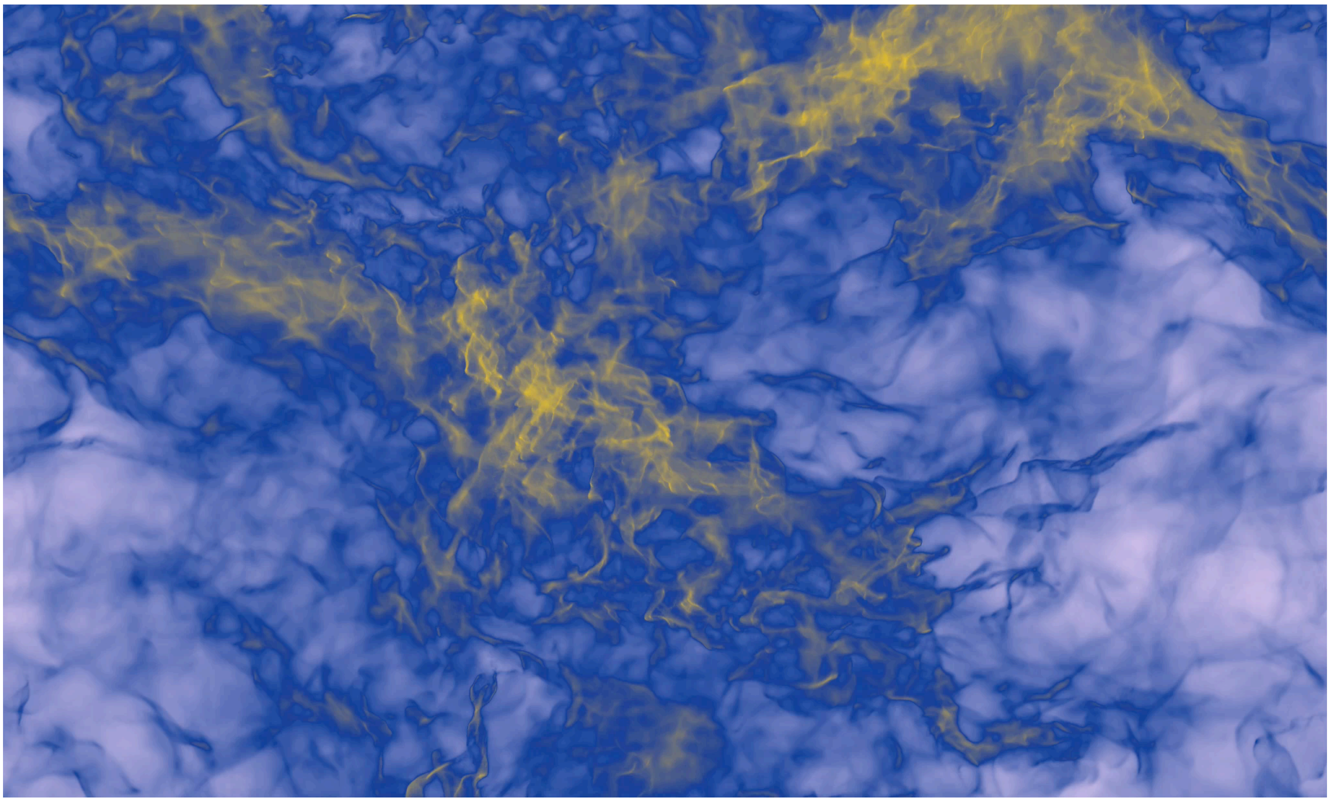


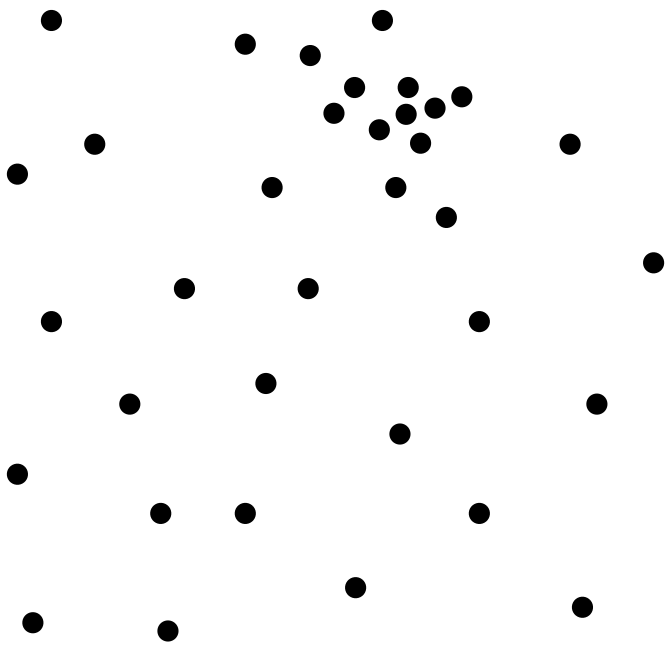
Smoothed Particle Hydrodynamics:

When you should, when you shouldn't
(or: things I wish my mother taught me)

Daniel Price
Monash Centre for Astrophysics
(MoCA) Melbourne, Australia

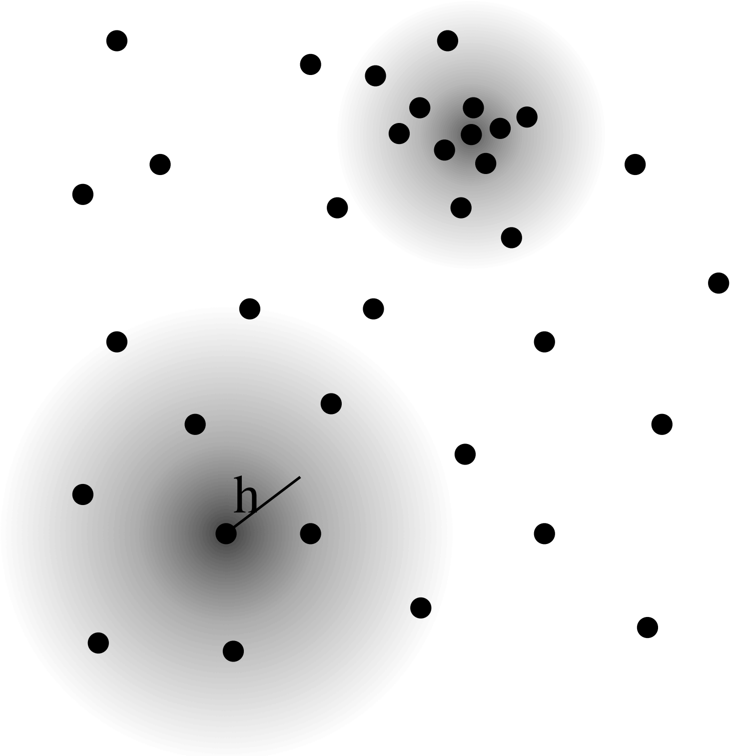
"Advances in Computational Astrophysics", June 13th-17th 2011, Cefalu, Italy

SPH starts here...



What is the density?

The SPH density estimate



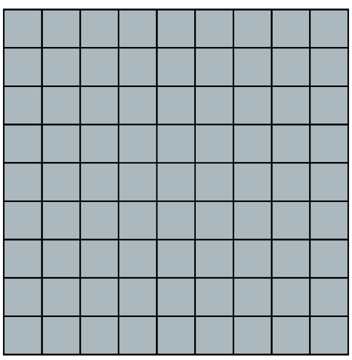
Kernel-weighted sum:

$$\rho(\mathbf{r}) = \sum_{j=1}^{N} m_j W(|\mathbf{r} - \mathbf{r}_j|, h)$$

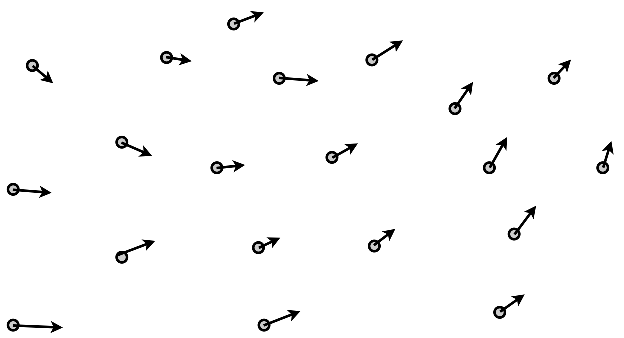
e.g. $W = \frac{\sigma}{h^3} e^{-r^2/h^2}$

Resolution follows mass

Grid



SPH



$$\frac{\mathrm{d}\mathbf{x}}{\mathrm{d}t} = \mathbf{v}$$

From density to hydrodynamics

$$L_{sph} = \sum_j m_j \left[\frac{1}{2} v_j^2 - u_j(\rho_j, s_j) \right]$$ ← Lagrangian

$+$

$$\mathrm{d}u = \frac{P}{\rho^2} \mathrm{d}\rho$$ ← 1st law of thermodynamics

$+$

$$\nabla \rho_i = \sum_j m_j \nabla W_{ij}(h)$$ ← density sum

$+$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \mathbf{v}}\right) - \frac{\partial L}{\partial \mathbf{r}} = 0$$ ← Euler-Lagrange equations

$=$

$$\frac{\mathrm{d}\mathbf{v}_i}{\mathrm{d}t} = -\sum_j m_j \left(\frac{P_i}{\rho_i^2} + \frac{P_j}{\rho_j^2} \right) \nabla_i W_{ij}(h)$$ ← equations of motion!

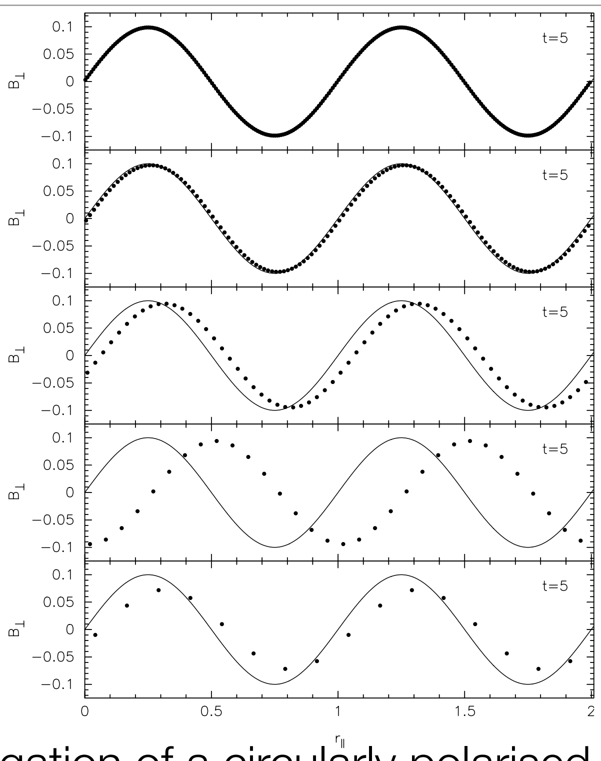
$$\left(\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = -\frac{\nabla P}{\rho} \right)$$

What this gives us: Advantages of SPH

- An exact solution to the continuity equation
- Resolution follows mass
- ZERO dissipation
- Advection done perfectly
- EXACT conservation of mass, momentum, angular momentum, energy and entropy
- A guaranteed minimum energy state

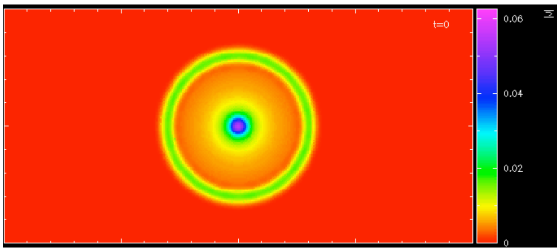
Zero dissipation

Zero dissipation - Example I.

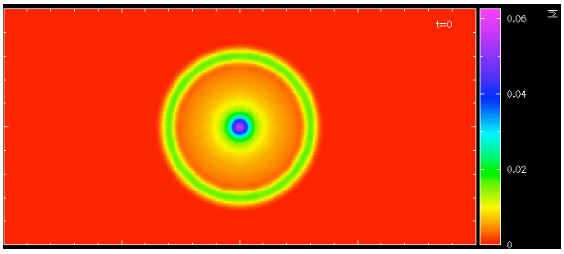


Propagation of a circularly polarised Alfven wave

Zero dissipation - II. Advection of a current loop



first 25 crossings



1000 crossings (Rosswog & Price 2007)

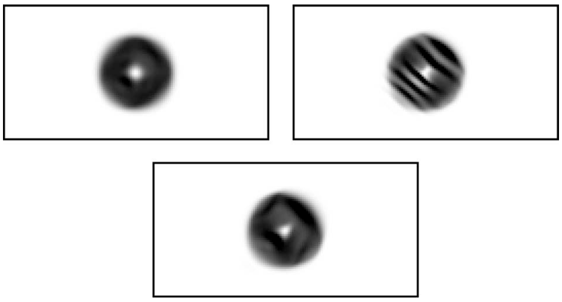


Fig. 3. Gray-scale images of the magnetic pressure ($B_x^2 + B_y^2$) at $t = 2$ for an advected field loop ($v_0 = \sqrt{5}$) using the $\mathcal{E}_z^\alpha$ (top left), (top right) and $\mathcal{E}_z^c$ (bottom) CT algorithm.

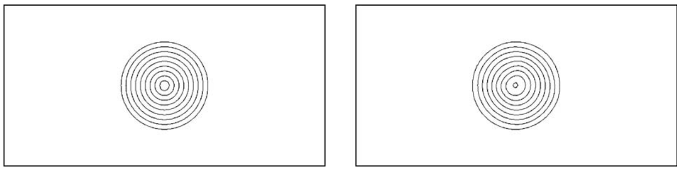


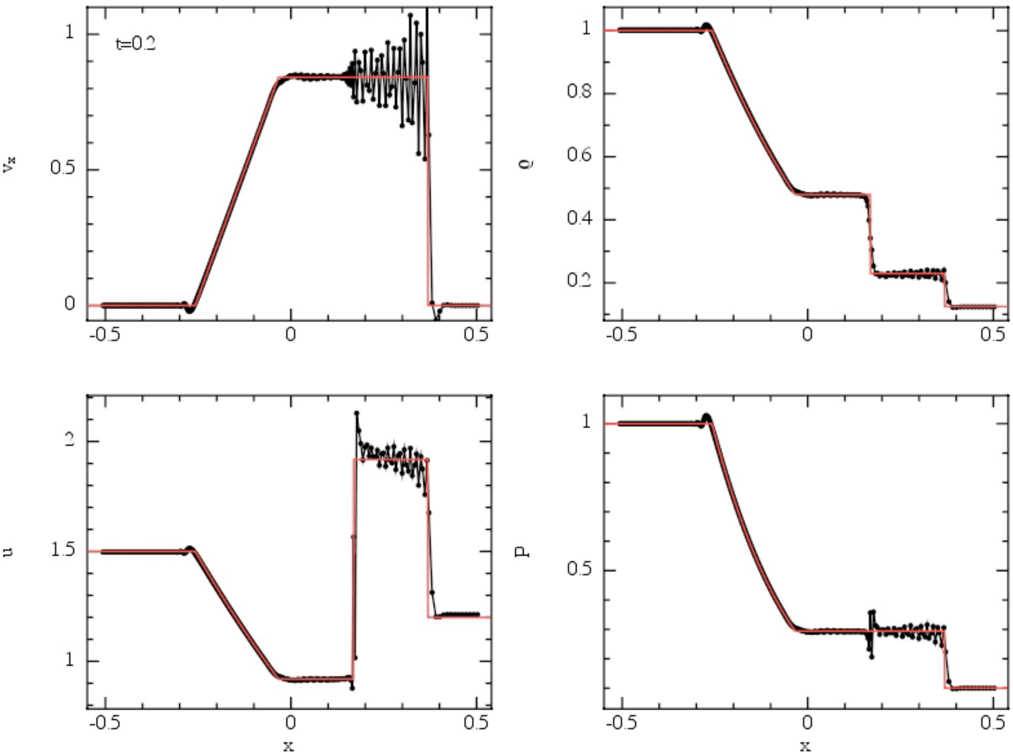
Fig. 8. Magnetic field lines at $t = 0$ (left) and $t = 2$ (right) using the CTU + CT integration algorithm.

2 crossings (Gardiner & Stone 2005)

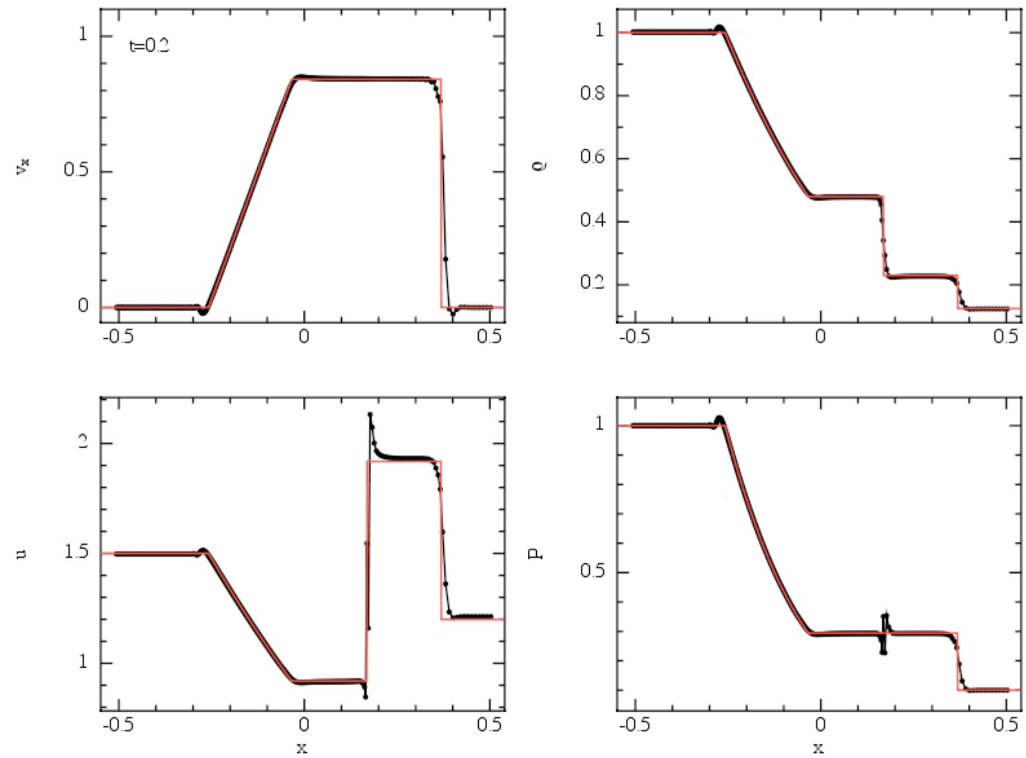
SPH

grid

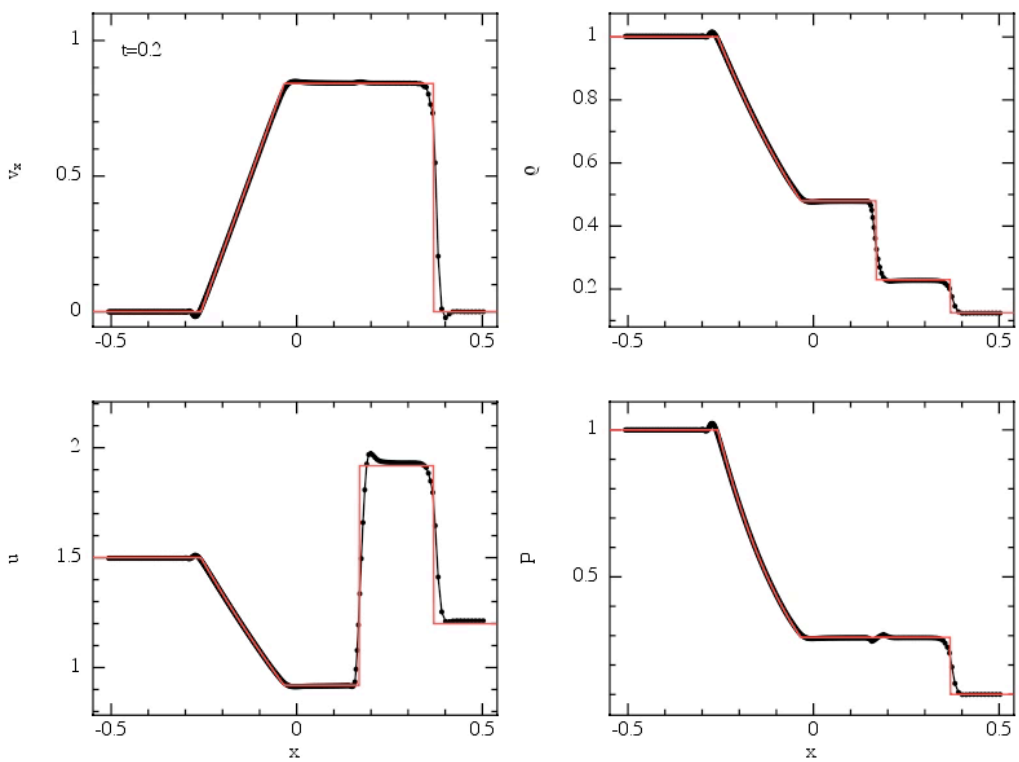
Zero dissipation...



Zero dissipation... so we have to add some

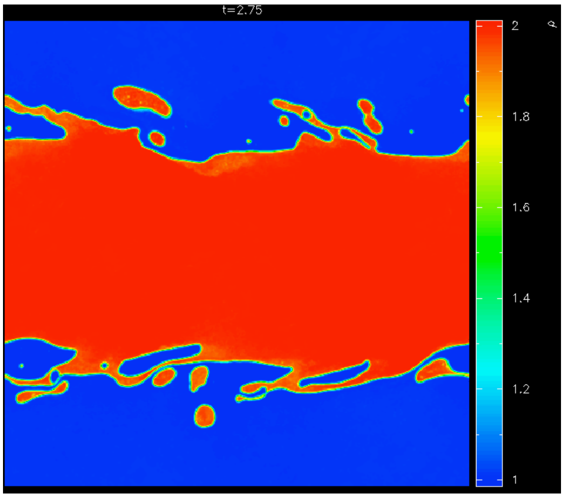


Must treat EVERY discontinuity

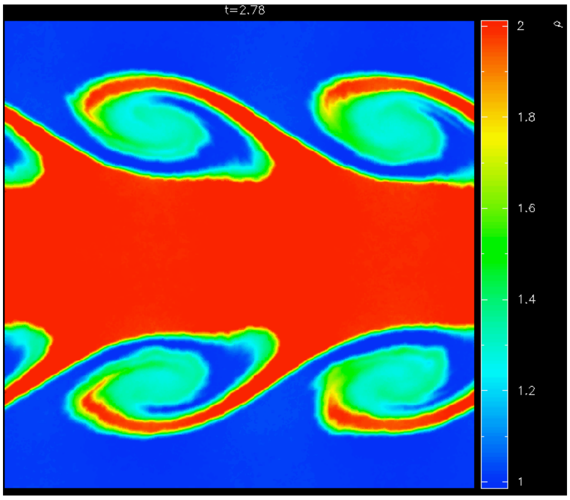


Viscosity
+
Conductivity

But must treat discontinuities properly...



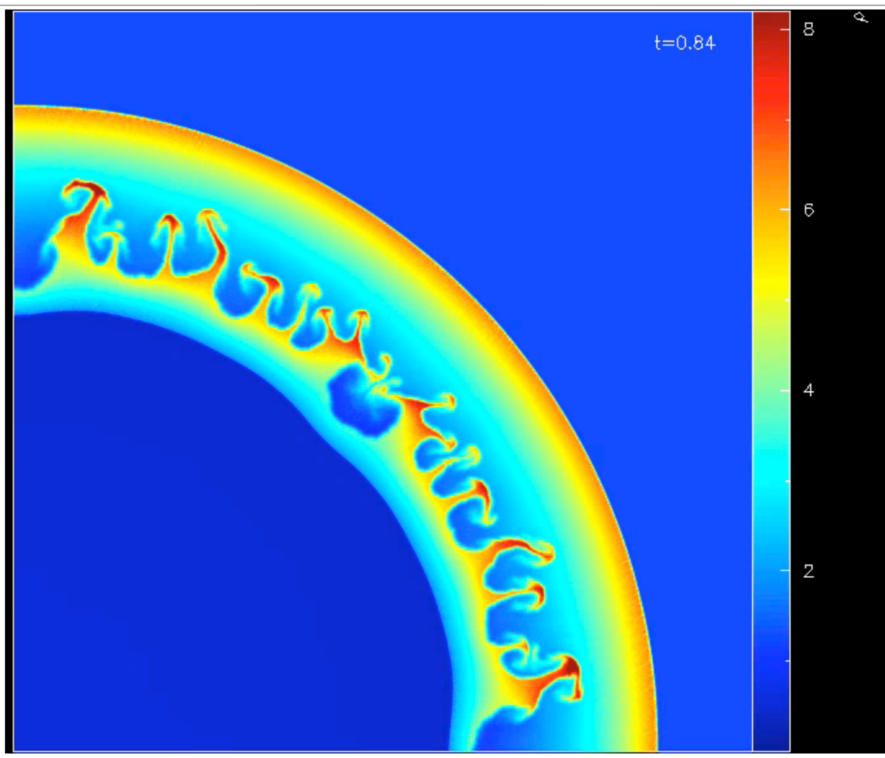
Viscosity only



Viscosity + conductivity

This issue has NOTHING to do with the Kelvin-Helmholtz instability

Richtmyer-Meshkov Instability



Exploding blob (Børve & Price 2010)

dissipation terms need to be explicitly added

The key is a good switch

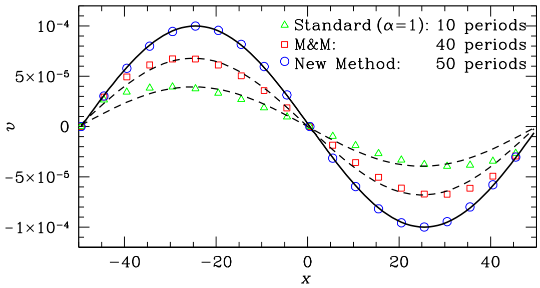


Figure 2. As Fig. 1, but for SPH with standard ($\alpha = 1$) or Morris & Monaghan (1997) artificial viscosity, as well as our new method (only every fifth particle is plotted). Also shown are the undamped wave (*solid*) and lower-amplitude sinusoidals (*dashed*). Only with our method the wave propagates undamped, very much like SPH without any viscosity, as in Fig. 1.

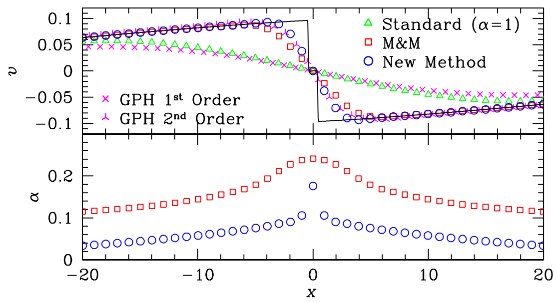
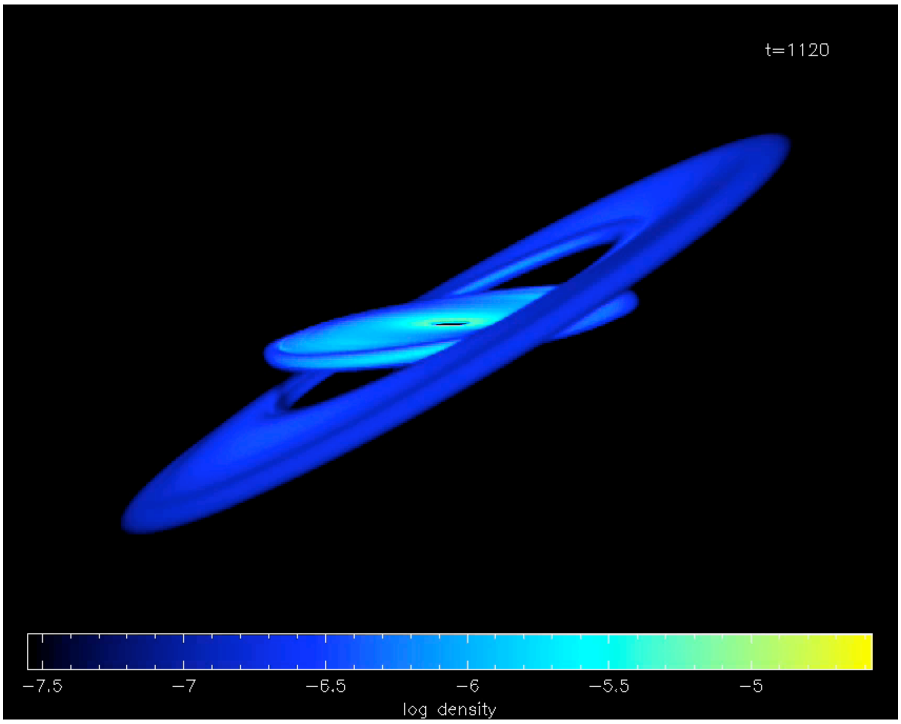


Figure 6. Steepening of a 1D sound wave: velocity and viscosity parameter vs. position for standard SPH, the M&M method, our new scheme, and Godunov particle hydrodynamics of first and second order (GPH, Cha & Whitworth 2003), each using 100 particles per wavelength. The solid curve in the top panel is the solution obtained with a high-resolution grid code.

Cullen & Dehnen (2010)

Exact conservation

Exact conservation: Advantages

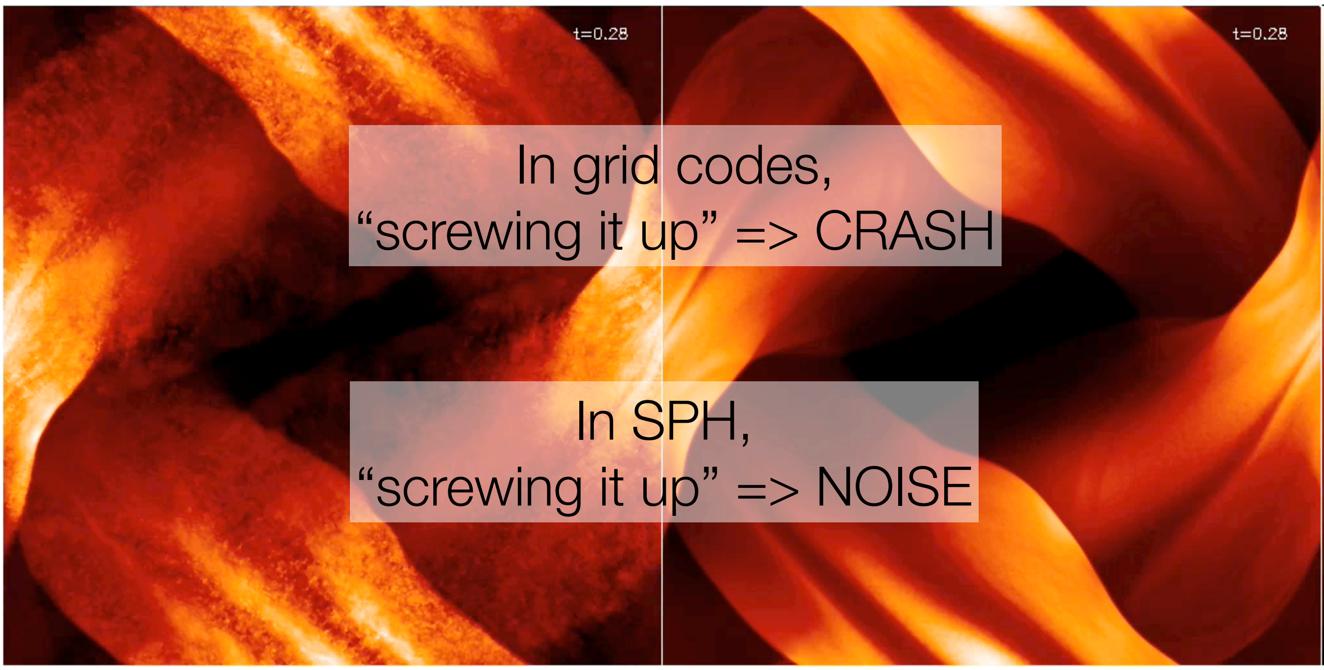


Lodato & Price (2010)

Orbits are orbits... even when they're not aligned with any symmetry axis.

Exact conservation: Disadvantages

- Calculations keep going, even when they're screwed up...



Orszag-Tang Vortex in MHD (c.f. Price & Monaghan 2005, Rosswog & Price 2007, Price 2010)

How to fix this

```
if (particles_are_noisy()) {
   die();
}

if (particles_are_noisy()) then
   stop
endif

if ( particles ^ AnyofP("noise") ):
   die('sorry, your SPH code crashed, we are not AMUSEd')
```

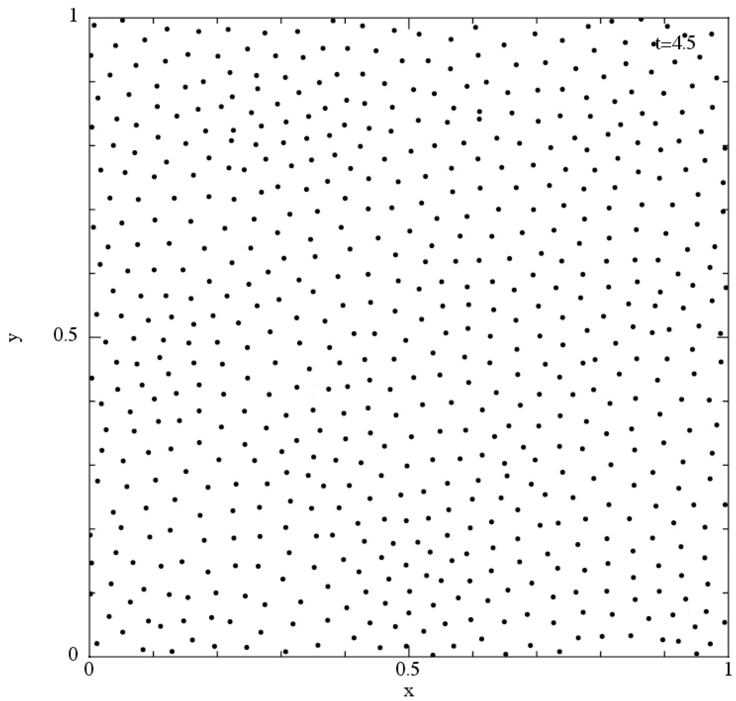
What this gives us: Advantages of SPH

- An exact solution to the continuity equation
- Resolution follows mass
- ZERO dissipation
- Advection done perfectly
- EXACT conservation of mass, momentum, angular momentum, energy and entropy
- A guaranteed minimum energy state

The minimum energy state

The “grid” in SPH...

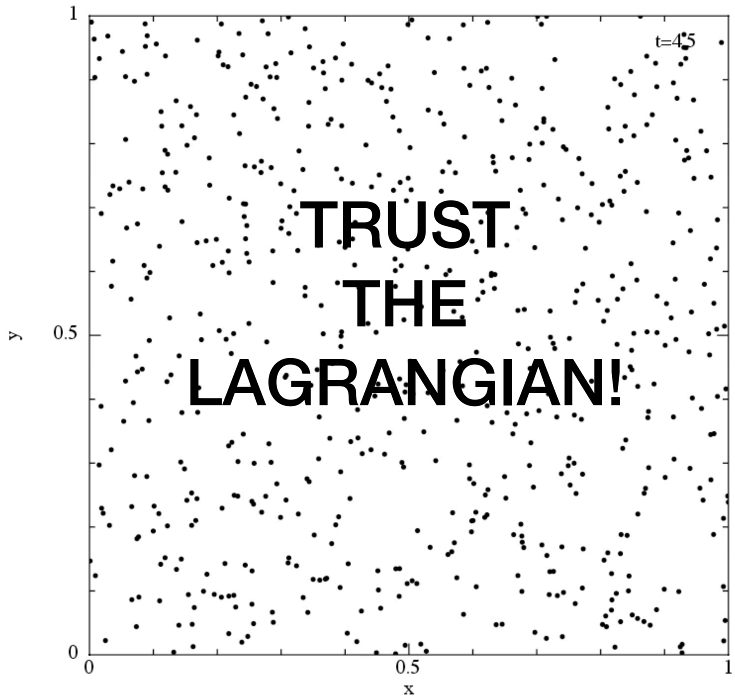
What happens to a random particle arrangement?



$$\frac{\mathrm{d}\mathbf{v}_i}{\mathrm{d}t} = -\sum_j m_j \left(\frac{P_i}{\rho_i^2} + \frac{P_j}{\rho_j^2} \right) \nabla_i W_{ij}$$

SPH particles know how to stay regular

Why “rpSPH” (Morris 1996, Abel 2010) is a bad idea

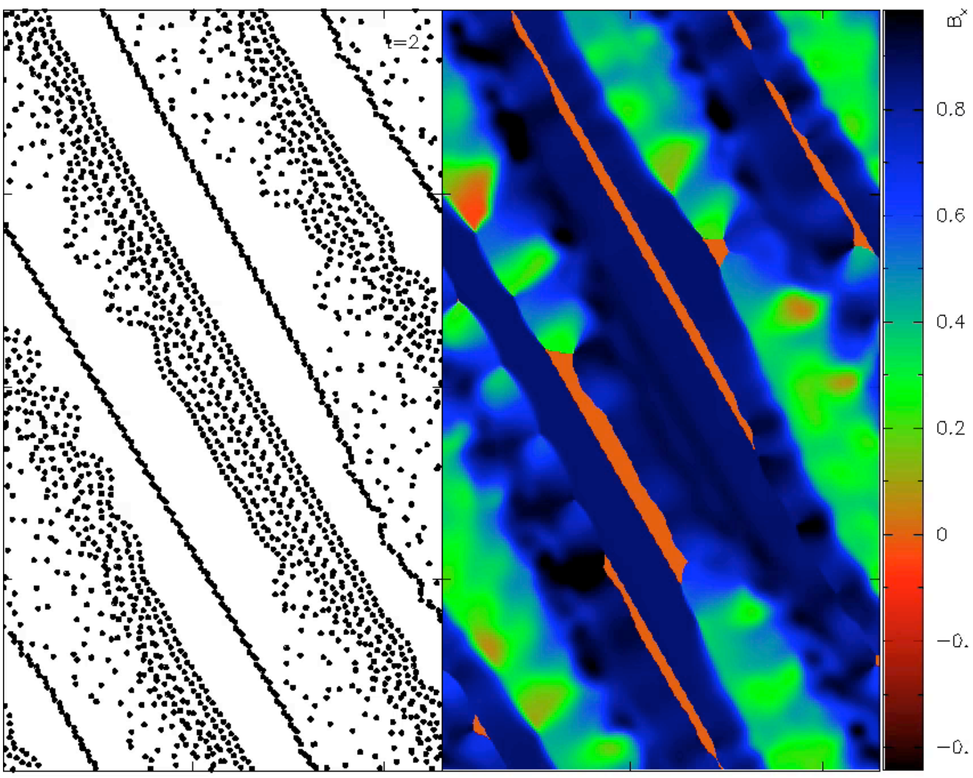


$$\frac{\mathrm{d}\mathbf{v}_i}{\mathrm{d}t} = \sum_j m_j \left(\frac{P_i - P_j}{\rho_j^2} \right) \nabla_i W_{ij}$$

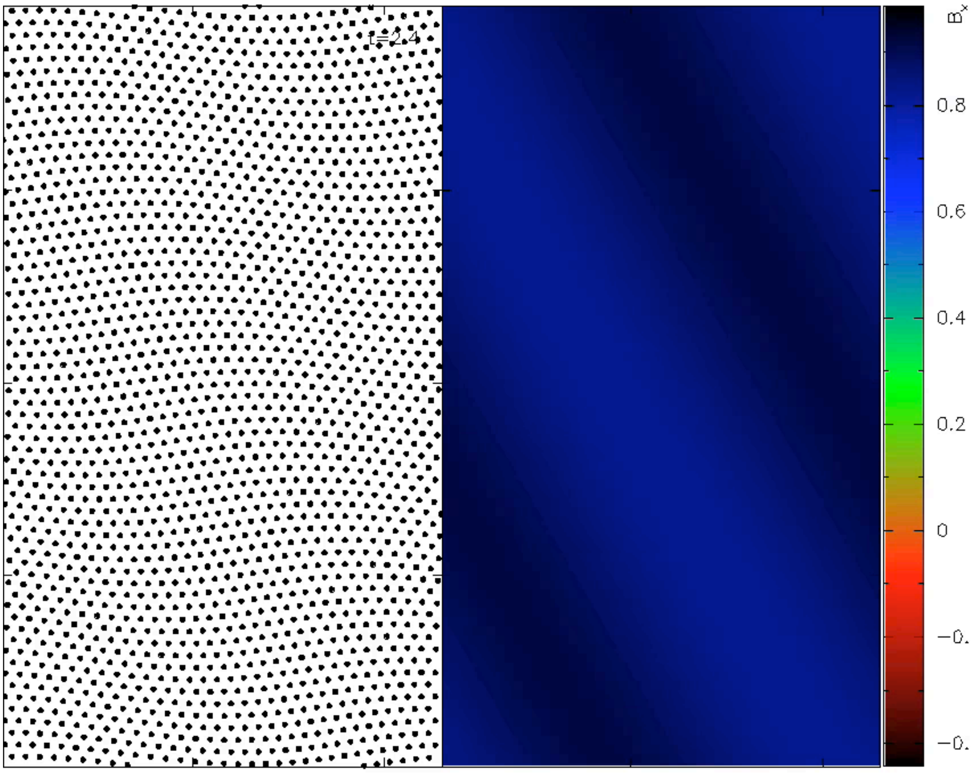
Improving the gradient operator leads to WORSE results

Corollary: Better to use a worse but conservative gradient operator

Corollary: Need positive pressures



Compromise approach gives stability



Subtract

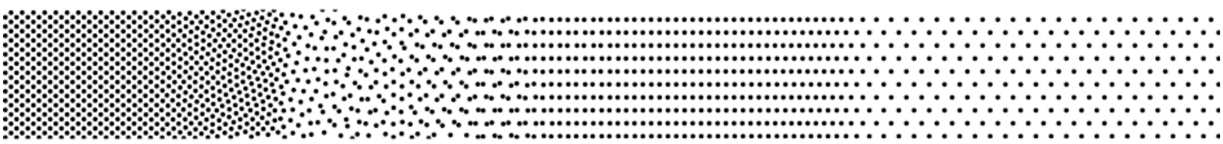
$$-\mathbf{B}(\nabla \cdot \mathbf{B})$$

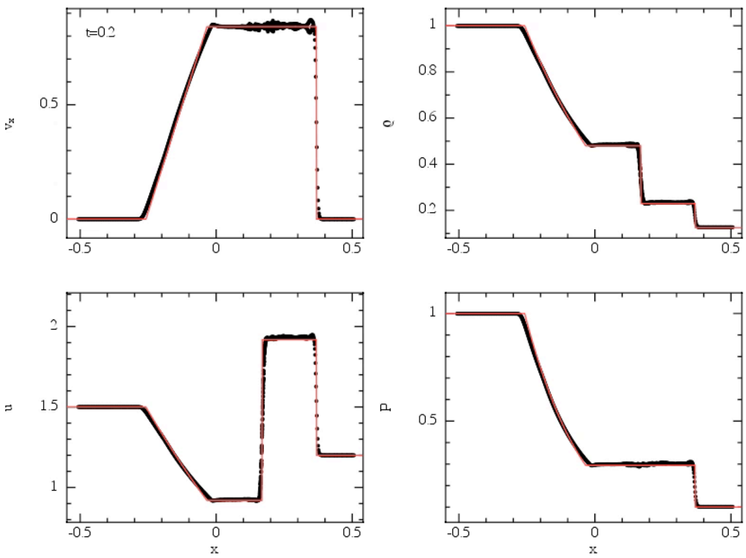
from MHD force:

Stable but nonconservative

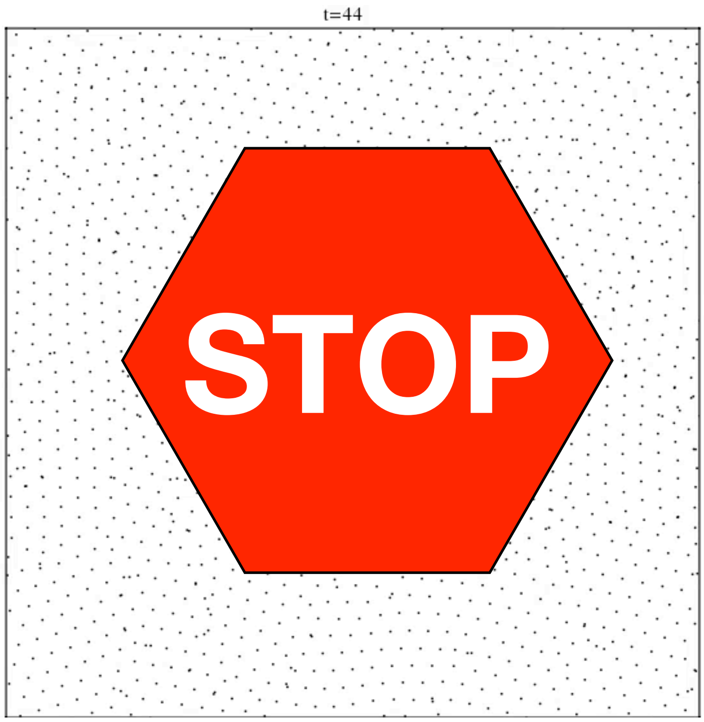
2D shock tube

- intrinsic “remeshing” of particles





Why you cannot use "more neighbours" (or: How to halve your resolution)



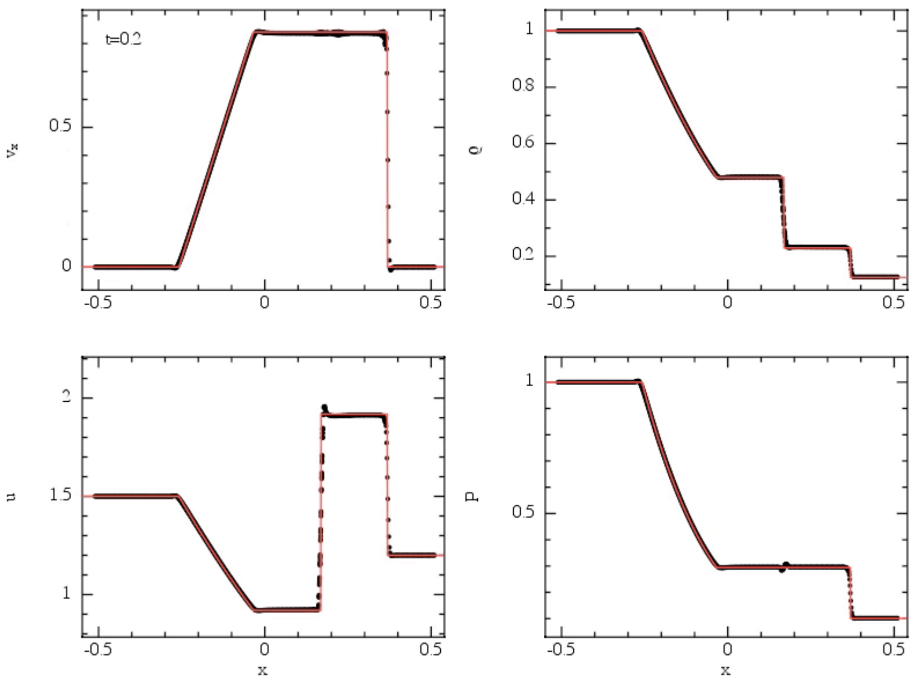
pairing occurs for > 65 neighbours for the cubic spline in 3D

N_{neigh} should NOT be a free parameter!

i.e., should not change the ratio of smoothing length to particle spacing

2D shock tube

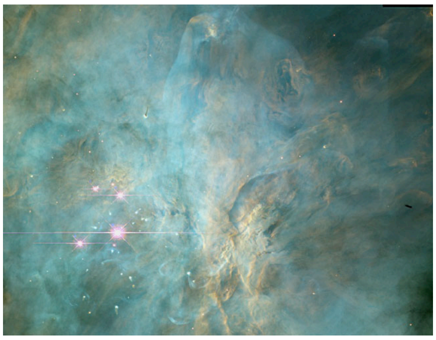
- use smoother quintic kernel - truncated at 3h instead of 2h (NOT the same as "more neighbours" with the cubic spline)

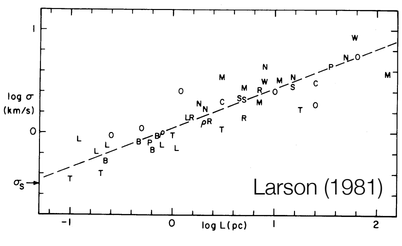


Grid vs. SPH: Turbulence

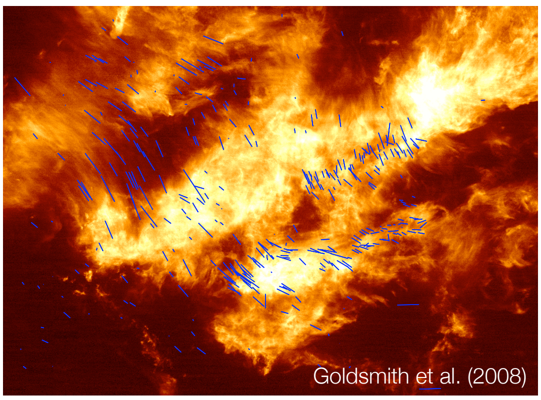
Turbulence in the Interstellar Medium

- highly supersonic, Mach numbers ~ 5-20
- isothermal to good approximation
- unknown driving mechanism, but “large scale”
- super-Alfvenic - magnetic fields mildly important
- statistics of turbulence may determine statistics of star formation (e.g. Padoan & Nordlund 2002, Hennebelle & Chabrier 2008)





Larson (1981)



Goldsmith et al. (2008)

GRID vs. SPH

Padoan et al. (2007), commenting on Ballesteros-Paredes et al. (2006):

THE MASS SPECTRA OF CORES IN TURBULENT MOLECULAR CLOUDS AND IMPLICATIONS FOR THE INITIAL MASS FUNCTION

Javier Ballesteros-Paredes,[1] Adriana Gazol,[1] Jongsoo Kim,[2] Ralf S. Klessen,[3] Anne-Katharina Jappsen,[3] and Epimenio Tejero[1]

Received 2005 January 27; accepted 2005 September 20

"The complete absence of an inertial range with a reasonable slope, or with a reasonable dependence of the slope on the Mach number, makes their SPH simulations totally inadequate for testing the turbulent fragmentation model..."

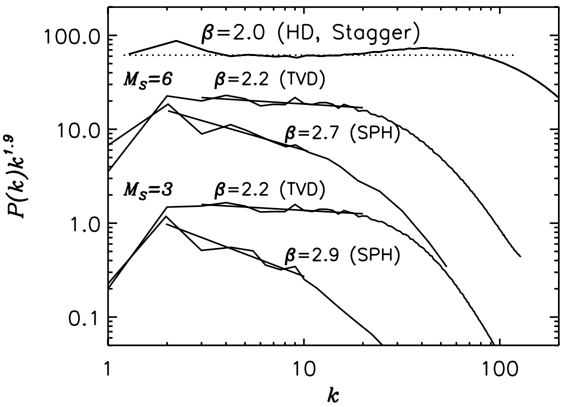
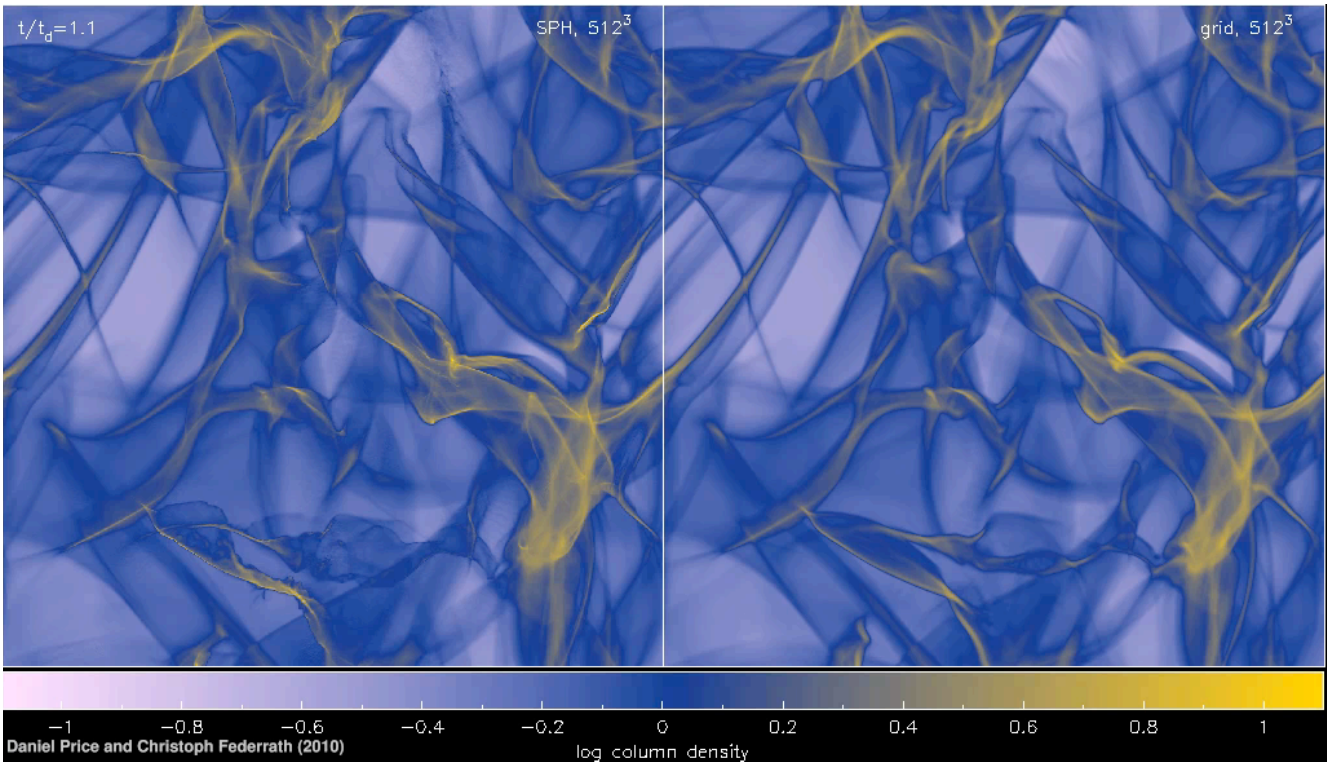


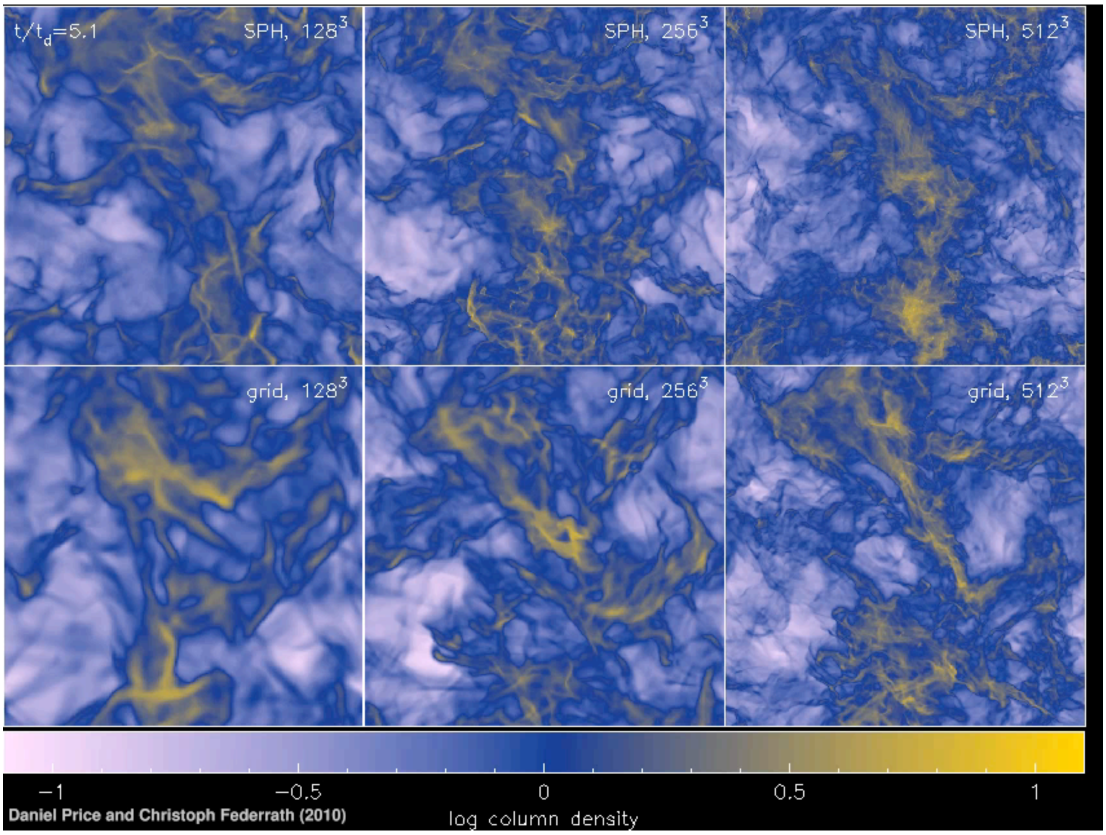
Fig. 8.—Power spectra compensated for the slope of the Stagger code HD run, $\beta = 1.9$. The TVD and SPH power spectra are the same as in Fig. 2 of Ballesteros-Paredes et al. (2006) for the Mach numbers 3 and 6.

...but low resolution SPH (58^3)

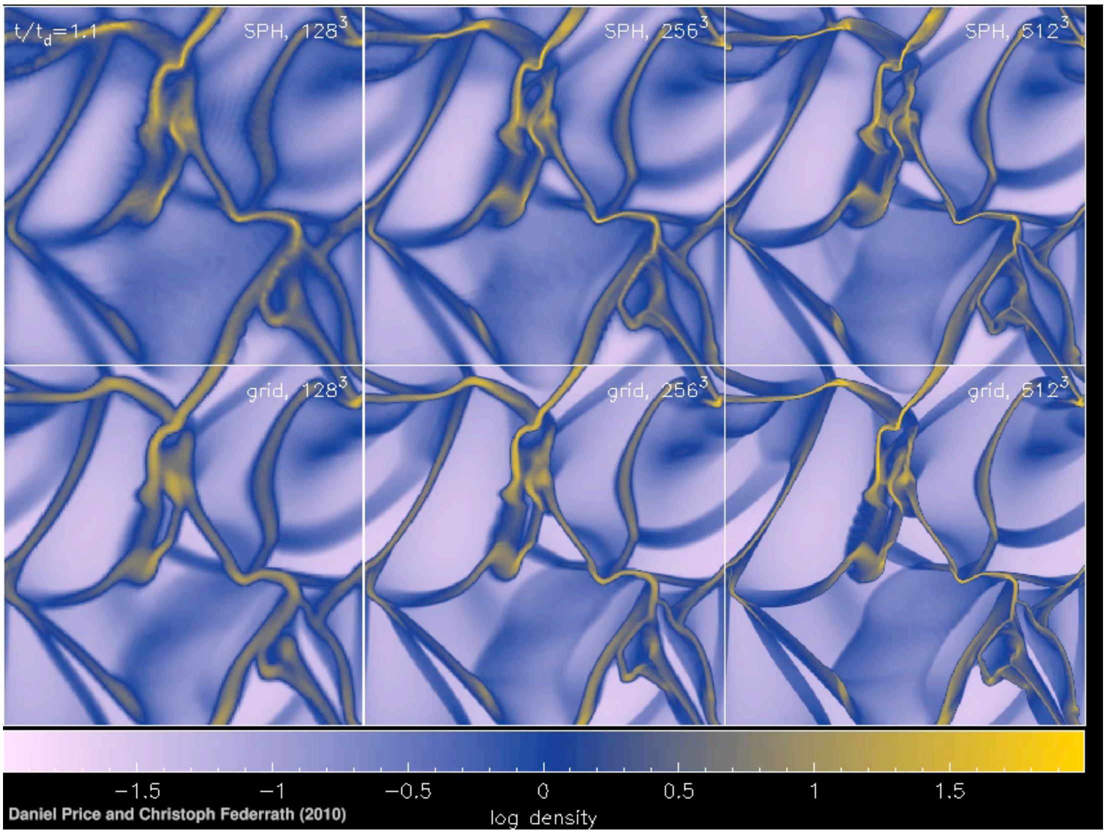
Price & Federrath (2010): Comparison of driven turbulence



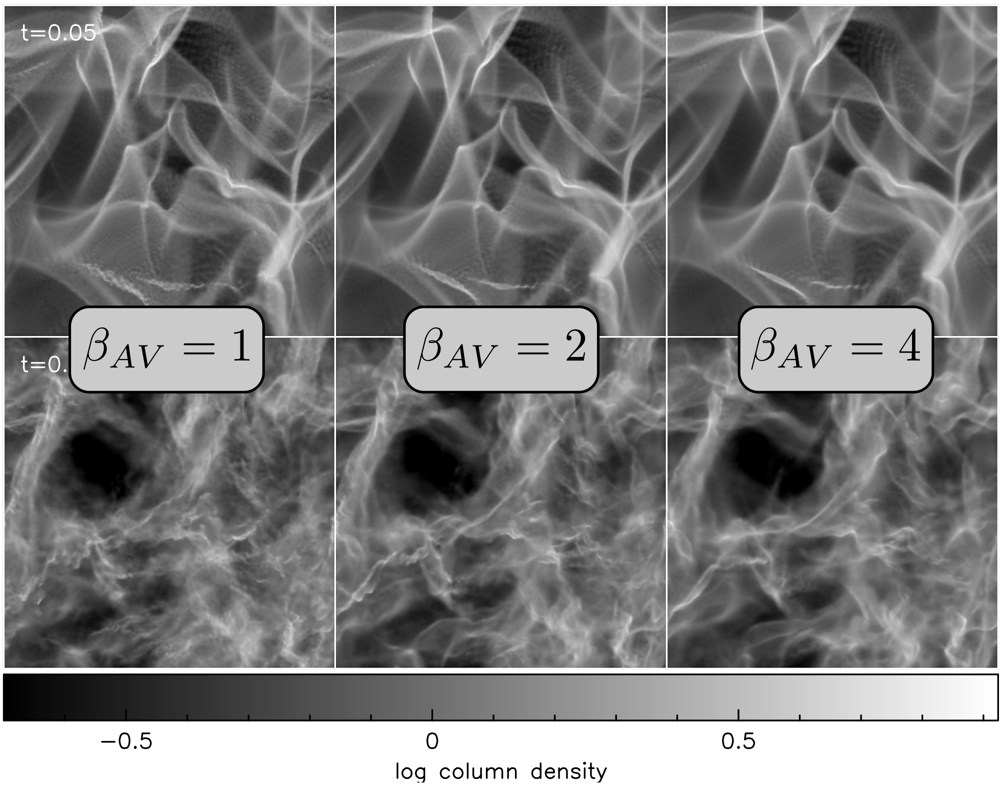
PHANTOM FLASH


t/t_d=5.1
SPH, 128^3
SPH, 256^3
SPH, 512^3
grid, 128^3
grid, 256^3
grid, 512^3
−1
−0.5
0
0.5
1
Daniel Price and Christoph Federrath (2010)
log column density

Slice:


t/t_d=1.1
SPH, 128^3
SPH, 256^3
SPH, 512^3
grid, 128^3
grid, 256^3
grid, 512^3
−1.5
−1
−0.5
0
0.5
1
1.5
Daniel Price and Christoph Federrath (2010)
log density

Particle penetration and high Mach number shocks



Take care with viscosity at high Mach numbers!

TURBULENCE: Theory

- Kolmogorov (1941):

$$\dot{E} = \frac{\eta v_L^3}{L} = \mathrm{const}$$

$$v_L \propto L^{1/3}$$

$$E_{kin} \propto v_L^2 \propto L^{2/3} \propto k^{-2/3}$$

$$E(k) = \frac{dE_{kin}}{dk} \propto k^{-5/3}$$

(for incompressible turbulence)

- Kritsuk et al. (2007):

$$\dot{E} = \frac{\eta \rho v_L^3}{L} = \mathrm{const}$$

$$\rho^{1/3} v_L \propto L^{1/3}$$

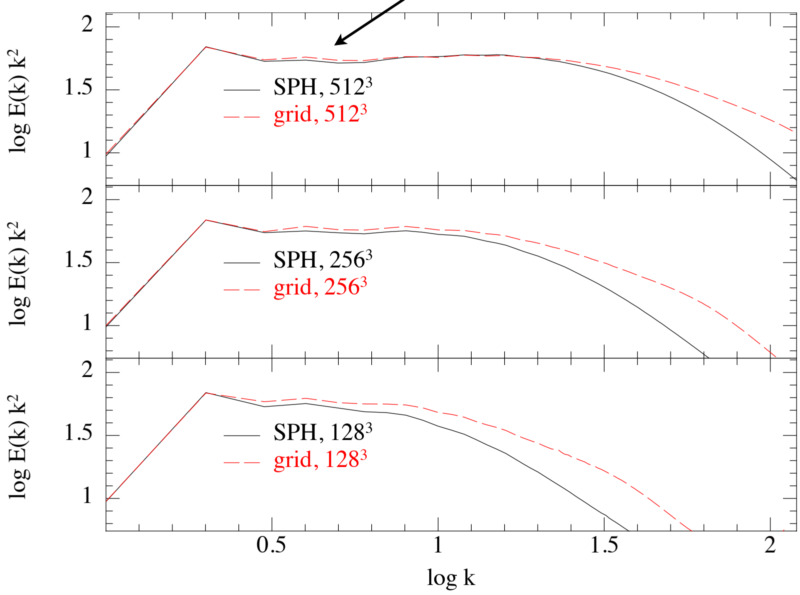
$$(\rho^{1/3} v_L)^2 \propto L^{2/3} \propto k^{-2/3}$$

$$\mathcal{E}(k) = \frac{d(\rho^{1/3} v_L)^2}{dk} \propto k^{-5/3}$$

(for compressible and supersonic turbulence)

Kinetic energy spectra (time averaged)

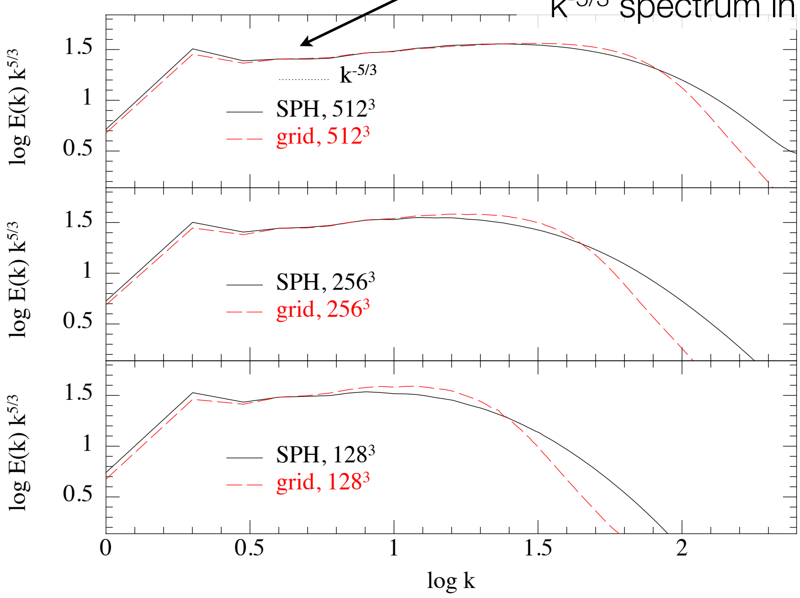
Burgers-like k^{-2} spectrum in the kinetic energy for Mach 10 hydro



Price & Federrath (2010)

Density-weighted energy spectra $(\rho^{1/3}\mathbf{v})$

Confirms Kritsuk et al. (2007) suggestion of Kolmogorov-like $k^{-5/3}$ spectrum in this variable

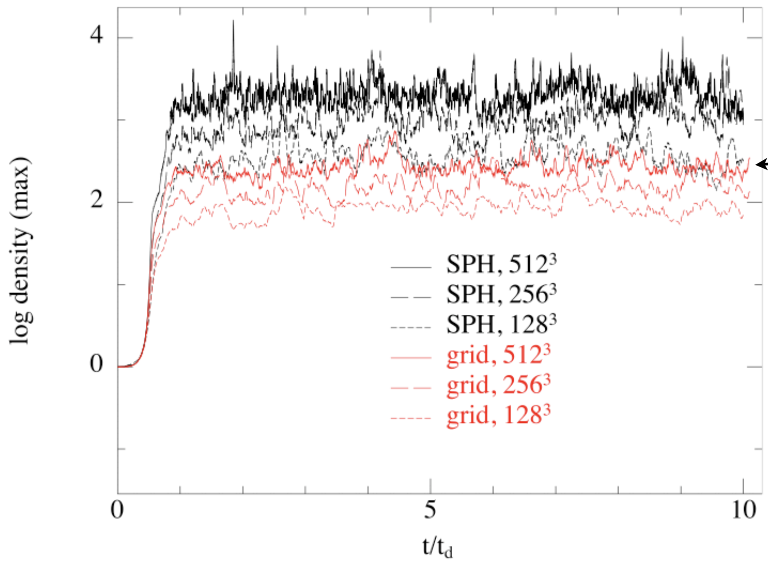


Price & Federrath (2010)

Summary:

You get what you pay for
(i.e., need high resolution in any method)

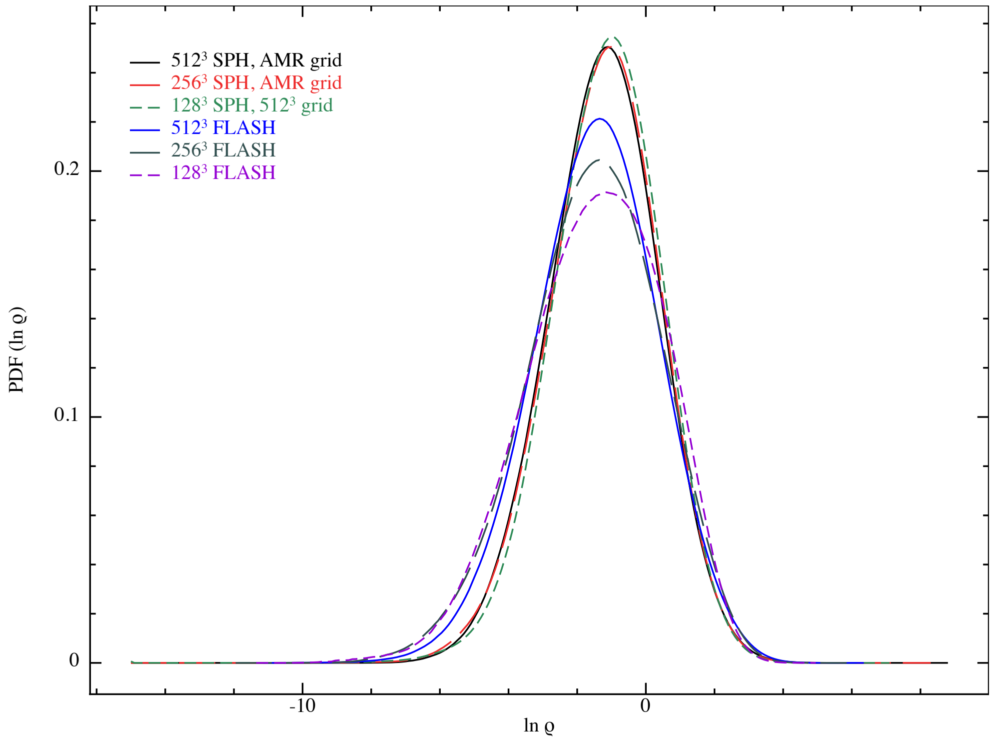
But SPH resolution is in density field



max density in SPH at 128^3 similar to max grid density at 512^3

Price & Federrath (2010)

Density PDFs:



Summary: Advantages and disadvantages of SPH

Advantages:

- Resolution follows mass
- Zero dissipation until explicitly added
- Exact and simultaneous conservation of all physical quantities is possible
- Intrinsic remeshing procedure
- Does not crash

Disadvantages:

- Resolution follows mass
- Dissipation terms must be explicitly added to treat discontinuities
  - methods can be crude (need a good switch)
- Exact conservation no guarantee of accuracy
- Need to be careful with effects from particle remeshing
- Screw-ups indicated by noise rather than code crash

But remember: You get what you pay for!

NDSPMHD code and test problems available from
http://users.monash.edu.au/~dprice/ndspmhd/

SPLASH visualisation tool available from:
http://users.monash.edu.au/~dprice/splash/