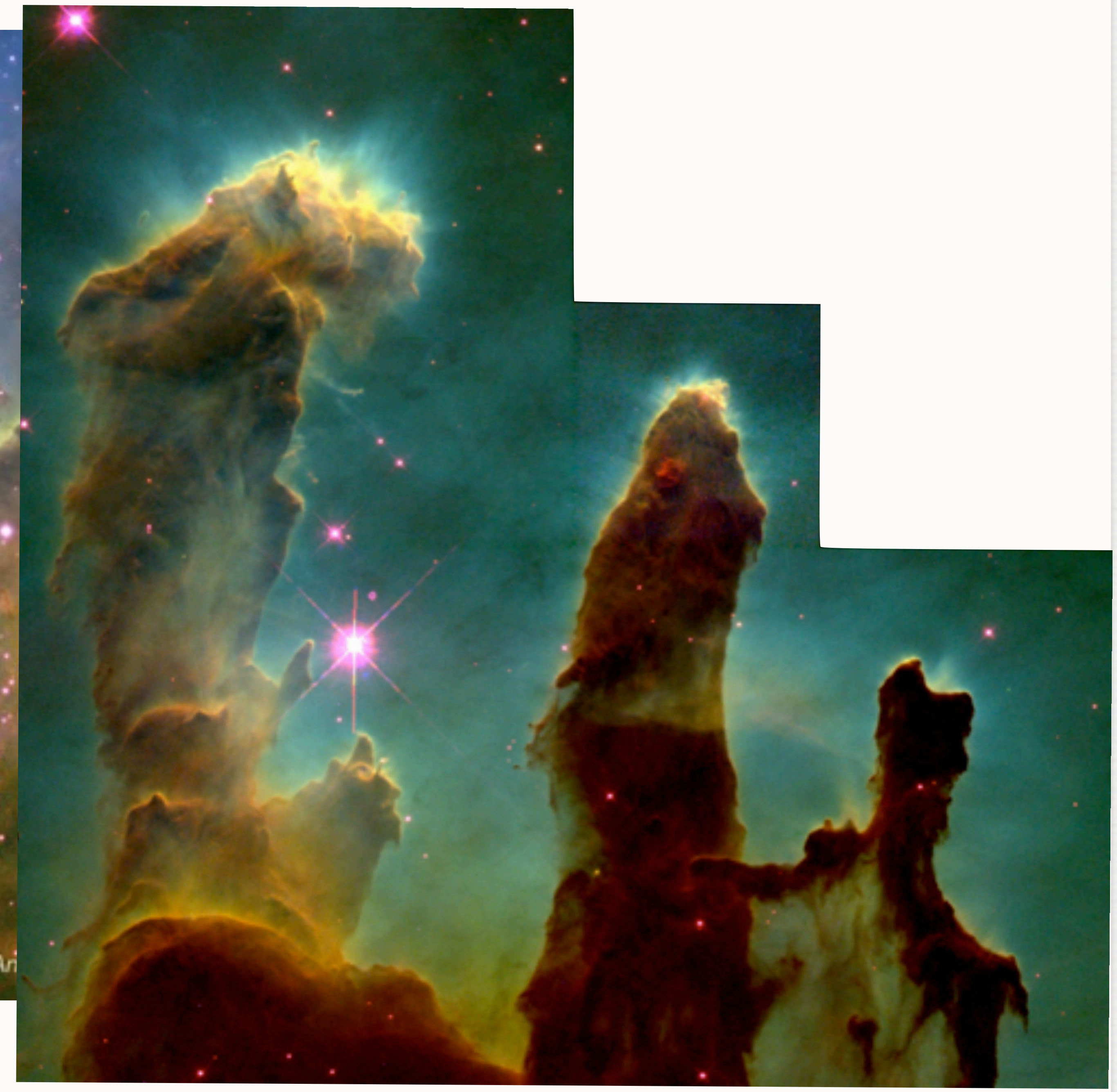
DUST BLOCKS (ABOVE THE GAP)

DUST GLOWS


Ophiucus Region
Lupus Region
Cygnus Region
Vela Region
Rosette Nebula
Galactic Center
Andromeda
Galaxy
Large Magellanic
Cloud
Betelgeuse
Taurus Region
Orion Region

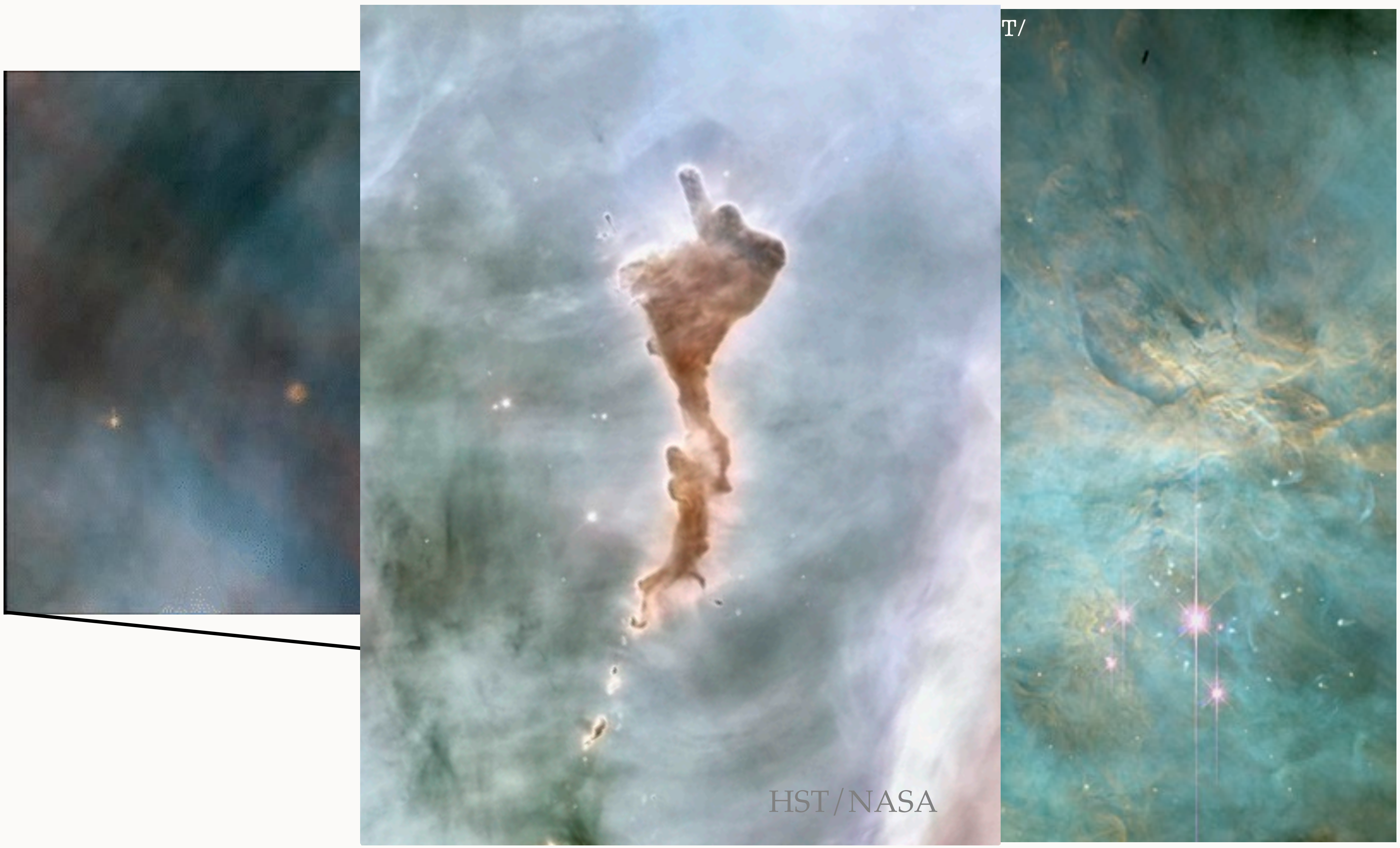
DUST IS KEY TO STAR FORMATION



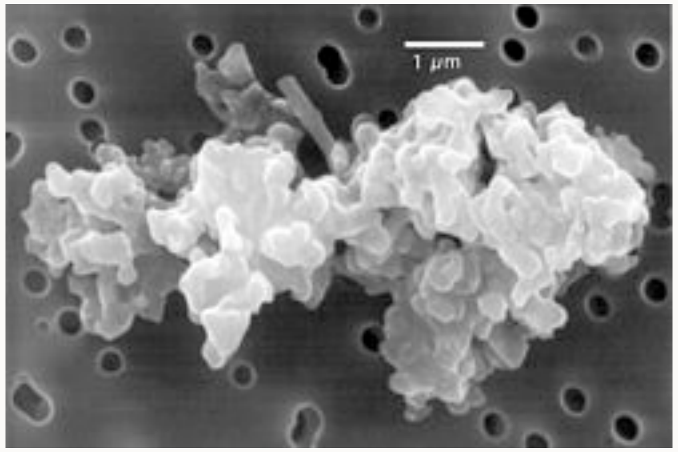


Pillars of Creation (the Eagle Nebula)

DUST EVAPORATES



DUST GROWS TO FORM PLANETS



en.wikipedia.org

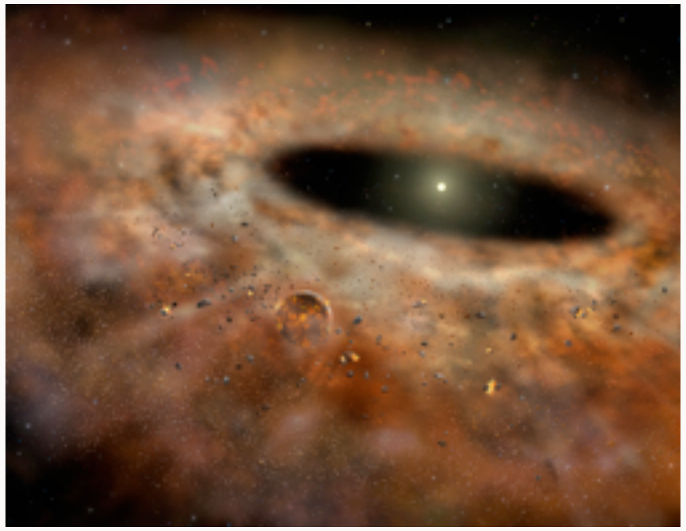
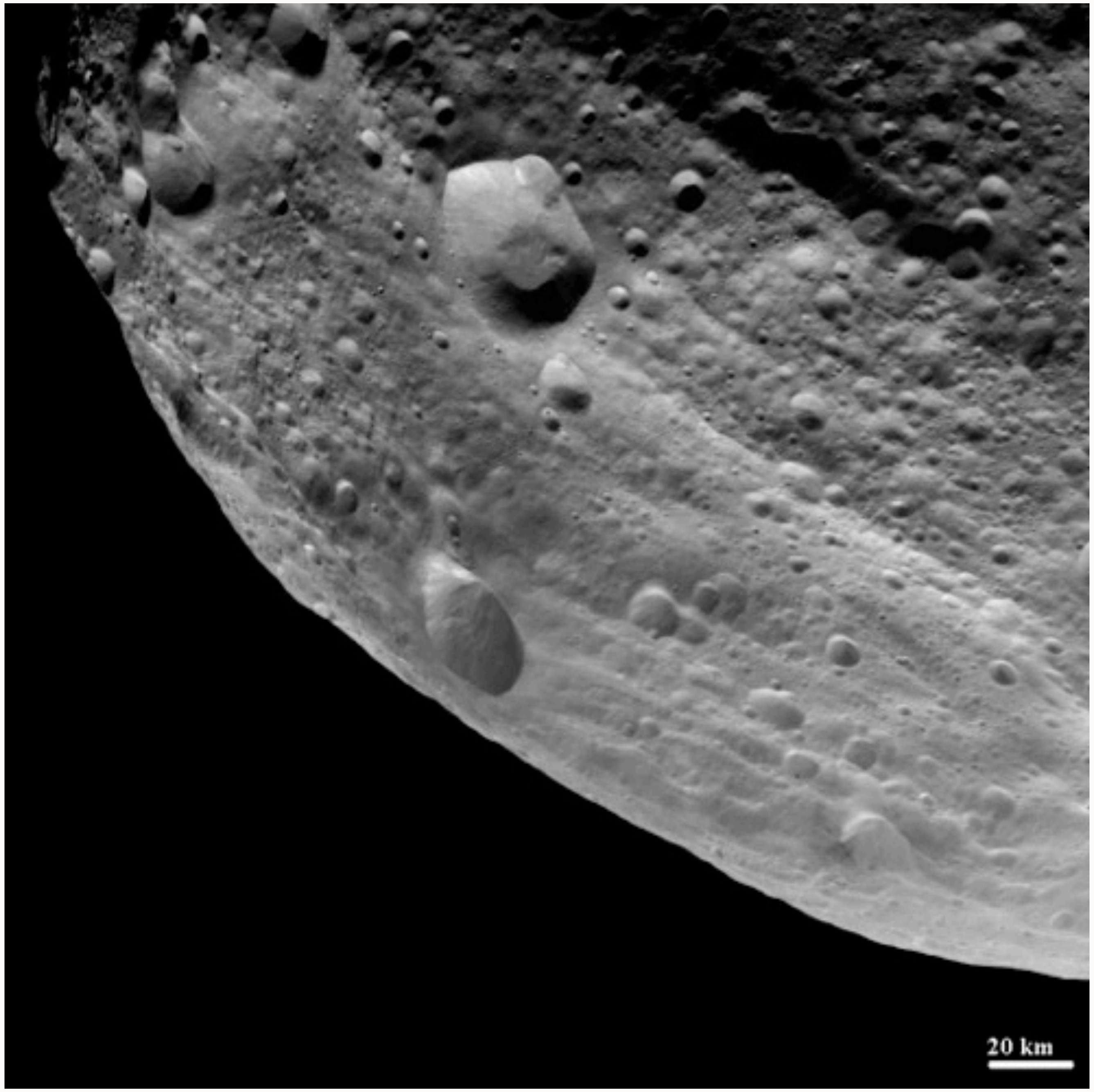


Image: Gemini Observatory/AURA
Artwork by Lynette Cook



MODELLING DUST+GAS

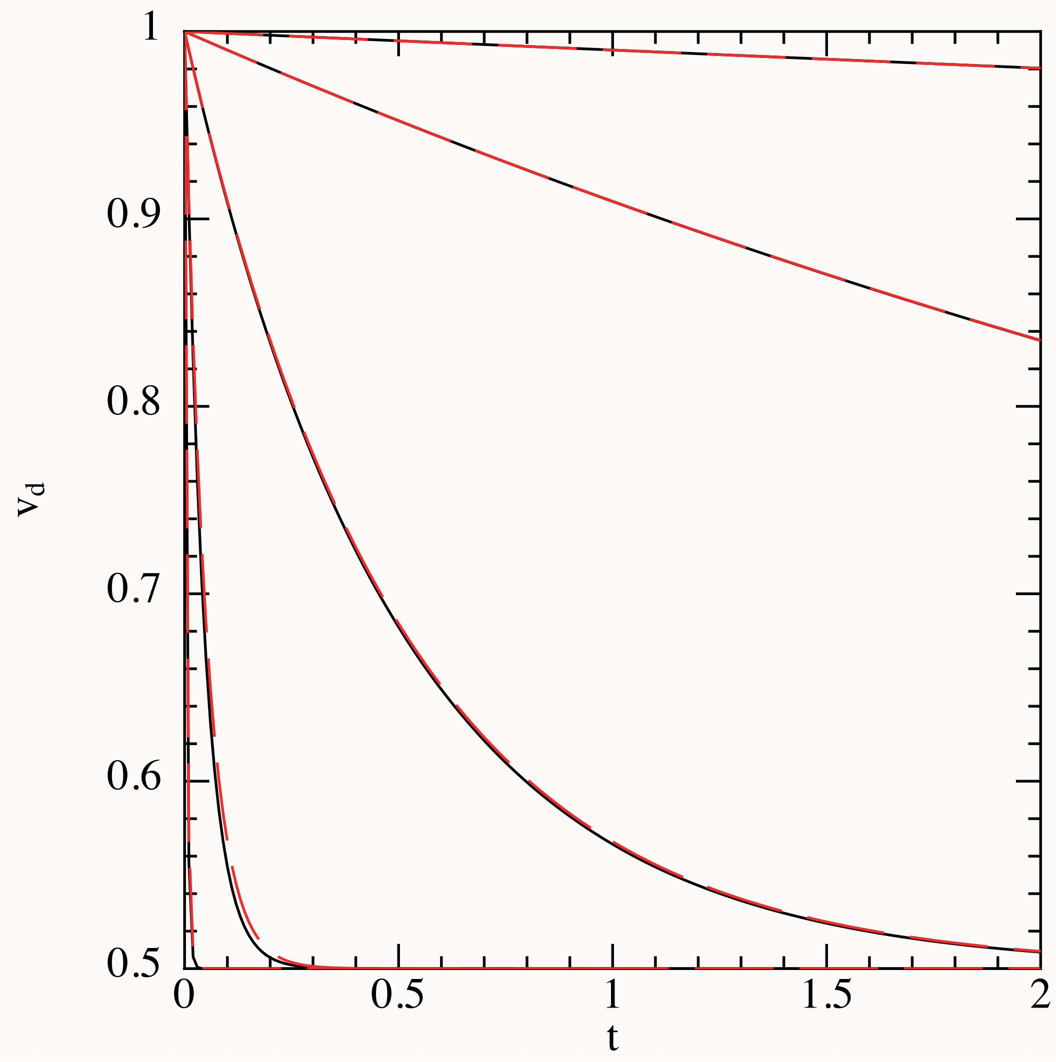
SPH gas+dust particles: Monaghan & Kocharyan (1995), Monaghan (1997), Maddison et al. (2003), Rice et al. (2004), Barriere-Fouchet et al. (2005), Ayliffe et al. (2011), Cha & Nayakshin (2011), Nayakshin & Cha (2012)...
Grid+dust particles: Fromang & Papaloizou (2006), Pardekooper & Mellema (2006), Johansen et al. (2007), Miniati (2010), Bai & Stone (2010)...

- Two fluids coupled by a drag term

$$\begin{aligned}
\frac{\partial \rho_{\rm g}}{\partial t} + \nabla . \left(\rho_{\rm g} \mathbf{v}_{\rm g}\right) &= 0, \\
\frac{\partial \rho_{\rm d}}{\partial t} + \nabla . \left(\rho_{\rm d} \mathbf{v}_{\rm d}\right) &= 0, \\
\frac{\partial \mathbf{v}_{\rm g}}{\partial t} + \left(\mathbf{v}_{\rm g} . \nabla\right) \mathbf{v}_{\rm g} &= -\frac{\nabla P_{\rm g}}{\rho_{\rm g}} + K(\mathbf{v}_{\rm d} - \mathbf{v}_{\rm g}) + \mathbf{f}, \\
\frac{\partial \mathbf{v}_{\rm d}}{\partial t} + \left(\mathbf{v}_{\rm d} . \nabla\right) \mathbf{v}_{\rm d} &= -K(\mathbf{v}_{\rm d} - \mathbf{v}_{\rm g}) + \mathbf{f},
\end{aligned}$$

DUSTYBOX

- Drag induces decay of differential velocity between fluids



K=0.01->100

"Stopping time"

$$t_{\rm stop} \equiv \frac{\rho_{\rm d}\rho_{\rm g}}{K(\rho_{\rm d}+\rho_{\rm g})}$$

Laibe & Price (2011, 2012a)

DUSTYWAVES

Laibe & Price, 2011, MNRAS 418, 1491

$$\delta v = A e^{i(kx - \omega t)}$$

Dispersion relation:

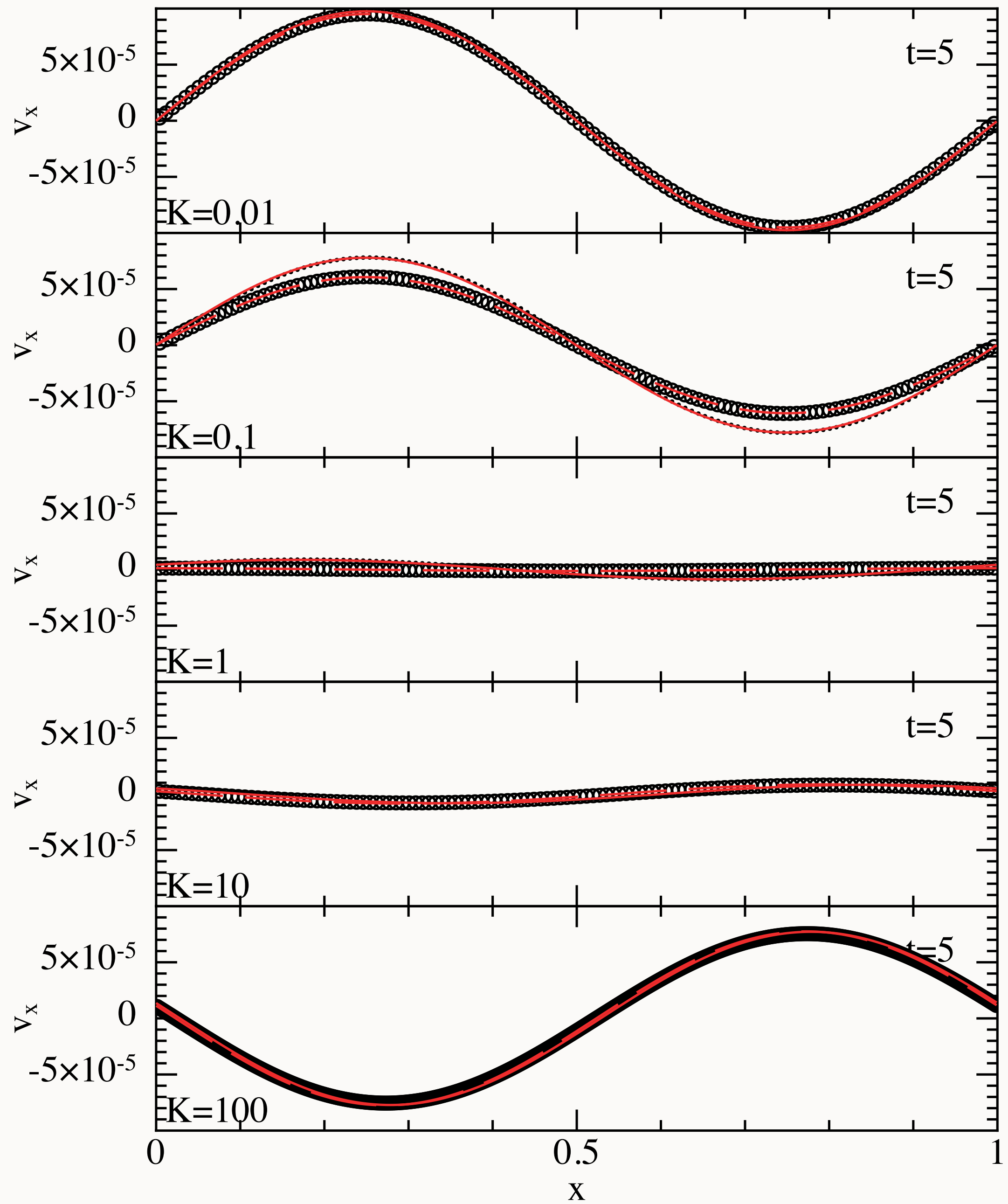
$$\omega^3 + iK\left(\frac{1}{\hat{\rho}_{\rm g}} + \frac{1}{\hat{\rho}_{\rm d}}\right)\omega^2 - k^2 c_{\rm s}^2 \omega - iK\frac{k^2 c_{\rm s}^2}{\hat{\rho}_{\rm d}} = 0$$

For strong drag:

$$\omega = \pm k\tilde{c}_{\rm s} - i\frac{\hat{\rho}_{\rm g}\hat{\rho}_{\rm d}}{K\left(\hat{\rho}_{\rm g} + \hat{\rho}_{\rm d}\right)} k^2 c_{\rm s}^2 \left(\frac{1 - A^2}{2}\right)$$

Effective sound speed:

$$\tilde{c}_{\rm s} \equiv c_{\rm s} A = c_{\rm s}\left(1 + \frac{\hat{\rho}_{\rm d}}{\hat{\rho}_{\rm g}}\right)^{-\frac{1}{2}}$$



DUSTYWAVES: ANALYTIC SOLUTION

Laibe & Price, 2011, MNRAS 418, 1491

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!-------------------------------
!DUST VELOCITIES
!-------------------------------
vd3r = - (rhogeq*cs**2*k**2*w2r**2*w1r**2*rhogsol + rhogeq**2*w3i**4*k*w1r*vgsol - w3i*cs**2*k**3*rhogeq*Kdrag*vdsol*w2r - w3i*cs**2*k**3*rhogeq*Kdrag*vdsol*w1r + w3i*cs**2*k**3*rhogeq*Kdrag*vgsol*w2r +
w3i*cs**2*k**3*rhogeq*Kdrag*vgsol*w1r -rhogeq**2*cs**2*k**3*w2i**2*w1r*vgsol + rhogeq*cs**4*k**4*w2r*w1r*rhogsol - rhogeq**2*cs**2*k**3*w2r**2*w1r*vgsol - rhogeq*cs**2*k**3*w2r*w1i*Kdrag*vgsol -
rhogeq**2*cs**2*k**3*w2r*w1i**2*vgsol + rhogeq*cs**2*k**3*w2r*w1i*Kdrag*vdsol -    rhogeq**2*cs**2*k**3*w2r*w1r**2*vgsol - rhogeq*w3i**4*cs**2*k**2*rhogsol - rhogeq*cs**4*k**4*w2i*w1i*rhogsol +
rhogeq*cs**2*k**2*w1i**2*w2r**2*rhogsol + rhogeq**2*w3i**4*w2r*k*vgsol - rhogeq*w3i**3*k*Kdrag*vdsol*w1r +  2*rhogeq*w3i**3*k*Kdrag*vgsol*w2r + rhogeq*w2i**2*w1i**2*cs**2*k**2*rhogsol +
rhogeq*w2i**2*w1r**2*cs**2*k**2*rhogsol - rhogeq*w3i**4*w2r*w1r*rhogsol + rhogeq*w3i**4*w1i*rhogsol*w2i +  w3i*cs**4*k**4*rhogeq*rhogsol*w2i + w3i*cs**4*k**4*rhogeq*rhogsol*w1i + w3i**2*Kdrag**2*k*vgsol*w1r -
w3i**3*Kdrag*rhogsol*k**2*cs**2 - w3i**3*Kdrag*rhogsol*w2r*w1r + w3i**3*Kdrag*rhogsol*w2i*w1i - w3i*cs**2*k**2*rhogeq*rhogsol*w2r**2*w1i - w3i*cs**2*k**2*rhogeq*rhogsol*w1i*w2i**2 +
rhogeq*w3i**3*rhogsol*k**2*cs**2*w2i + rhogeq*w3i**3*rhogsol*k**2*cs**2*w1i - rhogeq*w3i**3*rhogsol*w2r**2*w1i  - rhogeq*w3i**3*rhogsol*w1i*w2i**2 - rhogeq*w3i**3*rhogsol*w2i*w1r**2 -
w3i*cs**2*k**2*rhogeq*rhogsol*w2i*w1r**2 - w3i*cs**2*k**2*rhogeq*rhogsol*w2i*w1i**2 - rhogeq*w3i**3*rhogsol*w2i*w1i**2 - rhogeq*w3i**3*k*Kdrag*vdsol*w2r + 2*rhogeq*w3i**3*k*Kdrag*vgsol*w1r +
w3r*rhogeq**2*cs**2*k**3*w2i**2*vgsol + w3r*rhogeq*w2r*w3i**2*cs**2*k**2*rhogsol - w3r*rhogeq**2*w3i**2*k*w1r**2*vgsol + w3r*rhogeq*w3i**2*cs**2*k**2*rhogsol*w1r +  2*w3r*rhogeq*cs**2*k**3*w3i*Kdrag*vdsol -
w3r*rhogeq*cs**4*k**4*w2r*rhogsol +  w3r*rhogeq**2*cs**2*k**3*w2r**2*vgsol + w3r**4*rhogeq**2*w2r*vgsol*k - w3r**4*rhogeq*w2r*w1r*rhogsol +  w3r**4*rhogeq**2*k*w1r*vgsol - w3r**4*rhogeq*rhogsol*k**2*cs**2 +
w3r**4*rhogeq*w2i*rhogsol*w1i +2*w3r*rhogeq*w2i*w3i*w1i*k*Kdrag*vgsol + 2*w3r*rhogeq**2*w2i*k*vgsol*w3i*w1r**2 +  2*w3r*rhogeq*w2r*w3i*w1r*k*Kdrag*vdsol - 2*w3r*rhogeq*w2i*w3i**2*k*Kdrag*vgsol -
w3r*w3i**2*Kdrag**2*k*vgsol + w3r*w3i**2*Kdrag**2*k*vdsol - w3r*rhogeq**2*w1i**2*k*vgsol*w3i**2 - 2*w3r*rhogeq**2*w3i**2*k*w2i*w1i*vgsol + 2*w3r*rhogeq**2*w2i*w1i**2*w3i*k*vgsol +
w3r*w3i**2*Kdrag*rhogsol*w1i*w2r + w3r*w3i**2*Kdrag*rhogsol*w2i*w1r - w3r*Kdrag*rhogsol*k**2*cs**2*w1i*w2r - w3r*Kdrag*rhogsol*k**2*cs**2*w2i*w1r +  w3r*Kdrag*k*rhogeq*vgsol*w2i*w1r**2 +
w3r*Kdrag*k*rhogeq*vgsol*w2i*w1i**2 + w3r*Kdrag**2*k*vdsol*w2r*w1r -w3r*Kdrag**2*k*vdsol*w2i*w1i - w3r*Kdrag**2*k*vgsol*w2r*w1r + w3r*Kdrag**2*k*vgsol*w2i*w1i + w3r*Kdrag*k*rhogeq*vgsol*w2r**2*w1i +
w3r*Kdrag*k*rhogeq*vgsol*w1i*w2i**2 - w3r*rhogeq**2*w2r**2*w3i**2*k*vgsol  - 2*w3r*rhogeq*w2r*w3i*w1r*k*Kdrag*vgsol + w3r*rhogeq*w2i*w3i**2*k*Kdrag*vdsol - 2*w3r*rhogeq**2*w2r*w3i**2*k*w1r*vgsol +
w3r*rhogeq*w2r**2*w3i**2*w1r*rhogsol -  2*w3r*rhogeq*w1i*w3i**2*k*Kdrag*vgsol + w3r*rhogeq*w1i*w3i**2*k*Kdrag*vdsol + 2*w3r*rhogeq**2*w3i*w2r**2*w1i*k*vgsol + w3r*rhogeq*w1i**2*w3i**2*w2r*rhogsol -
w3r*rhogeq**2*w3i**2*w2i**2*k*vgsol - w3r*rhogeq*cs**2*k**2*w1i**2*w2r*rhogsol -   w3r*rhogeq*cs**2*k**2*w2r*w1r**2*rhogsol -w3r*rhogeq*cs**2*k**2*w2r**2*w1r*rhogsol + w3r*rhogeq*w2i**2*w3i**2*w1r*rhogsol +
w3r*rhogeq*w2r*w3i**2*w1r**2*rhogsol - 2*w3r*rhogeq**2*cs**2*k**3*w3i*w2i*vgsol - w3r*rhogeq*w2i**2*w1r*cs**2*k**2*rhogsol +  2*w3r*rhogeq**2*w2i**2*w3i*w1i*k*vgsol + w3r*rhogeq*cs**2*k**3*w1i*Kdrag*vgsol -
w3r*rhogeq*cs**2*k**3*w2i*Kdrag*vdsol + w3r*rhogeq*cs**2*k**3*w2i*Kdrag*vgsol - w3r*rhogeq*cs**2*k**3*w1i*Kdrag*vdsol + w3r*rhogeq**2*cs**2*k**3*w1i**2*vgsol -  2*w3r*rhogeq**2*cs**2*k**3*w3i*w1i*vgsol +
w3r*rhogeq**2*cs**2*k**3*w1r**2*vgsol - w3r*rhogeq*cs**4*k**4*w1r*rhogsol - 2*w3r*rhogeq*cs**2*k**3*w3i*Kdrag*vgsol + 2*w3r*rhogeq**2*cs**2*k**3*w2r*w1r*vgsol - 2*w3r*rhogeq*w2i*w3i*w1i*k*Kdrag*vdsol +
2*w3r*rhogeq**2*cs**2*k**3*w2i*w1i*vgsol + w3r**2*w3i*rhogeq*rhogsol*k**2*cs**2*w1i - w3r**2*w3i*rhogeq*rhogsol*w2r**2*w1i - w3r**2*w3i*rhogeq*rhogsol*w1i*w2i**2 - w3r**2*w3i*rhogeq*rhogsol*w2i*w1r**2 -
w3r**2*w3i*rhogeq*rhogsol*w2i*w1i**2 - w3r**2*w3i*rhogeq*k*Kdrag*vdsol*w2r - w3r**2*Kdrag*rhogsol*w3i*w2r*w1r + w3r**2*Kdrag*rhogsol*w2i*w3i*w1i - w3r**2*w3i*rhogeq*k*Kdrag*vdsol*w1r +
w3r**2*w3i*rhogeq*rhogsol*k**2*cs**2*w2i - w3r**2*Kdrag*rhogsol*k**2*cs**2*w3i + w3r**2*Kdrag*rhogsol*k**2*cs**2*w2i + w3r**2*Kdrag*rhogsol*k**2*cs**2*w1i + 2*w3r**2*Kdrag*k*rhogeq*vgsol*w2r*w3i +
2*w3r**2*Kdrag*k*rhogeq*vgsol*w3i*w1r + 2*w3r**2*rhogeq**2*w2r*k*vgsol*w3i**2 - w3r**2*rhogeq**2*cs**2*k**3*w2r*vgsol + w3r**2*rhogeq*w2r*w1i*k*Kdrag*vgsol + w3r**2*rhogeq**2*w2r*w1i**2*k*vgsol -
w3r**2*rhogeq*w2r*w1i*k*Kdrag*vdsol + w3r**2*rhogeq**2*w2r*w1r**2*k*vgsol - w3r**2*rhogeq**2*cs**2*k**3*w1r*vgsol - w3r**2*Kdrag**2*k*vdsol*w2r - w3r**2*rhogeq*w2i**2*w1i**2*rhogsol -
w3r**2*rhogeq*w1r**2*w2i**2*rhogsol - w3r**2*rhogeq*w2r**2*w1r**2*rhogsol + w3r**2*rhogeq*cs**4*k**4*rhogsol - w3r**2*rhogeq*w1i**2*w2r**2*rhogsol - w3r**2*Kdrag*rhogsol*w1i*w2i**2 -
w3r**2*Kdrag*rhogsol*w2i*w1r**2 - w3r**2*Kdrag*rhogsol*w2i*w1i**2 - w3r**2*Kdrag*rhogsol*w2r**2*w1i - w3r**2*Kdrag**2*k*vdsol*w1r + w3r**2*Kdrag**2*k*vgsol*w2r + w3r**2*Kdrag**2*k*vgsol*w1r +
w3r**3*Kdrag*rhogsol*w2i*w1r + w3r**3*rhogeq*w2i*k*Kdrag*vdsol - w3r**3*rhogeq**2*w2i**2*k*vgsol + w3r**2*rhogeq**2*w2i**2*w1r*k*vgsol - 2*w3r**2*rhogeq*w2r*w3i**2*w1r*rhogsol +
2*w3r**2*rhogeq**2*w3i**2*k*w1r*vgsol + 2*w3r**2*rhogeq*w1i*w3i**2*rhogsol*w2i - w3r**2*rhogeq*w2i*w1r*k*Kdrag*vdsol + w3r**2*rhogeq*w2i*w1r*k*Kdrag*vgsol - w3r**3*rhogeq**2*w1i**2*k*vgsol -
2*w3r**3*rhogeq**2*w2r*k*w1r*vgsol + w3r**3*rhogeq*w2r**2*w1r*rhogsol + w3r**3*rhogeq*w2r*cs**2*k**2*rhogsol - 2*w3r**3*rhogeq*w2i*k*Kdrag*vgsol - w3r**3*rhogeq**2*w1r**2*k*vgsol +
w3r**3*rhogeq*w1r*cs**2*k**2*rhogsol - w3r**3*rhogeq**2*w2r**2*k*vgsol + w3r**3*rhogeq*w1i*k*Kdrag*vdsol - 2*w3r**3*rhogeq**2*k*w2i*w1i*vgsol + w3r**3*rhogeq*w2r*w1r**2*rhogsol +
w3r**3*rhogeq*w1i**2*w2r*rhogsol + w3r**3*rhogeq*w2i**2*w1r*rhogsol - 2*w3r**3*rhogeq*w1i*k*Kdrag*vgsol + w3r**3*Kdrag*rhogsol*w1i*w2r + w3r**2*rhogeq**2*w2r**2*k*w1r*vgsol -
2*w3r**2*rhogeq*w3i**2*cs**2*k**2*rhogsol + w3r**3*Kdrag**2*k*vdsol + w3i**2*rhogeq**2*cs**2*k**3*w2r*vgsol - w3r**3*Kdrag**2*k*vgsol + w3i**2*rhogeq*w2i*w1r*k*Kdrag*vdsol - w3i**2*Kdrag**2*k*vdsol*w2r +
w3i**2*Kdrag**2*k*vgsol*w2r - w3i**2*Kdrag*rhogsol*w2r**2*w1i - w3i**2*Kdrag*rhogsol*w1i*w2i**2 - w3i**2*Kdrag*rhogsol*w2i*w1r**2 - w3i**2*Kdrag*rhogsol*w2i*w1i**2 + w3i**2*Kdrag*rhogsol*k**2*cs**2*w2i +
w3i**2*Kdrag*rhogsol*k**2*cs**2*w1i - Kdrag*k*rhogeq*vgsol*w3i*w1r*w2r**2 - Kdrag*k*rhogeq*vgsol*w3i*w2r*w1r**2 - Kdrag*k*rhogeq*vgsol*w3i*w2r*w1i**2 - Kdrag*k*rhogeq*vgsol*w3i*w1r*w2i**2 +
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Kdrag*rhogsol*w2i**2*w1r**2*w3i + Kdrag**2*k*vdsol*w2r*w1i*w3i + Kdrag**2*k*vdsol*w2i*w3i*w1r - Kdrag**2*k*vgsol*w2r*w1i*w3i - Kdrag**2*k*vgsol*w2i*w3i*w1r - w3i**2*Kdrag**2*k*vdsol*w1r +
rhogeq*cs**2*k**3*w2i*w1r*Kdrag*vdsol - rhogeq*cs**2*k**3*w2i*w1r*Kdrag*vgsol - w3i**2*rhogeq**2*w2i**2*w1r*k*vgsol - w3i**2*rhogeq**2*w2r**2*k*w1r*vgsol - w3i**2*rhogeq*w2i*w1r*k*Kdrag*vgsol -
w3i**2*rhogeq*w2r*w1i*k*Kdrag*vgsol - w3i**2*rhogeq**2*w2r*w1i**2*k*vgsol + w3i**2*rhogeq*w2r*w1i*k*Kdrag*vdsol - w3i**2*rhogeq**2*w2r*w1r**2*k*vgsol + w3i**2*rhogeq**2*cs**2*k**3*w1r*vgsol +
w3i**2*rhogeq*w2i**2*w1i**2*rhogsol + w3i**2*rhogeq*w1r**2*w2i**2*rhogsol + w3i**2*rhogeq*w2r**2*w1r**2*rhogsol - w3i**2*rhogeq*cs**4*k**4*rhogsol + w3i**2*rhogeq*w1i**2*w2r**2*rhogsol)/(w1i**2 - 2*w3i*w1i +
w3r**2 + w1r**2 + w3i**2 - 2*w3r*w1r)/(w2r**2 - 2*w3r*w2r + w2i**2 + w3i**2 - 2*w2i*w3i + w3r**2)/k/rhogeq/Kdrag
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RESOLUTION STUDY

Laibe & Price, 2012, MNRAS 420, 2345

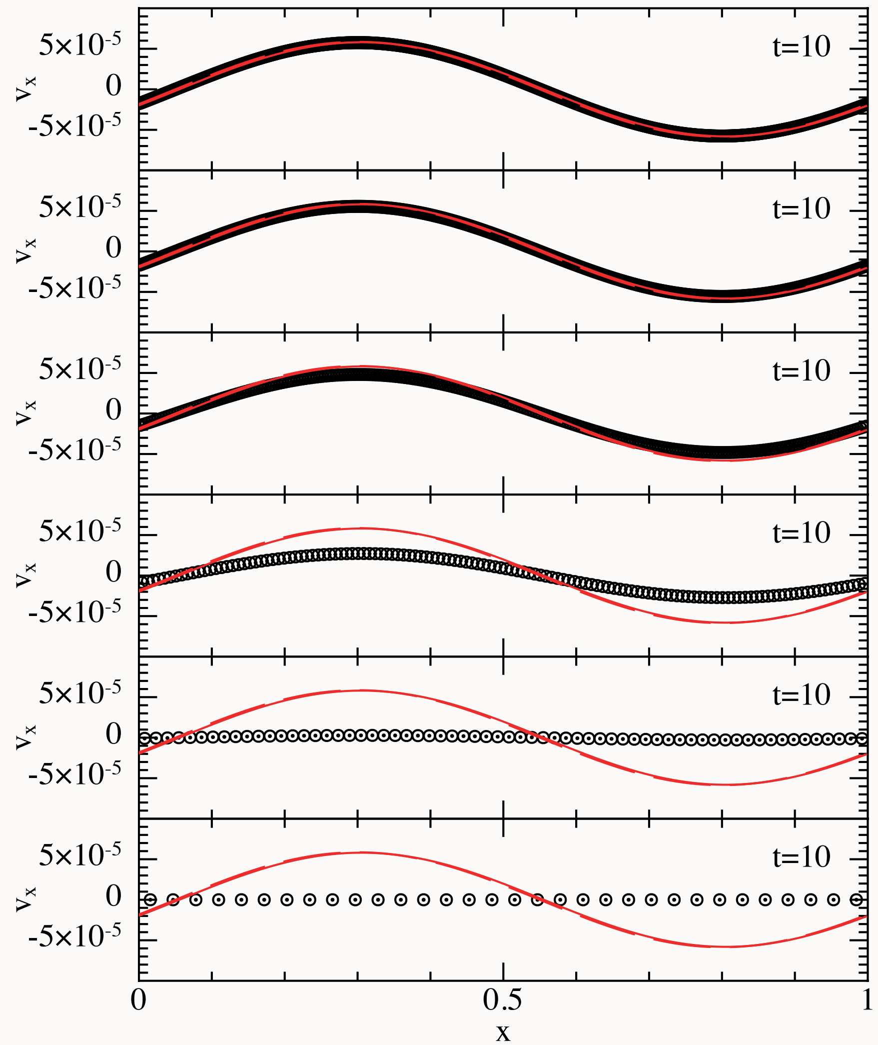
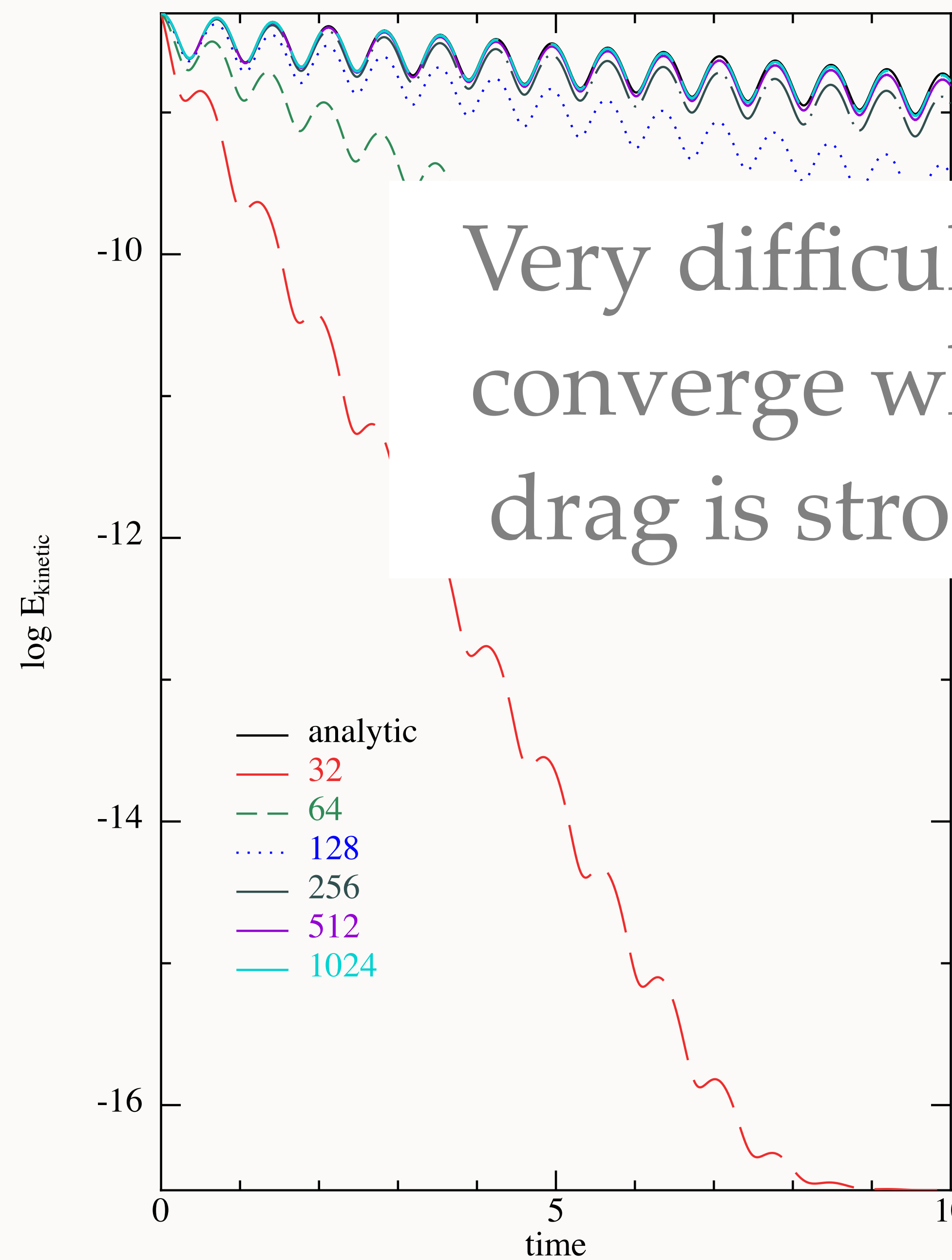


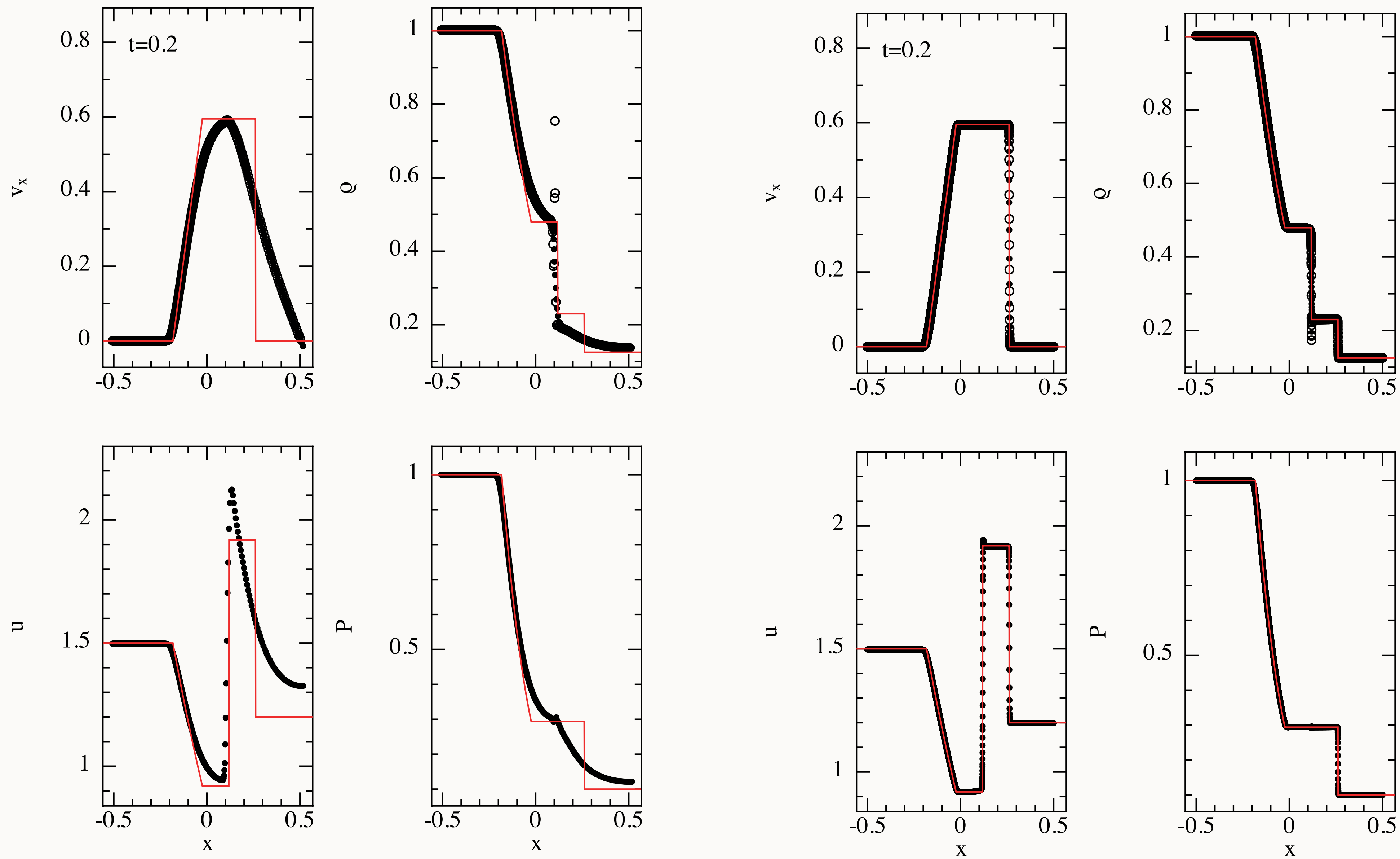
Figure 8. Resolution study for the DUSTYWAVE test in 1D using a high drag coefficient ($K = 100$) and a dust-to-gas ratio of unity using 32, 64, 128, 256, 512 and 1024 particles from bottom to top. At large drag high resolution is required to resolve the small differential motions between the fluids and thus prevent over-damping of the numerical solution, corresponding to the criterion $h \lesssim c_s t_s$, here implying $\gtrsim 240$ particles. See also Fig. 9.



Very difficult to converge when drag is strong!

DUSTYSHOCK

Laibe & Price, 2012, MNRAS 420, 2345



sensible resolution ludicrous resolution

RESOLUTION CRITERION

Laibe & Price, 2012, MNRAS 420, 2345

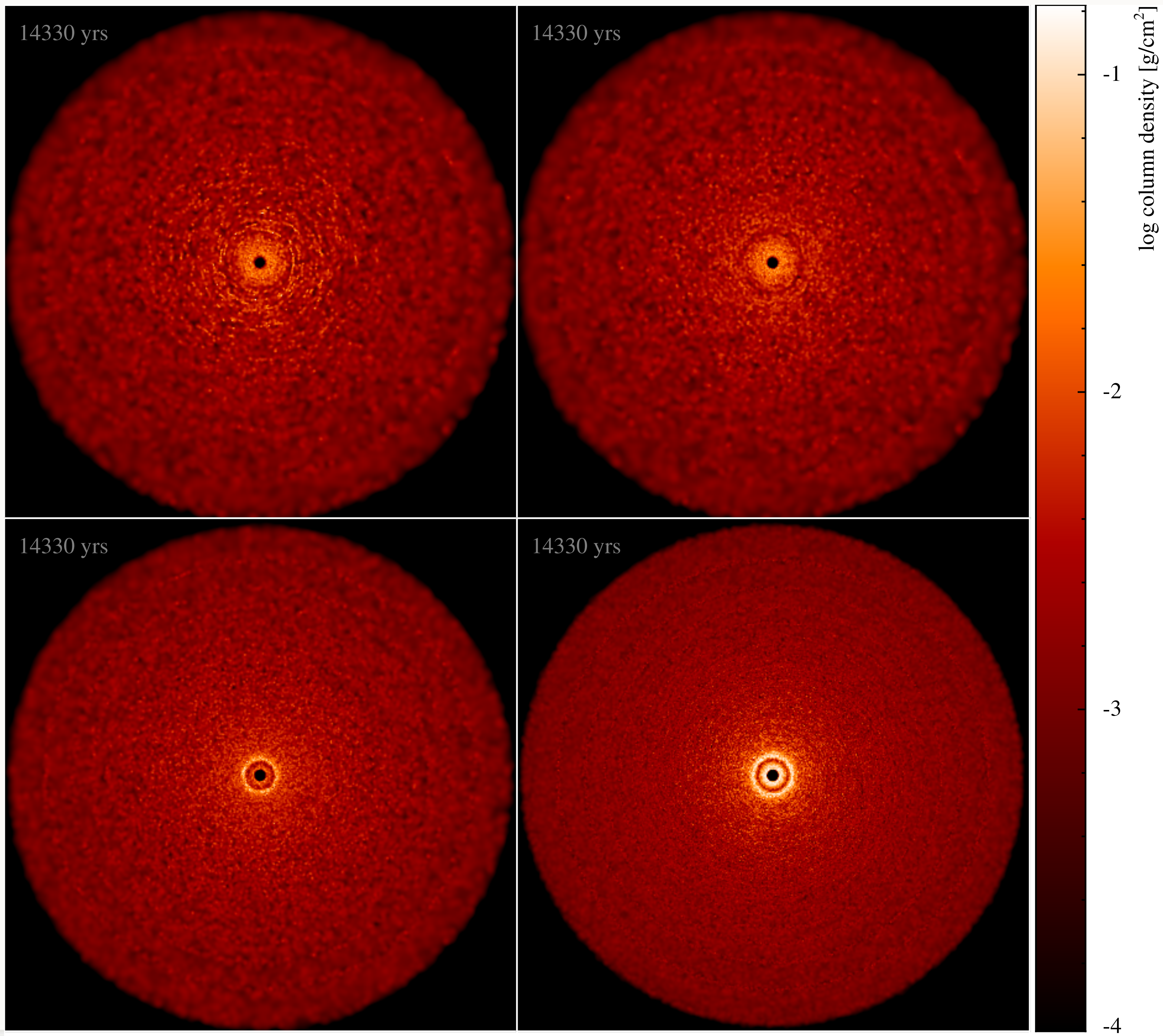
Temporal: $\Delta t < t_{\rm stop}$ (can be fixed with implicit timestepping methods)

Spatial: $\Delta x \lesssim t_{\rm stop} c_{\rm s}$ (cannot be fixed)

$t_{\rm stop} \to 0$ $(K \to \infty)$ implies $\Delta t \to 0$, $\Delta x \to 0$

- Require infinite timesteps AND infinite resolution in the obvious limit of perfect coupling!

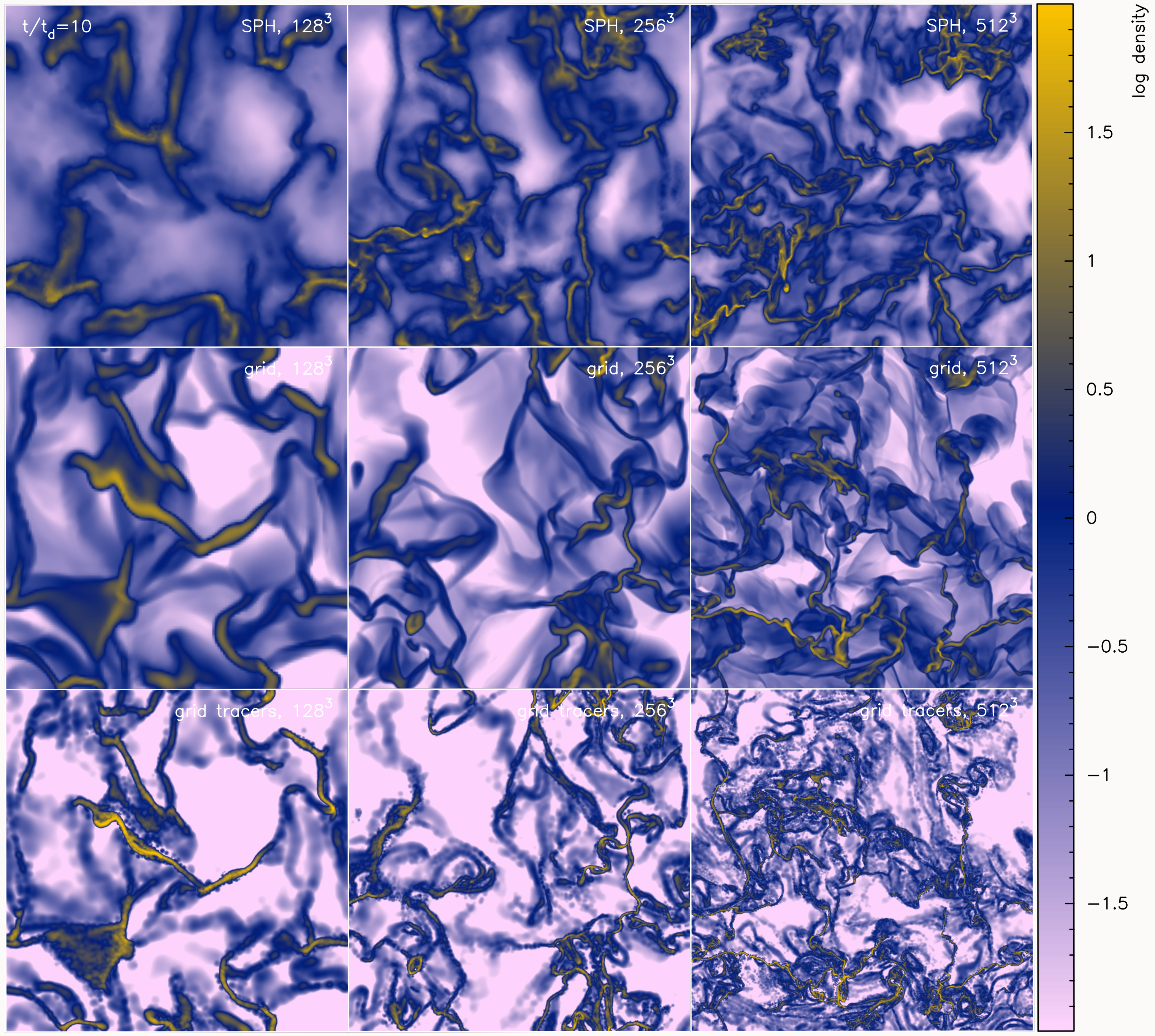
DUSTYDISC



Laibe & Price (2012a,b)

- Problems when the dust is concentrated below the resolution of the gas
- Need to ensure that h_gas < h_dust at all times

SAME ISSUE WITH GRIDS + DUST PARTICLES



Price & Federrath (2010)

SO WE ARE STUFFED

- Can’t use same formulation to solve both small grains (strong coupling) and large grains (weak coupling). Need a better approach to the strong coupling regime.
- Two fluid approach always ends up in trouble when dust collapses/condenses below resolution of the gas.
- Without analytic solutions, none of this would be obvious

DUSTY GAS WITH ONE FLUID?

- Reformulate equations on the barycentre of both fluids

$$\mathbf{v} \equiv \frac{\rho_{\rm g}\mathbf{v}_{\rm g} + \rho_{\rm d}\mathbf{v}_{\rm d}}{\rho_{\rm g} + \rho_{\rm d}}$$

- Change of variables, from $\mathbf{v}_{\rm g}, \mathbf{v}_{\rm d}, \rho_{\rm g}, \rho_{\rm d}$

  to $\mathbf{v}, \Delta\mathbf{v}, \rho, \rho_{\rm d}/\rho_{\rm g}$

TWO BECOME ONE

A phoenix from the ashes

- Two fluids coupled by a drag term

$$
\begin{aligned}
\frac{\partial \rho_{\rm g}}{\partial t} + \nabla . \left(\rho_{\rm g} \mathbf{v}_{\rm g}\right) &= 0, \\
\frac{\partial \rho_{\rm d}}{\partial t} + \nabla . \left(\rho_{\rm d} \mathbf{v}_{\rm d}\right) &= 0, \\
\frac{\partial \mathbf{v}_{\rm g}}{\partial t} + \left(\mathbf{v}_{\rm g} . \nabla\right) \mathbf{v}_{\rm g} &= -\frac{\nabla P_{\rm g}}{\rho_{\rm g}} + K(\mathbf{v}_{\rm d} - \mathbf{v}_{\rm g}) + \mathbf{f}, \\
\frac{\partial \mathbf{v}_{\rm d}}{\partial t} + \left(\mathbf{v}_{\rm d} . \nabla\right) \mathbf{v}_{\rm d} &= -K(\mathbf{v}_{\rm d} - \mathbf{v}_{\rm g}) + \mathbf{f},
\end{aligned}
$$

TWO BECOME ONE

A phoenix from the ashes

- One mixture with decay of differential velocity

$$
\begin{aligned}
\frac{\mathrm{d}\rho}{\mathrm{d}t} &= -\rho(\nabla.\mathbf{v}), \\
\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} &= \mathbf{f} - \frac{\nabla P_{\mathrm{g}}}{\rho} - \frac{1}{\rho}\nabla\left(\frac{\rho_{\mathrm{g}}\rho_{\mathrm{d}}}{\rho}\Delta\mathbf{v}^2\right), \\
\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\rho_{\mathrm{d}}}{\rho_{\mathrm{g}}}\right) &= -\frac{\rho}{\rho_{\mathrm{g}}^2}\nabla\cdot\left(\frac{\rho_{\mathrm{g}}\rho_{\mathrm{d}}}{\rho}\Delta\mathbf{v}\right), \\
\frac{\mathrm{d}\Delta\mathbf{v}}{\mathrm{d}t} &= -\frac{\Delta\mathbf{v}}{t_{\mathrm{s}}} + \frac{\nabla P_{\mathrm{g}}}{\rho_{\mathrm{g}}} - (\Delta\mathbf{v}\cdot\nabla)\mathbf{v} + \frac{1}{2}\nabla\left(\frac{\rho_{\mathrm{d}} - \rho_{\mathrm{g}}}{\rho_{\mathrm{d}} + \rho_{\mathrm{g}}}\Delta\mathbf{v}^2\right).
\end{aligned}
$$

Laibe & Price (2013, submitted to MNRAS)

WHAT DOES THIS SOLVE?

- Only one resolution length
- Mixture described by evolution of gas-to-dust ratio
- Only minor extension to usual fluid equations
- Strong drag limit is trivial!
- Completely general: can simulate both weak and strong coupling (small and large grains) with same formulation*

* if no counter-streaming

STRONG DRAG/SMALL GRAINS

- Equations simplify further in t

$$\Delta \mathbf{v} = \frac{\nabla P_{\rm g}}{\rho_{\rm g}} t_{\rm s}$$

$$\frac{{\rm d}\rho}{{\rm d}t} = -\rho(\nabla . \mathbf{v}),$$

$$\frac{{\rm d}\mathbf{v}}{{\rm d}t} = \mathbf{f} - \frac{\nabla P_{\rm g}}{\rho},$$

$$\frac{\rm d}{{\rm d}t}\left(\frac{\rho_{\rm d}}{\rho_{\rm g}}\right) = -\frac{\rho}{\rho_{\rm g}^2}\nabla . \left(\frac{\rho_{\rm g}\rho_{\rm d}}{\rho}\left[\frac{\nabla P_{\rm g}}{\rho_{\rm g}} t_{\rm s}\right]\right).$$

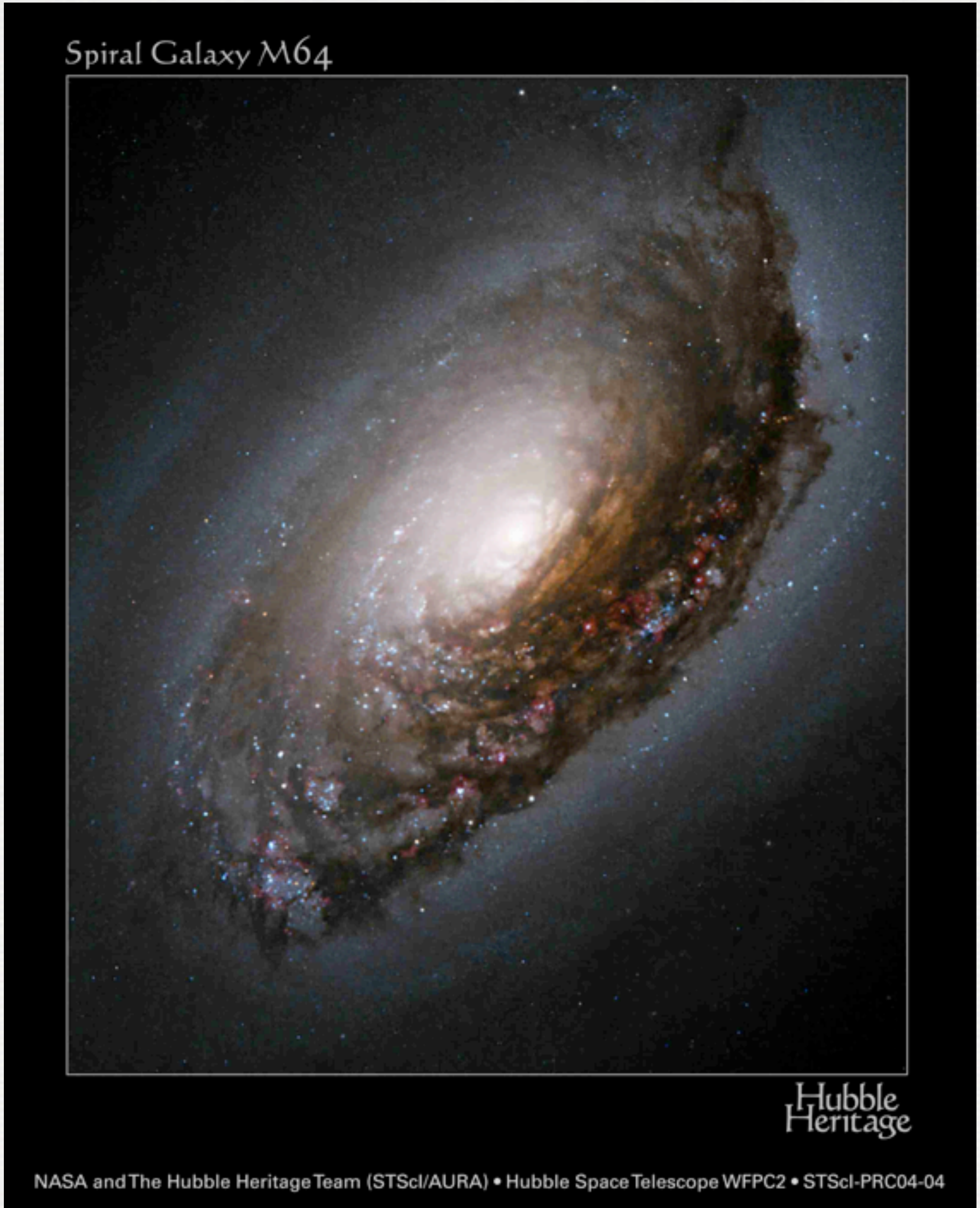
Valid when $t_{\rm stop} < \Delta t$

Hey, aren't we the usual equations of fluid dynamics?

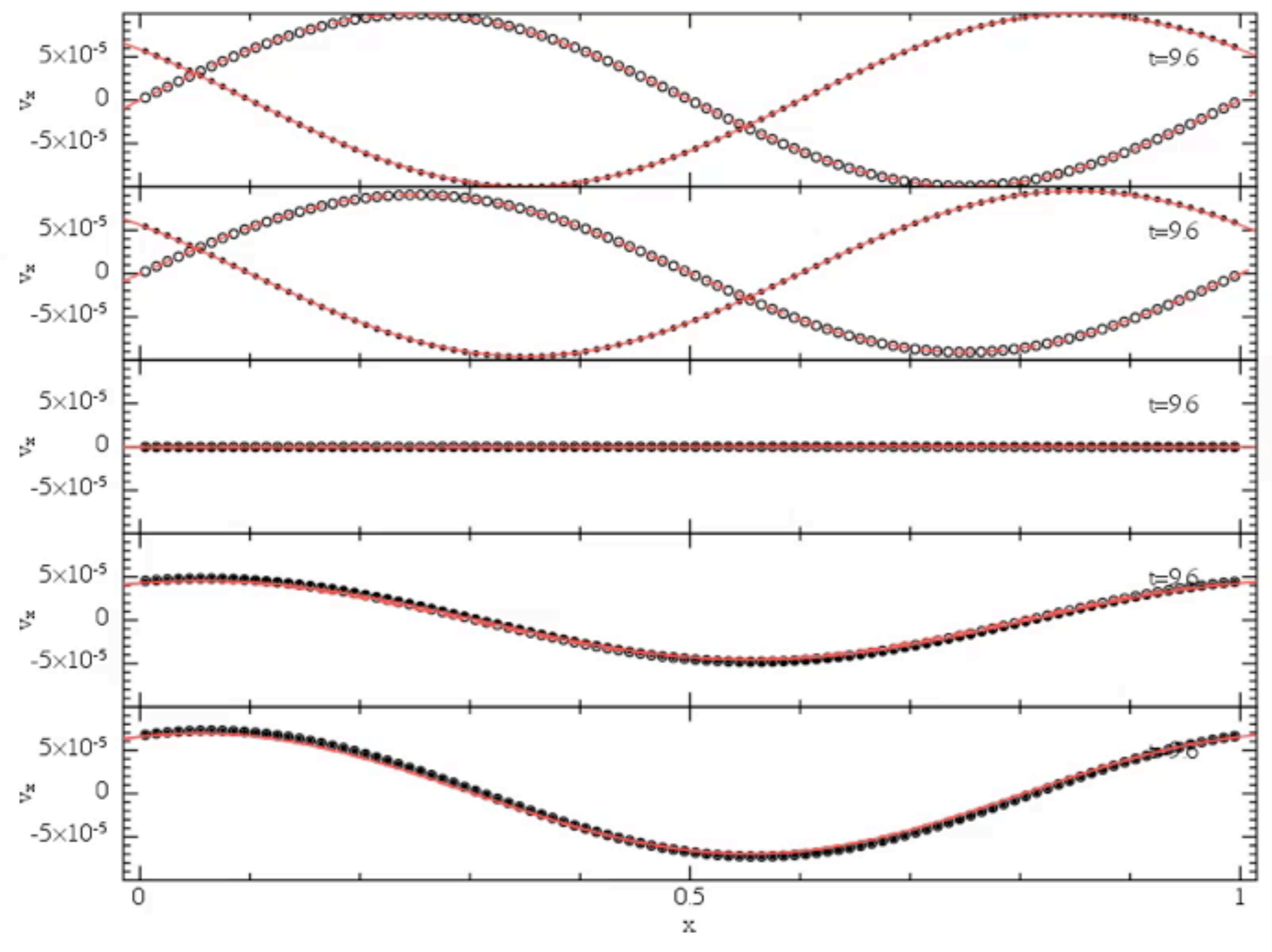
yes, but $P_{\rm g} = \tilde{c}_{\rm s}\rho$

diffusion equation for the dust-to-gas ratio

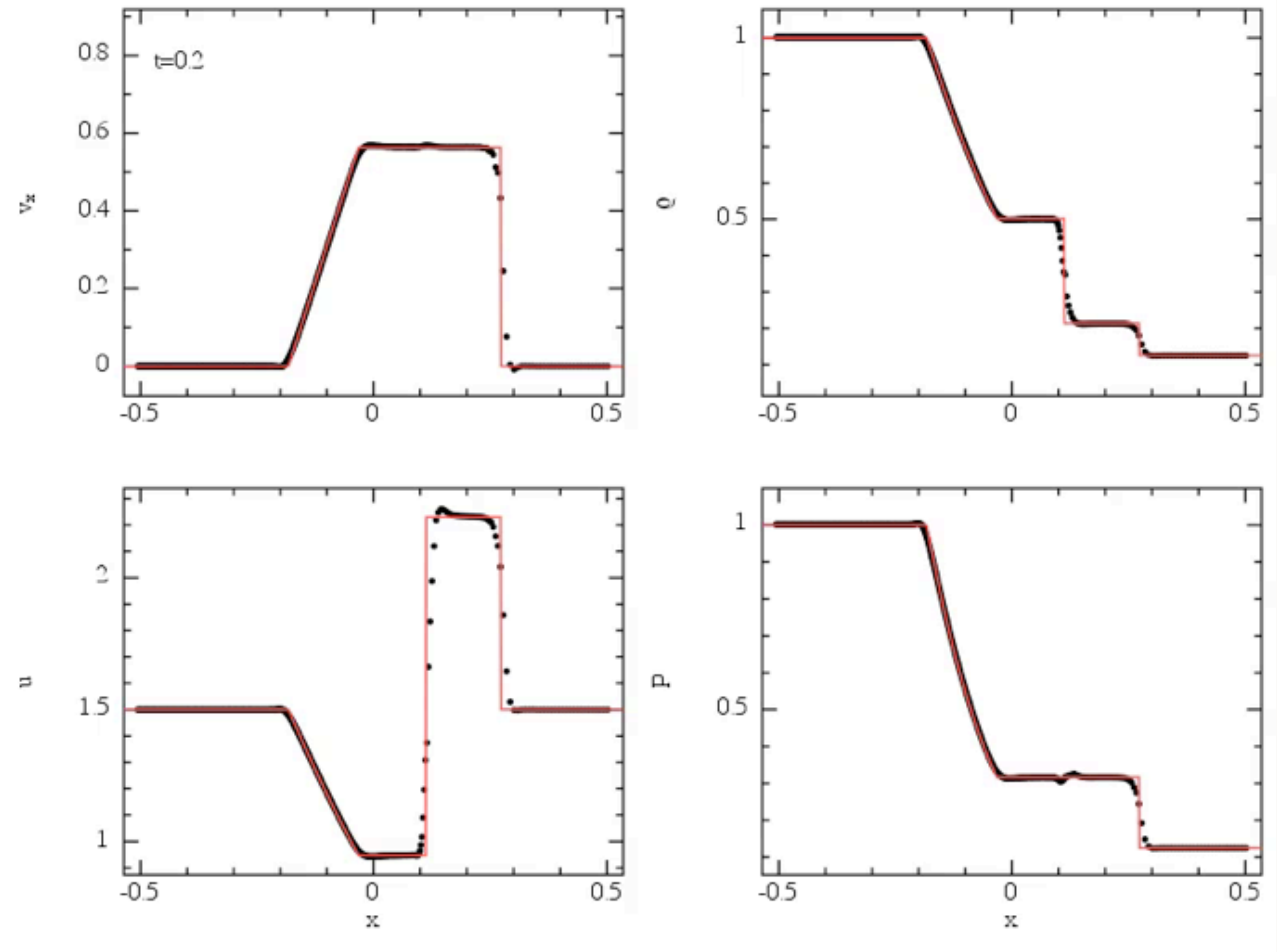
Laibe & Price (2013, submitted)


Spiral Galaxy M64
Hubble Heritage
NASA and The Hubble Heritage Team (STScI/AURA) • Hubble Space Telescope WFPC2 • STScI-PRC04-04

DUSTY WAVES: ONE FLUID



DUSTYSHOCK WITH ONE FLUID



CONCLUSION

WHAT DRAG HAS JOINED TOGETHER, LET MAN NOT SEPARATE