SPH +/- MHD

The state of the art in the dark arts

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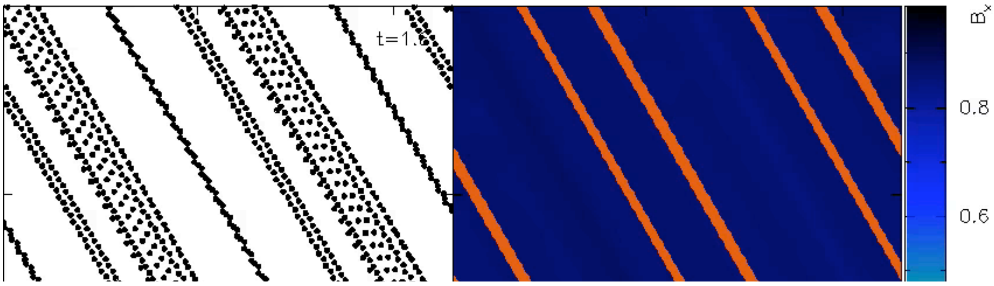
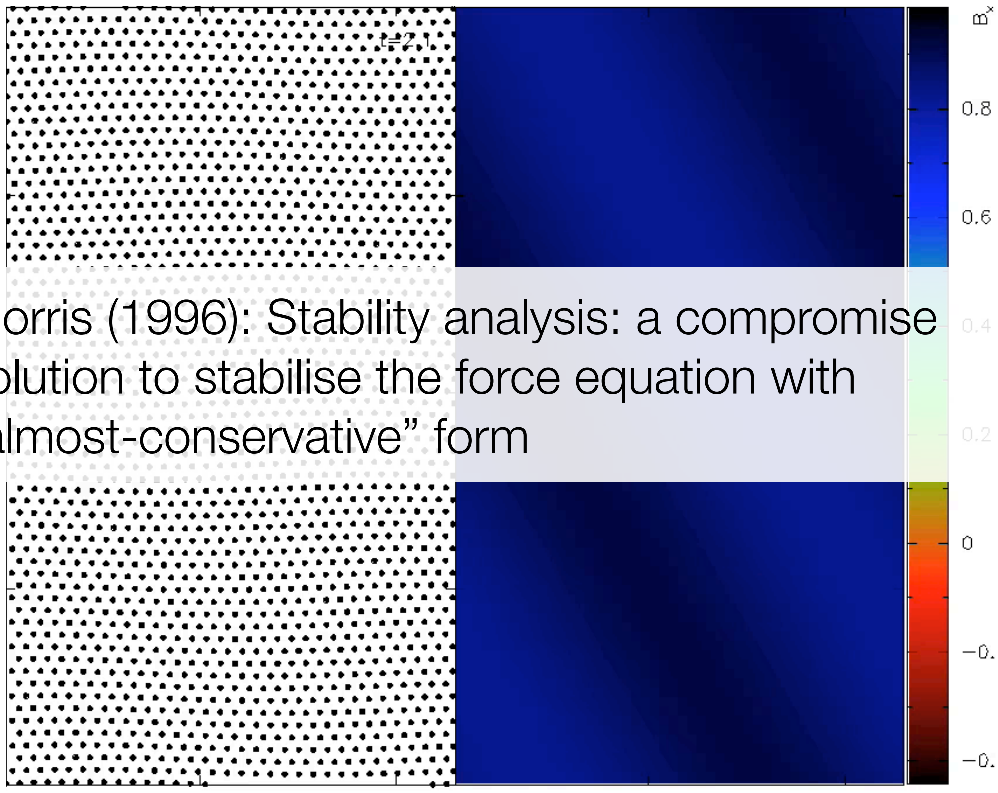
Gingold & Monaghan (1977):
Magnetic polytropes with SPH



Phillips & Monaghan (1985): SPH with MHD in conservative form is unstable when beta < 1



Morris (1996): Stability analysis: a compromise solution to stabilise the force equation with “almost-conservative” form

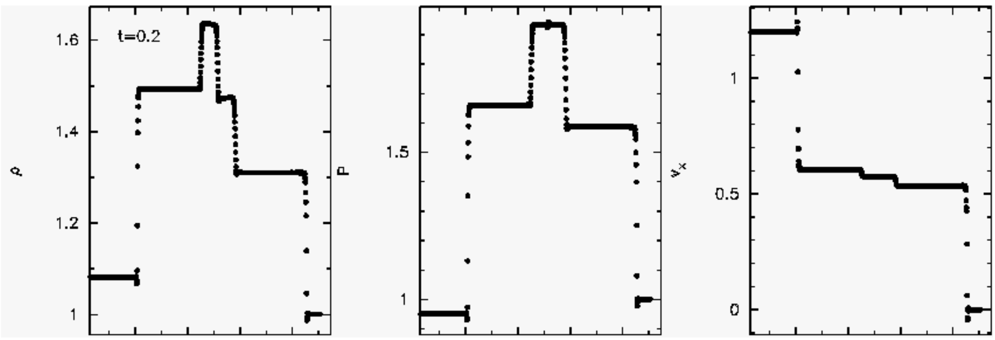
The “Middle Years”

Dolag, Bartelmann & Lesch (1999): SPH+MHD applied to galaxy clusters (beta >> 1)

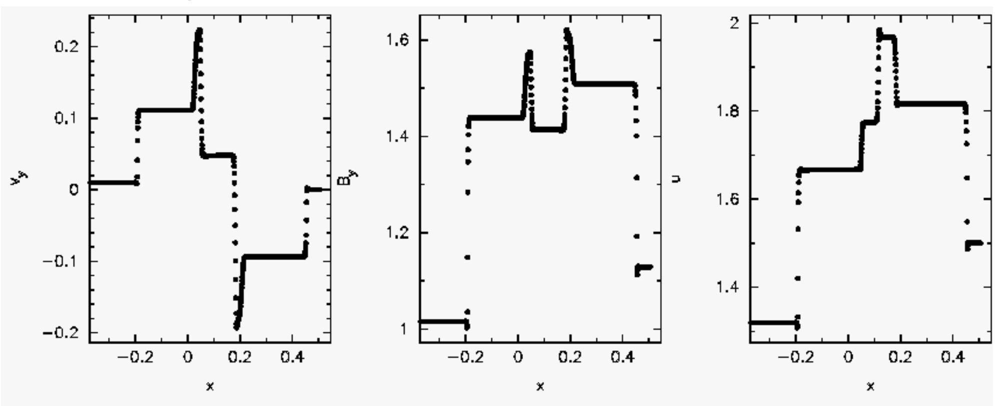
Børve, Omang & Tru ... sed SPH: very nice MHD shoc ... particles

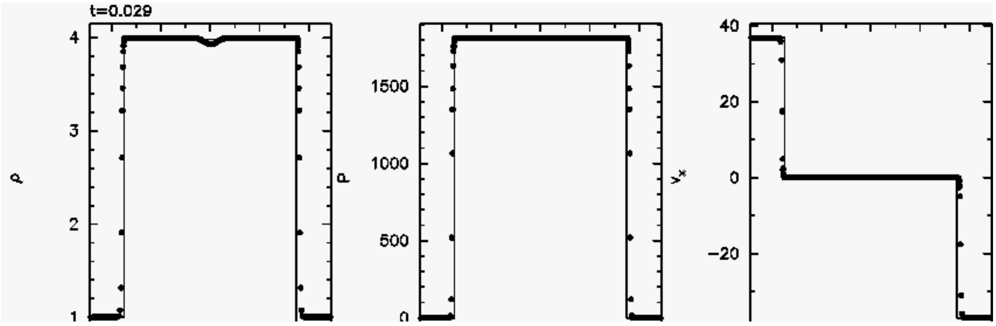
Screw conservative ... le Magnetohydrodyna ... es 2003)

Hosking & Whitwort ... nservative formulation but first ... d (ion/ neutral)

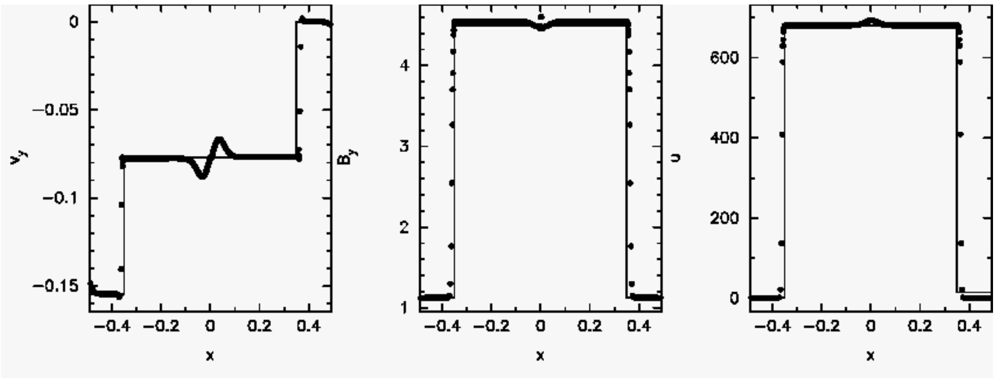


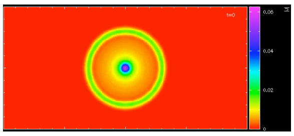
Price & Monaghan 2004a (paper I): dissipative terms for MHD shocks





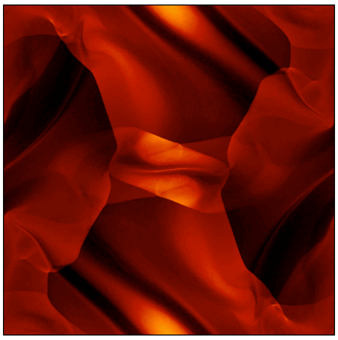
Price & Monaghan 2004b (paper II): we could do strong shocks (variable smoothing length formulation)



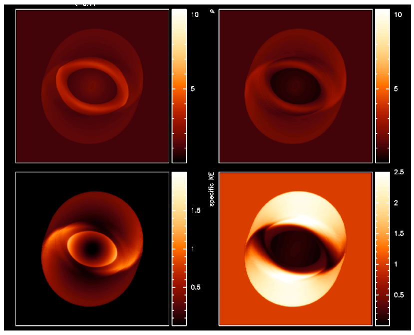


advection of a current loop (Gardiner & Stone 2006, Rosswog & Price 2007)

good results on test problems...



Orszag-Tang vortex problem (PM05, Rosswog & Price 2007)

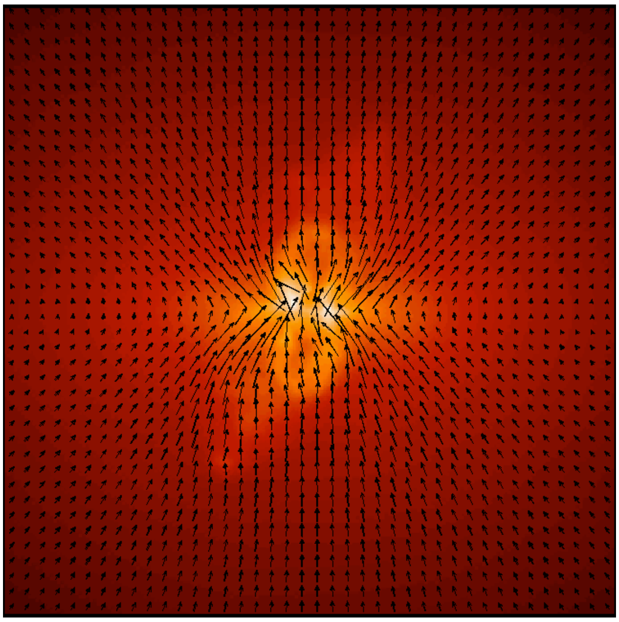


Magnetic rotor problem (PM05)

Price & Monaghan 2005 (paper III): How to handle the divergence constraint (using IGNORE or CLEAN approach)

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) \qquad \nabla \cdot \mathbf{B} = 0$$

...but didn't work so well for star formation



$$\mathbf{B} = \nabla\alpha \times \nabla\beta$$

(satisfies $\nabla \cdot \mathbf{B} = 0$ by construction: the PREVENT approach)

Price & Bate (2007), Rosswog & Price (2007): Enter the Euler potentials

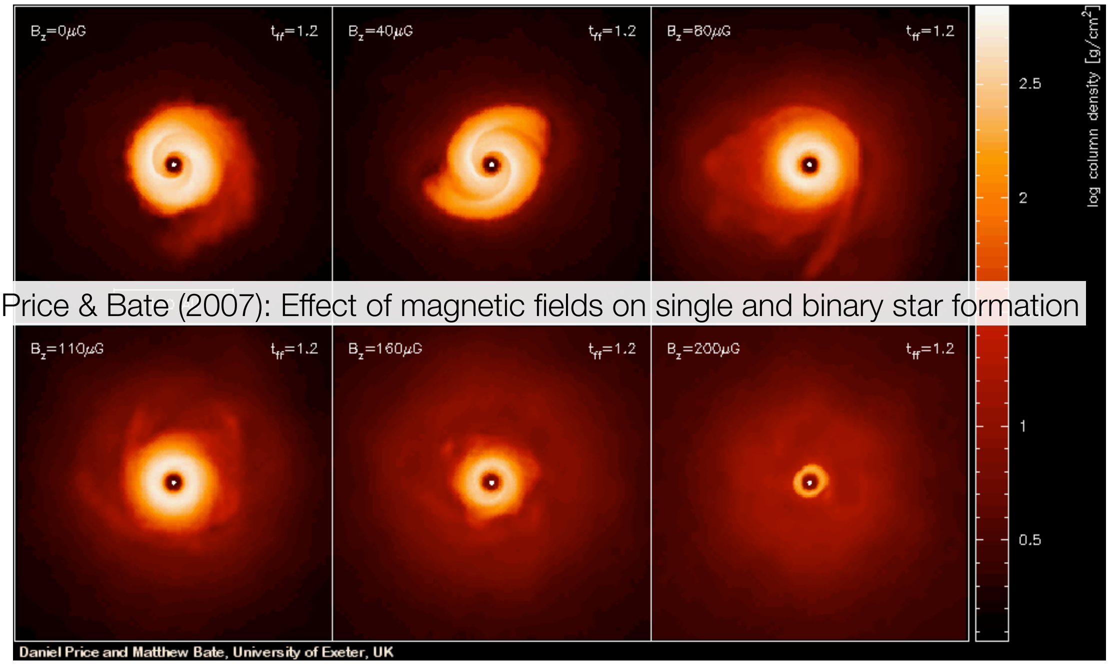
(cf. Phillips & Monaghan 1985)

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) \quad \Rightarrow \quad \frac{d\alpha}{dt} = 0, \quad \frac{d\beta}{dt} = 0$$

(advection of magnetic field lines by Lagrangian particles)

(all with supercritical field strengths)

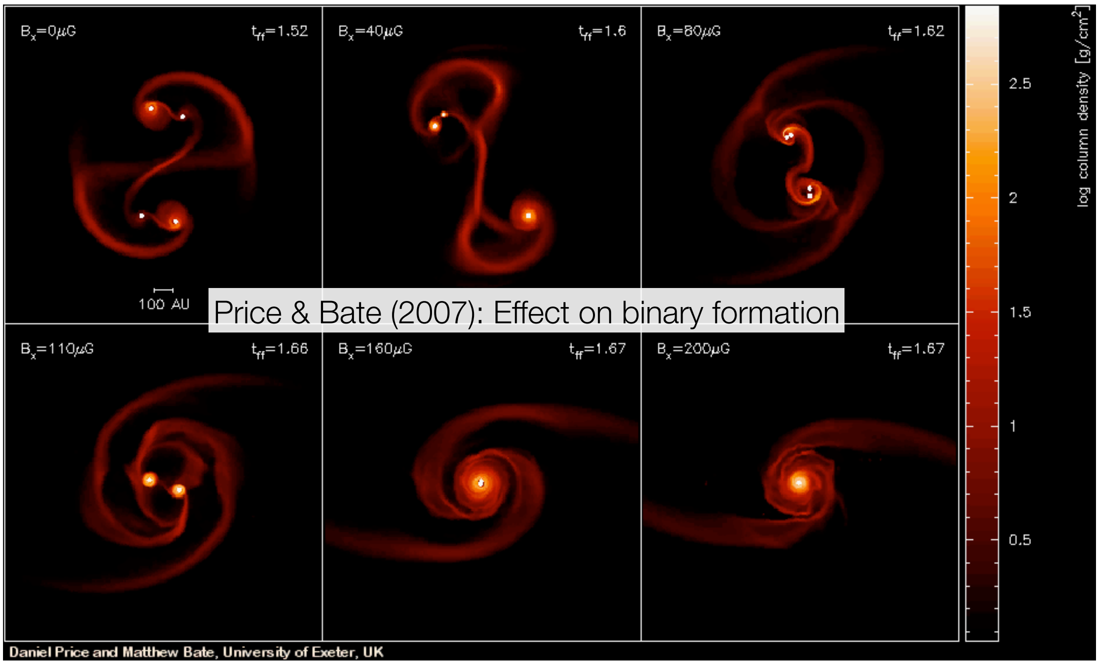
Price & Bate (2007): Effect of magnetic fields on single and binary star formation



...problem forming discs in the presence of magnetic fields?

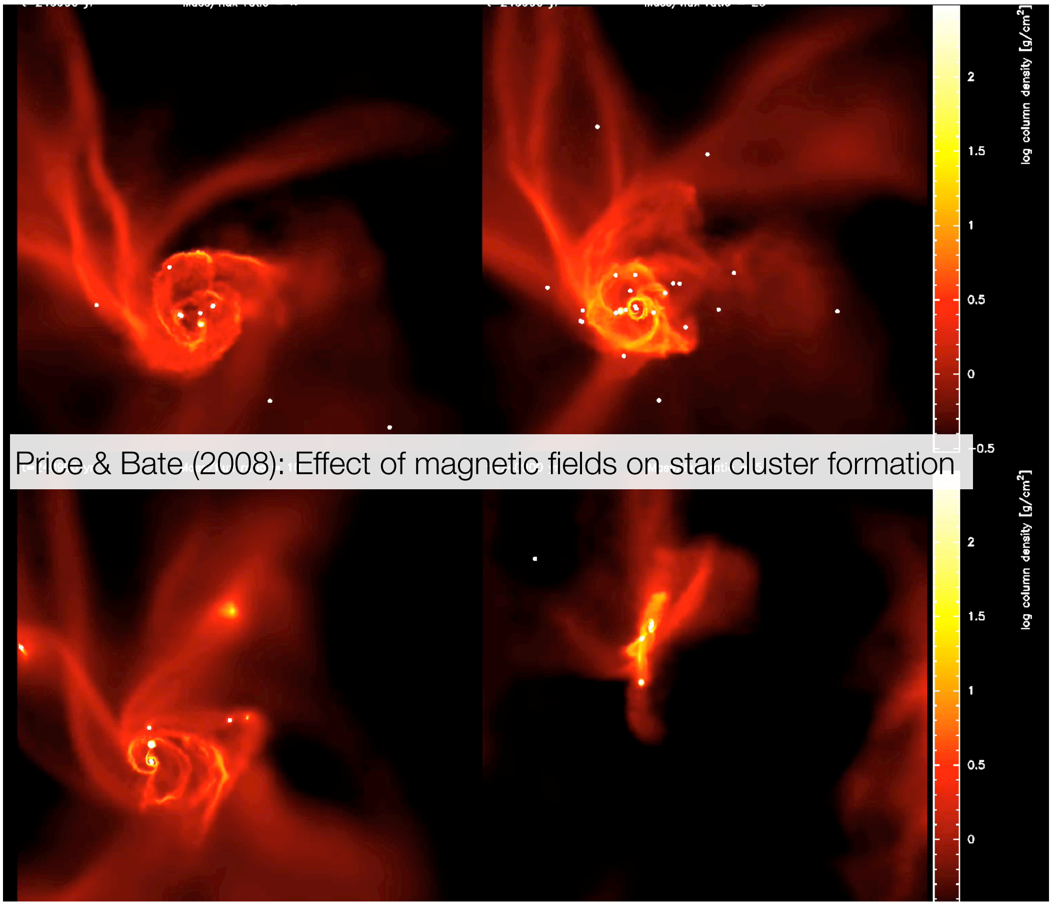
see also Hennebelle & Fromang (2008), Hennebelle & Ciardi (2009), Mellon & Li (2009), Duffin & Pudritz (2009)

Price & Bate (2007): Effect on binary formation

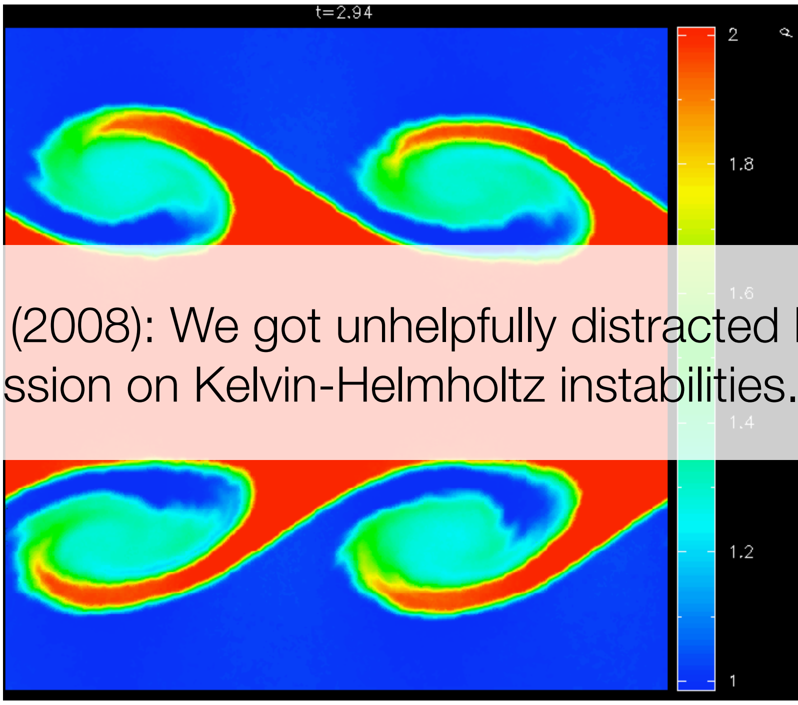


... a fragmentation crisis?

cf. also Hennebelle & Teyssier (2009), Mellon & Li (2008), Machida et al. (2008) and others

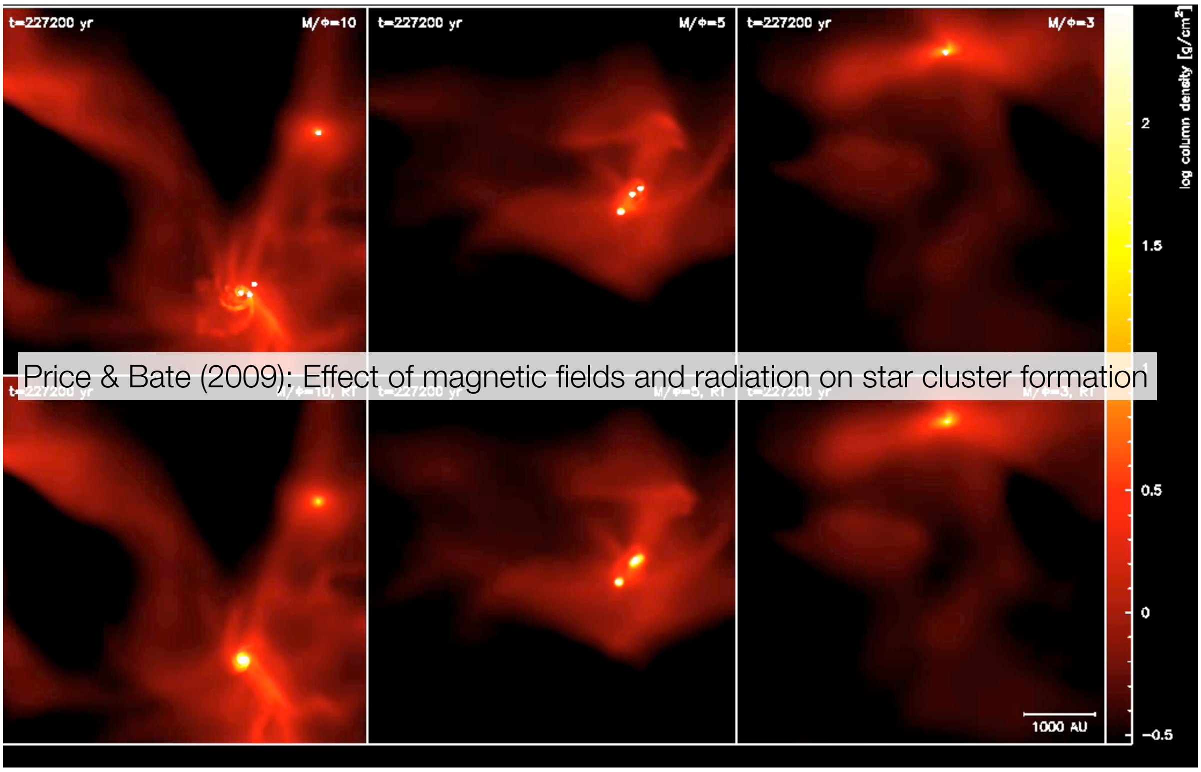


Price & Bate (2008): Effect of magnetic fields on star cluster formation



Price (2008): We got unhelpfully distracted by a discussion on Kelvin-Helmholtz instabilities...

Price & Bate (2009): Effect of magnetic fields and radiation on star cluster formation



net effect is a very much reduced star formation rate / efficiency per t_ff

Dolag & Stasyszyn (2009):
SPH+MHD makes it’s way into GADGET

application to galaxy clusters, magnetic field evolution and dynamics in spiral galaxies (Kotarba et al. 2009, Stasyszyn et al. 2010)

Limitations of the Euler potentials approach

(Rosswog & Price 2007, Price & Bate 2008, Brandenburg 2010)

$$\mathbf{B} = \nabla\alpha \times \nabla\beta$$

$$\frac{d\alpha}{dt} = 0$$

$$\frac{d\beta}{dt} = 0$$

- advection of magnetic fields: no change in topology (A.B = 0)
- does not follow wind-up of magnetic fields
- difficult to model resistive effects -- reconnection processes not treated correctly

Axel Brandenburg (at KITP 2007): “Why don’t you just use the vector potential?”

is $\mathbf{B} = \nabla \times \mathbf{A}$ a better approach?

$$\frac{\partial \mathbf{A}}{\partial t} = \mathbf{v} \times \mathbf{B} + \nabla\phi \quad \Rightarrow \quad \frac{d\mathbf{A}}{dt} = -A_i \nabla v^i$$

2.4.4 Equations of motion

Putting the perturbations (31) and (33) [the second term of which has been expanded into (38) and (39)] into (13) we have

$$\int \left\{ -m_a \frac{dv_a^i}{dt} - \sum_b \frac{m_b}{\Omega_b}\left[\frac{P_b}{\rho_b^2} - \frac{3}{2\mu_0}\left(\frac{B_b}{\rho_b}\right)^2 + \frac{\xi_b}{\rho_b^2}\right]\sum_c m_c \frac{\partial W_{bc}(h_b)}{\partial x_b^i}(\delta_{ba}-\delta_{ca}) - \frac{1}{\mu_0}\sum_b \frac{m_b}{\Omega_b}\frac{B_b^j}{\rho_b^2}\epsilon_{jkl}\sum_c m_c (A_k^b - A_k^c)\frac{\partial^2 W_{bc}(h_b)}{\partial x_b^i \partial x_b^l}(\delta_{ba}-\delta_{ca}) \right.$$

Price 2010 (paper IV): 25 pages* of pain later, we had derived the ultimate vector potential formulation in SPH...

...it was beautiful, derived elegantly from a Lagrangian variational principle, the method was exactly conservative, novel, the divergence was constrained...

action is satisfied by the equations of motion in the form

$$\frac{dv_a^i}{dt} = -\sum_b m_b \left[\frac{P_a - \frac{3}{2\mu_0}B_a^2 + \xi_a}{\rho_a^2 \Omega_a}\frac{\partial W_{ab}(h_a)}{\partial x_a^i} + \frac{P_b - \frac{3}{2\mu_0}B_b^2 + \xi_b}{\rho_b^2 \Omega_b}\frac{\partial W_{ab}(h_b)}{\partial x_a^i}\right] \quad \text{isotropic term}$$

$$- \frac{1}{\mu_0}\sum_b m_b \left[\frac{B_a^j}{\Omega_a \rho_a^2}\epsilon_{jkl}(A_k^a - A_k^b)\frac{\partial^2 W_{ab}(h_a)}{\partial x_a^i \partial x_a^l} + \frac{B_b^j}{\Omega_b \rho_b^2}\epsilon_{jkl}(A_k^a - A_k^b)\frac{\partial^2 W_{ab}(h_b)}{\partial x_a^i \partial x_a^l}\right] \quad \text{2D term}$$

$$- \frac{1}{\mu_0}\sum_b m_b \left[\frac{B_a^j B_{int,a}^j}{\Omega_a \rho_a}\frac{\partial h_a}{\partial \rho_a}\frac{\partial^2 W_{ab}(h_a)}{\partial x_a^i \partial h_a} + \frac{B_b^j B_{int,b}^j}{\Omega_b \rho_b}\frac{\partial h_b}{\partial \rho_b}\frac{\partial^2 W_{ab}(h_b)}{\partial x_a^i \partial h_b}\right] \quad \text{2D } \nabla h \text{ term}$$

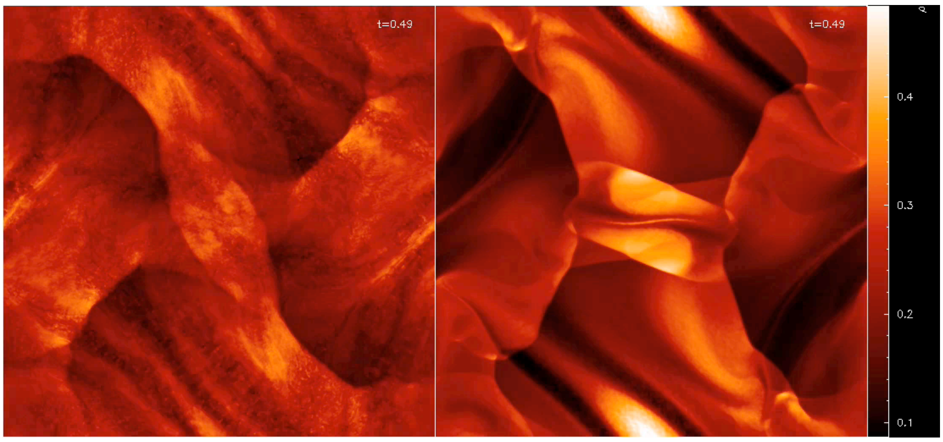
*in the published paper: ~80 in my notebook

...could this be the ultimate method for MHD in SPH?

$$- \sum_b m_b \left[\frac{A_a^i}{\Omega_a \rho_a^2}J_a^k \frac{\partial W_{ab}(h_a)}{\partial x_a^k} + \frac{A_b^i}{\Omega_b \rho_b^2}J_b^k \frac{\partial W_{ab}(h_b)}{\partial x_a^k}\right], \quad \text{3D term} \qquad (42)$$

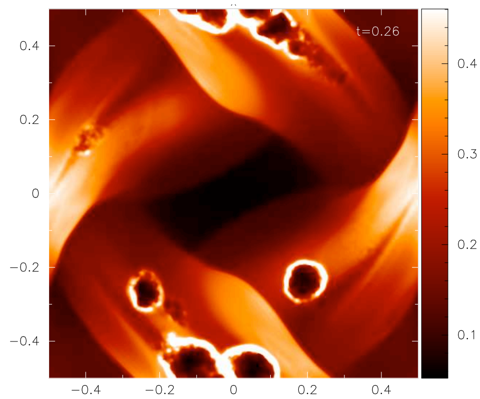
where the current J^k is defined according to

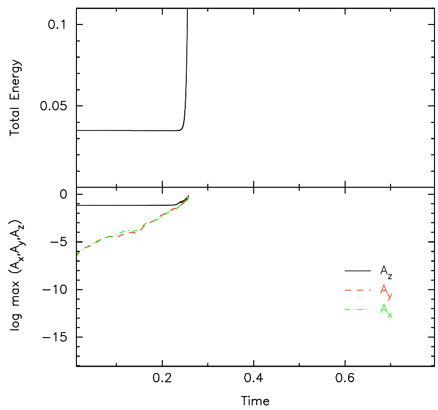
1st problem: same old numerical instability (in 2D and 3D)

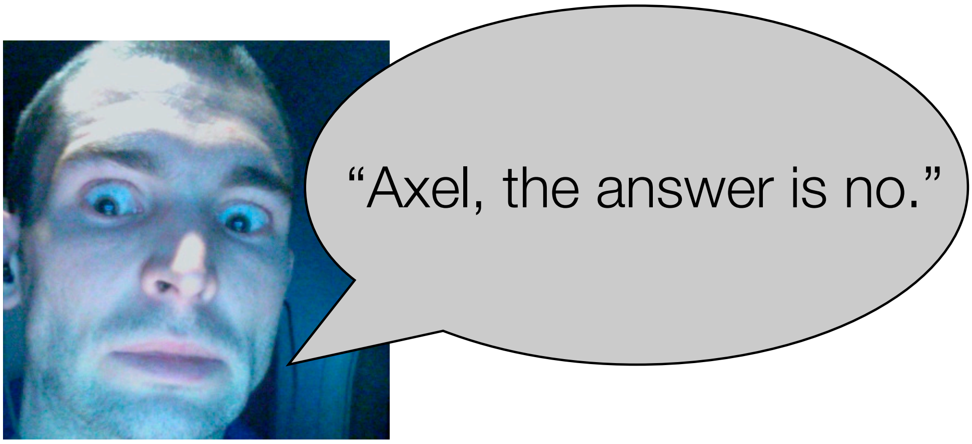


...and did it work?

2nd problem: unconstrained growth of non-physical components of **A** in 3D problems







(for an interesting reason)

...namely that the magnetic Galilean limit of Maxwell’s equations requires enforcement of div A = 0, similar to the original constraint on the B field.

(Price 2010, note submitted to J. Comp. Phys.)

Current directions on SPH+MHD

- generalised Euler potentials method

$$\mathbf{B} = \nabla\alpha_1 \times \nabla\beta_1 + \nabla\alpha_2 \times \nabla\beta_2 + \nabla\alpha_3 \times \nabla\beta_3$$
$$\mathbf{B} = \nabla\alpha_1 \times \nabla X_0 + \nabla\alpha_2 \times \nabla Y_0 + \nabla\alpha_3 \times \nabla Z_0$$

Allows remapping procedure (reconnection “by hand”):

$$\alpha_1^* = \alpha_1 \nabla\beta_1 \qquad \alpha_2^* = \alpha_2 \nabla\beta_2$$
$$[\beta_1^*, \beta_2^*, \beta_3^*] = [X_i, Y_i, Z_i]$$

- exact implementation of projection method for div B

$$\mathbf{B}^* = \mathbf{B} - \nabla\phi$$
$$\nabla^2\phi = \nabla\cdot\mathbf{B}$$

was tried by PM05, but with only approximate solution. With exact method might be better

- two fluid implementation (ions/neutrals)

completely independent of the ideal MHD implementation

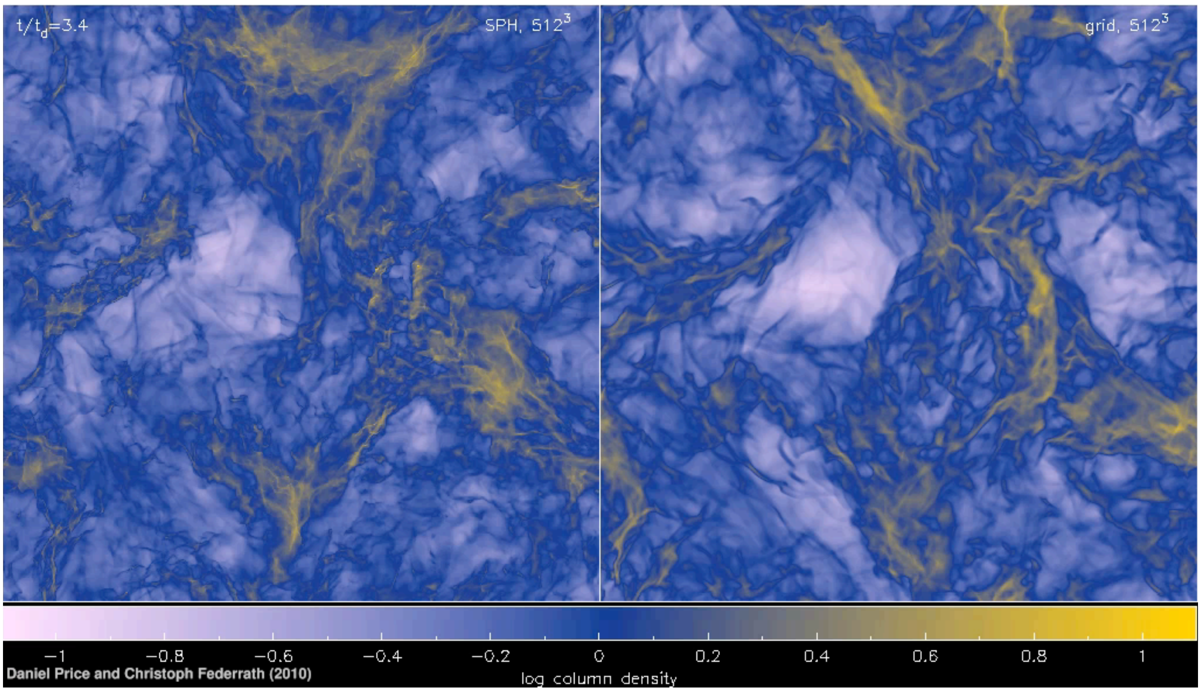
why haven’t we finished all this yet?

Padoan et al. (2007):

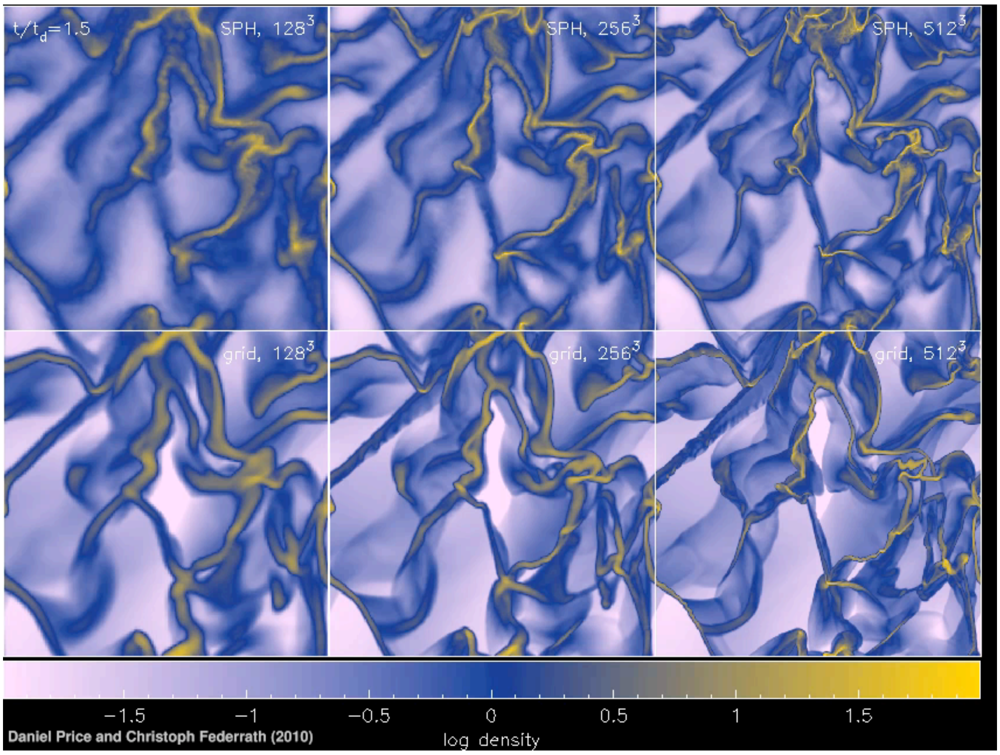
"Numerical simulations can ... account for ... turbulence in ... star formation only if they can generate an inertial range of turbulence, which requires both low numerical diffusivity and large numerical resolution. Furthermore... the magnetic field cannot be neglected"

"SPH simulations of large scale star formation to date fail in all three fronts: numerical diffusivity, numerical resolution, and presence of magnetic fields. This should cast serious doubts on the value of comparing predictions based on SPH simulations with observational data (see also **Agertz et al. 2006**)."

Comparison of Mach 10, hydro turbulence

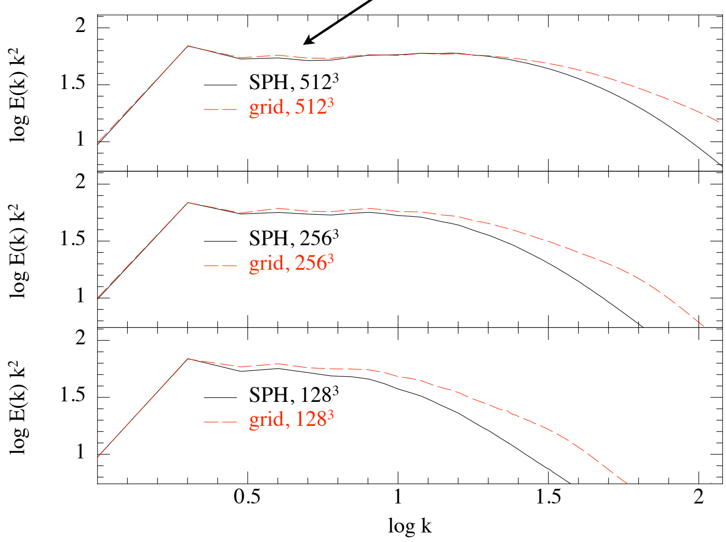


SPH=PHANTOM grid=FLASH



Kinetic energy spectra

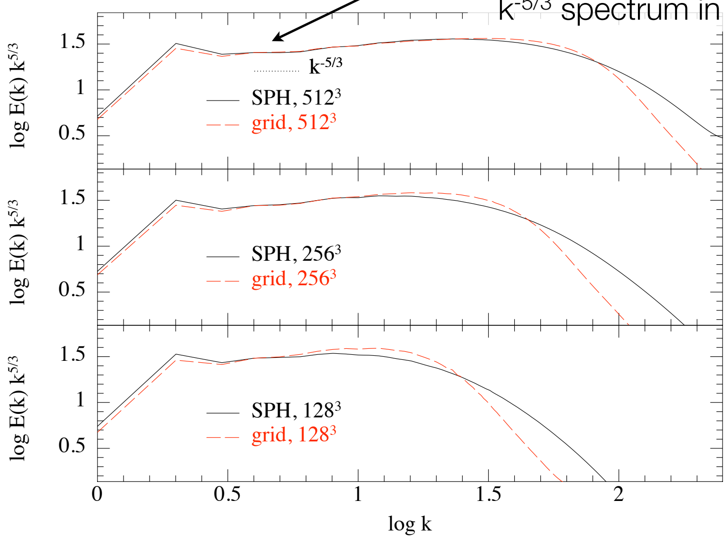
Burgers-like k^{-2} spectrum in the kinetic energy for Mach 10 hydro



Price & Federrath (2010)

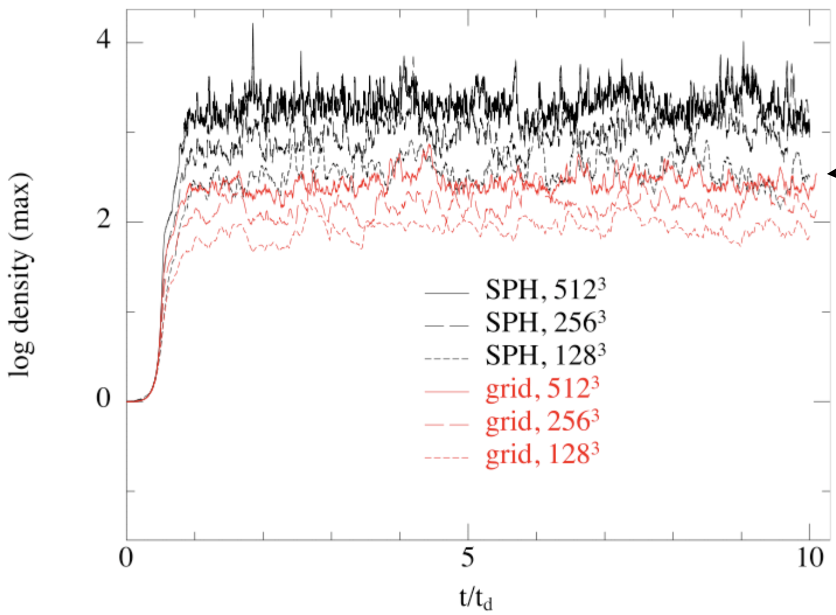
Density-weighted energy spectra $(\rho^{1/3}\mathbf{v})$

Confirms Kritsuk et al. (2007) suggestion of Kolmogorov-like $k^{-5/3}$ spectrum in this variable



Price & Federrath (2010)

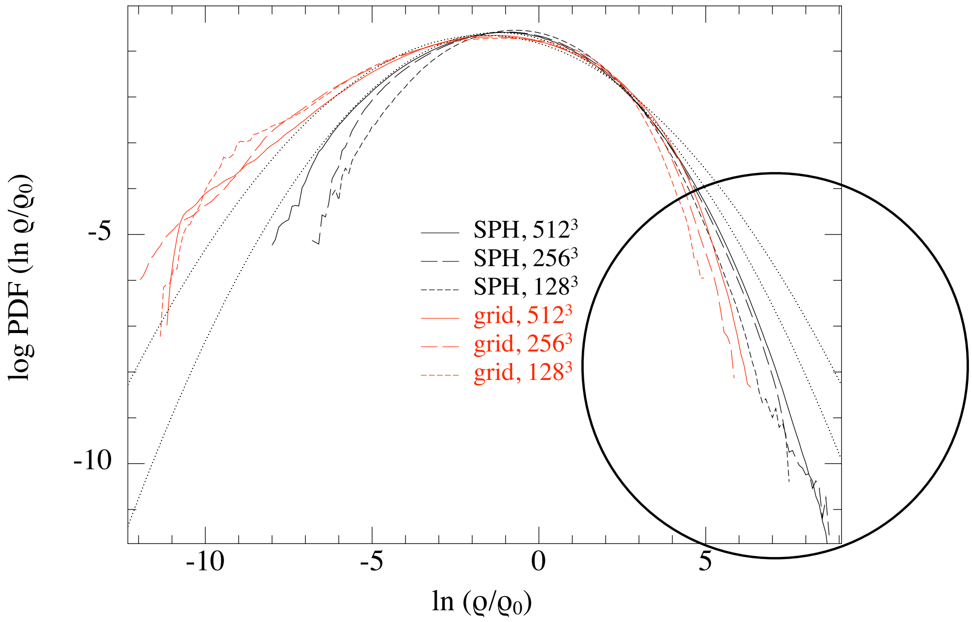
Density resolution



max density in SPH at 128^3 similar to max grid density at 512^3

Price & Federrath (2010)

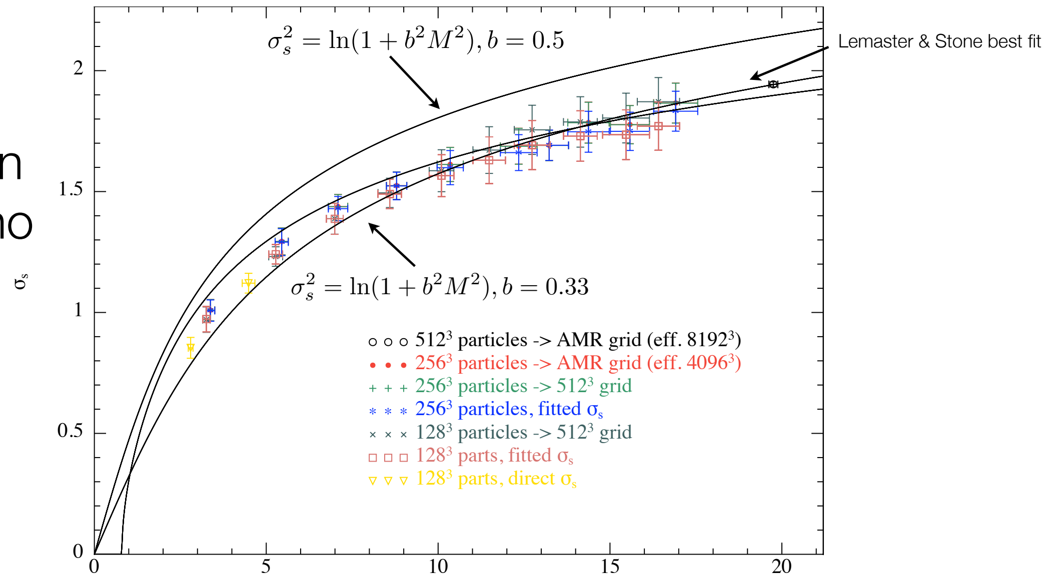
Probability Distribution Functions



Price & Federrath (2010)

Density variance -- Mach number relation

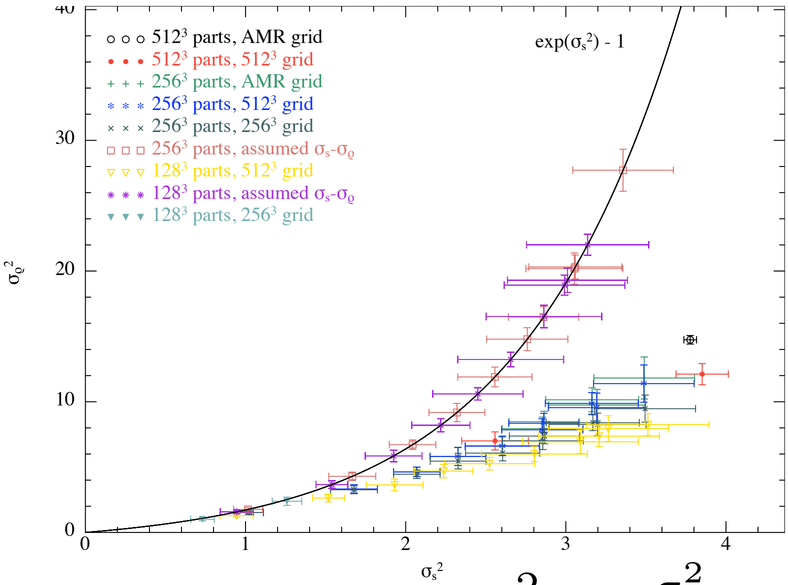
Std. dev in log rho



Mach number

Price, Federrath & Brunt (2010, in prep)

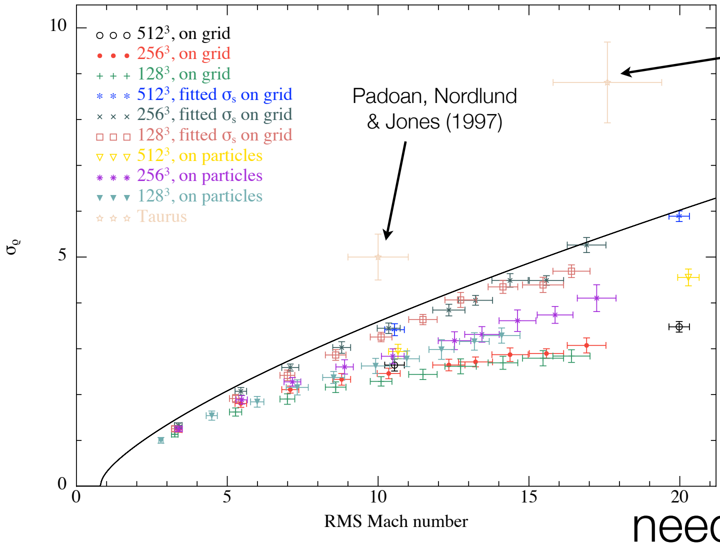
Trying to measure the (linear) density variance



If log normal, expect $\sigma_\rho^2 = e^{\sigma_s^2} - 1$

$s \equiv log(\rho/\overline{\rho})$

Comparison to observations



Brunt (2010) (based on new method for inferring 3D variance from 2D observations)

(see Brunt, Federrath and Price 2010)

need COMPRESSIVE DRIVING or GRAVITY

Price, Federrath & Brunt (2010, in prep)

Conclusions

- being a television presenter is easier than getting MHD in SPH to work
- MHD in SPH would work if people stopped making unsubstantiated swipes* at SPH
- Magnetic fields can significantly change star formation even at supercritical field strengths, so we need MHD in SPH
- SPH and grid codes agree very well on the statistics of turbulence when the resolutions are comparable: nparts = ncells to get similar spectra, but SPH much better at resolving dense structures.
- The standard-deviation-- Mach number relation in supersonic turbulence seems robust up to Mach 20, but observed density variances are much higher than can be produced with solenoidally-driven turbulence alone

*defined as any paper where the criticism is based purely on a citation to Agertz et al. (2006)