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NON-IDEAL SMOOTHED PARTICLE MAGNETOHYDRODYNAMICS

Can non-ideal MHD solve the magnetic braking catastrophe?

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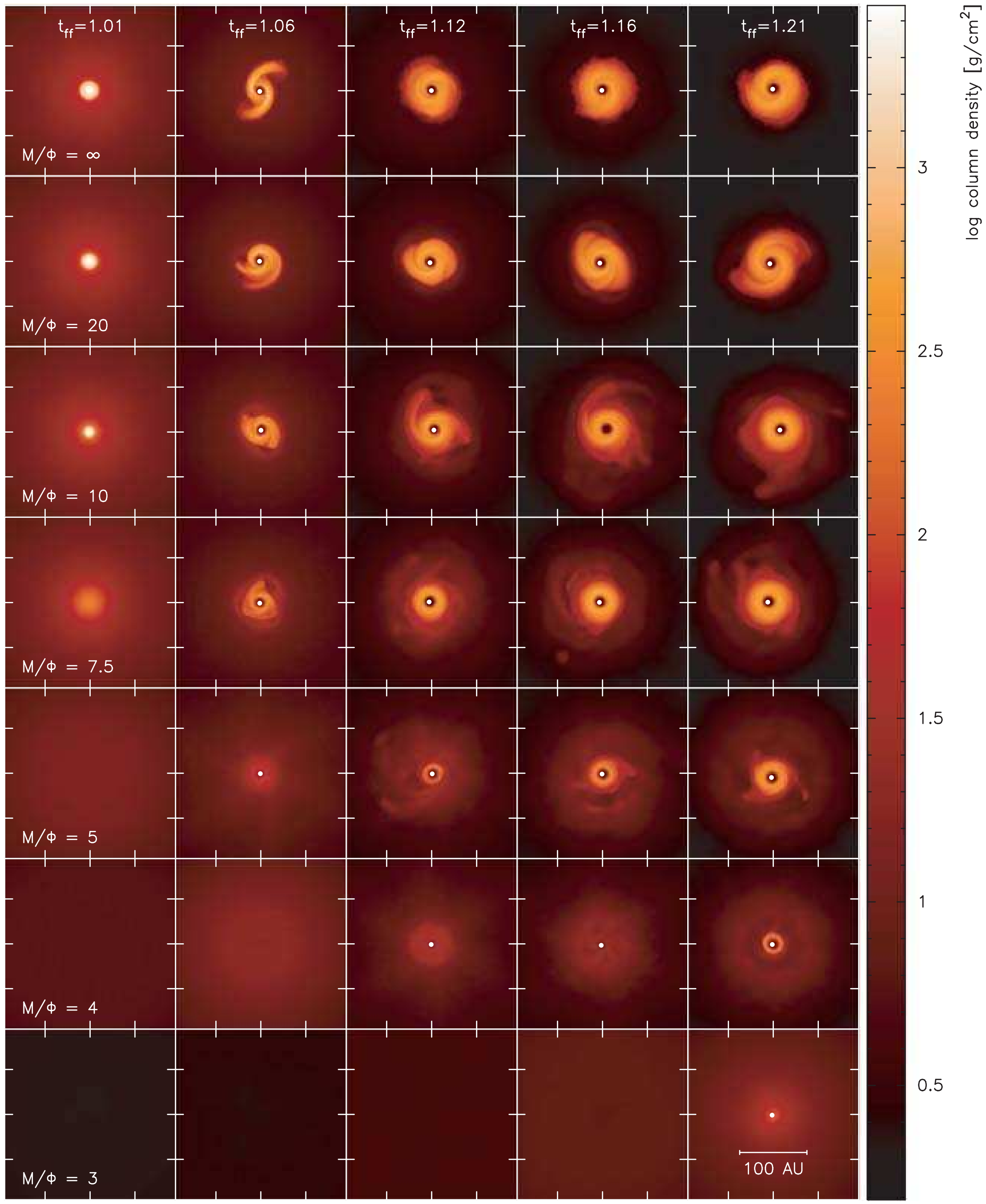
ASTRONUM-2016, 6th-10th June, Monterey, California

THE “MAGNETIC BRAKING CATASTROPHE” IN PROTOSTELLAR DISC FORMATION

Allen et al. (2003), Galli et al. (2006), Price & Bate(2007), Mellon & Li (2008), Hennebelle & Fromang (2008), Commerçon et al. (2010), Krasnopolsky et al. (2010) and many others



- Assumes ideal MHD (not true)
- Our previous work used Euler potentials to solve div B = 0

$$\mathbf{B} = \nabla\alpha \times \nabla\beta$$

(no outflows)

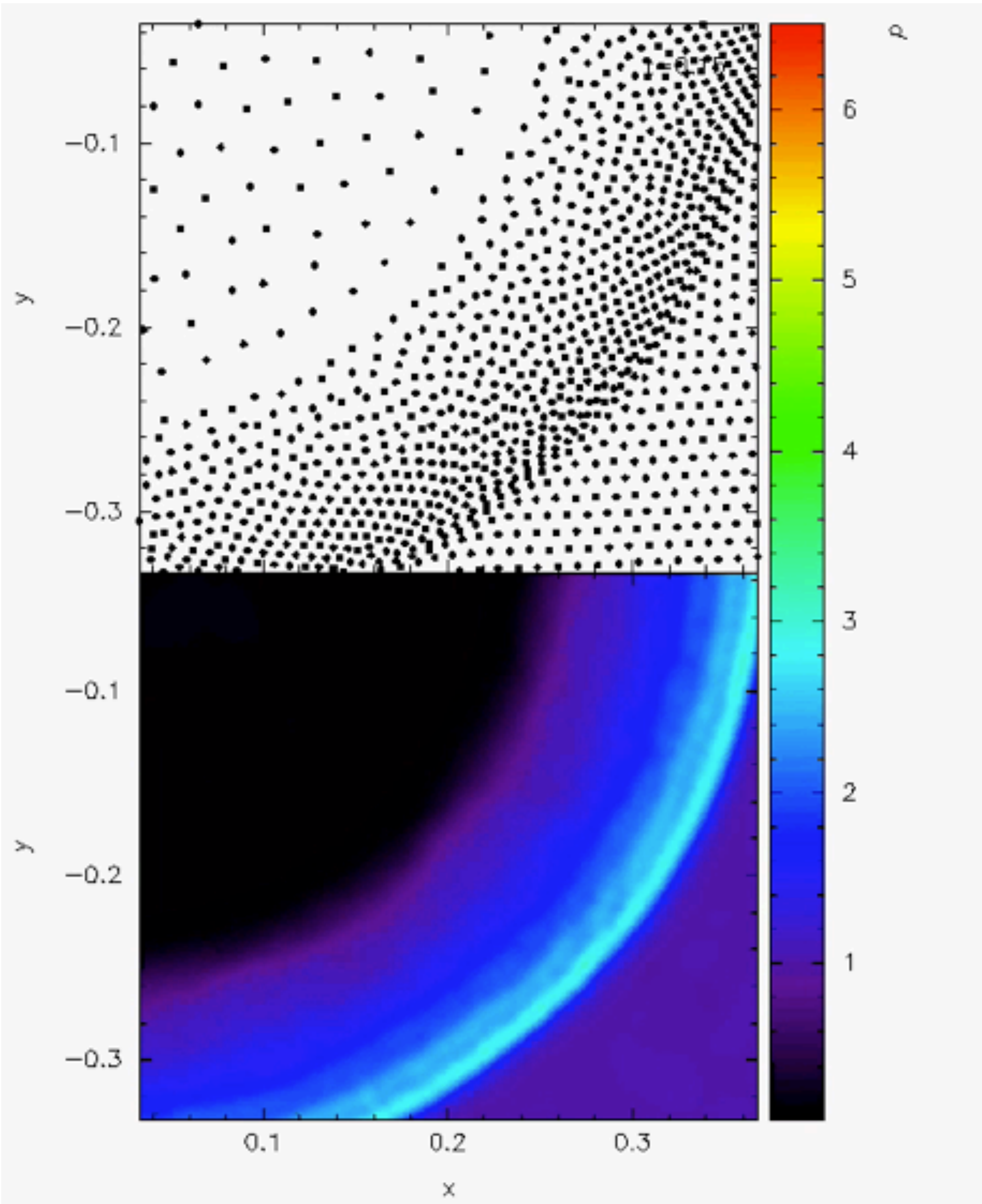
- ~~No turbulence~~

See Seifried et al. (2012), Joos et al. (2013) and others

Price & Bate (2007)

SMOOTHED PARTICLE HYDRODYNAMICS

Monaghan (1992, 2005)
Rosswog (2010); Springel (2010)
Price (2012) J. Comp. Phys. 231, 759



- Lagrangian/Hamiltonian particle method for solving equations of fluid dynamics
- Symmetry-preserving, maintain exact conservation of linear and angular momentum, energy, entropy and circulation in spatial discretisation
- Zero intrinsic dissipation
- Adaptive — resolution follows mass not volume
- No geometry restrictions, easily handle free surfaces

SMOOTHED PARTICLE MAGNETOHYDRODYNAMICS

see review by Price (2012) J. Comp. Phys. 231, 759

$$L = \int \left(\frac{1}{2}\rho v^2 - \rho u - \frac{1}{2\mu_0} B^2 \right) \mathrm{d}V$$

$$L = \sum_a m_a \left(\frac{1}{2} v_a^2 - u_a - \frac{B_a^2}{2\mu_0 \rho_a} \right)$$

Price & Monaghan (2004a,b, 2005)

Euler-Lagrange equations give discrete form of:

Subtract div B source term for stability

$$\frac{\mathrm{d}\boldsymbol{\rho}}{\mathrm{d}t} = -\boldsymbol{\rho}(\nabla \cdot \mathbf{v})$$

$$\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = -\frac{1}{\rho}\nabla \cdot \left[\left(P + \frac{1}{2}\frac{B^2}{\mu_0} \right) \mathbf{I} - \frac{\mathbf{B}\mathbf{B}}{\mu_0} \right] - \frac{\mathbf{B}(\nabla \cdot \mathbf{B})}{\mu_0 \rho}$$

$$\frac{\mathrm{d}u}{\mathrm{d}t} = -\frac{P}{\rho}(\nabla \cdot \mathbf{v})$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\mathbf{B}}{\rho} \right) = \left(\frac{\mathbf{B}}{\rho} \cdot \nabla \right) \mathbf{v}$$

Divergence advection test from Dedner et al. (2002)

Dissipationless: Must add dissipation terms to handle shocks and discontinuities

Need to separately handle div B = 0

These equations are equivalent to the 8-wave formulation of Powell et al. 1994

HYPERBOLIC/PARABOLIC DIVERGENCE CLEANING

Dedner et al. (2002)
Price & Monaghan (2005)
Mignone & Tzeferacos (2010)

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla\psi$$

$$\frac{\partial \psi}{\partial t} = -c_h^2(\nabla\cdot\mathbf{B}) - \frac{c_h^2}{c_p^2}\psi$$

$$\frac{\partial \psi}{\partial t} = -c_h^2(\nabla\cdot\mathbf{B}) - \frac{\sigma c_h}{h}\psi$$

$$\frac{\partial \psi}{\partial t} = -c_h^2(\nabla\cdot\mathbf{B}) - \frac{c_h}{\lambda}\psi$$

Hyperbolic

Parabolic

Use dimensionless parameter

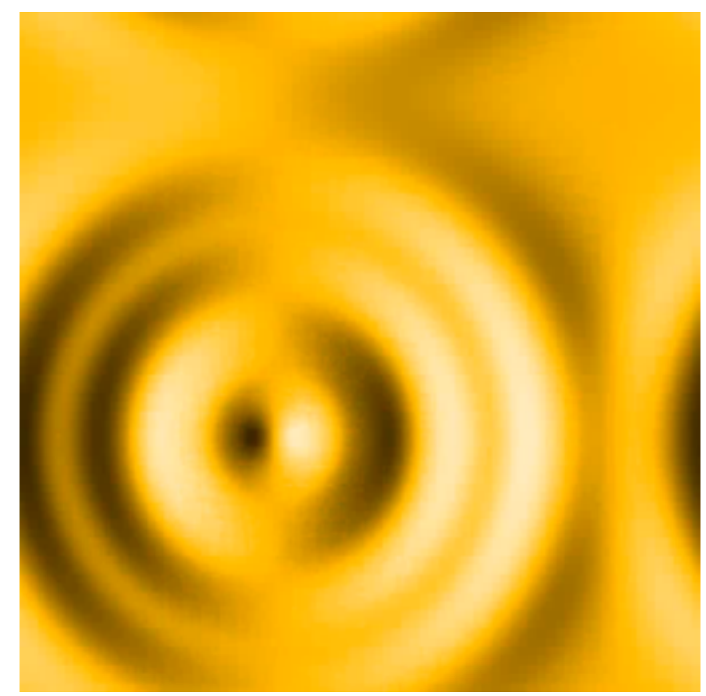
Price & Monaghan (2005); Mignone & Tzeferacos (2010)

Critical damping on resolution length

Price & Monaghan (2005); Mignone & Tzeferacos (2010)

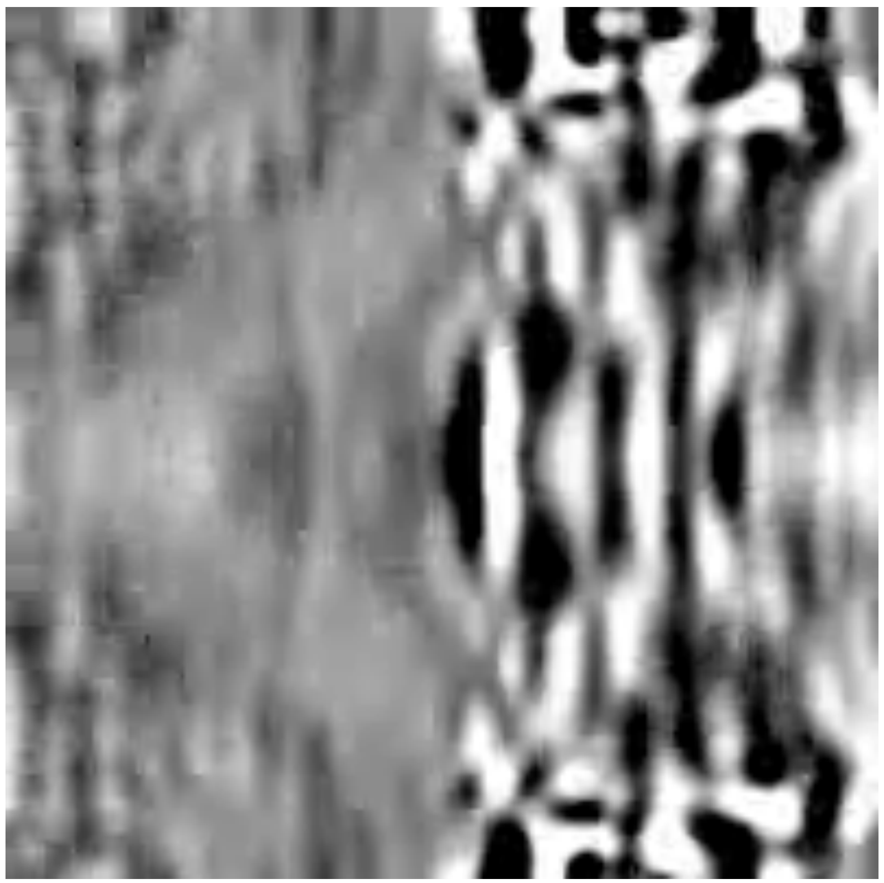
$$\frac{1}{c_h^2}\frac{\partial^2(\nabla\cdot\mathbf{B})}{\partial t^2} + \nabla^2(\nabla\cdot\mathbf{B}) + \frac{1}{\lambda c_h}\frac{\partial(\nabla\cdot\mathbf{B})}{\partial t} = 0$$

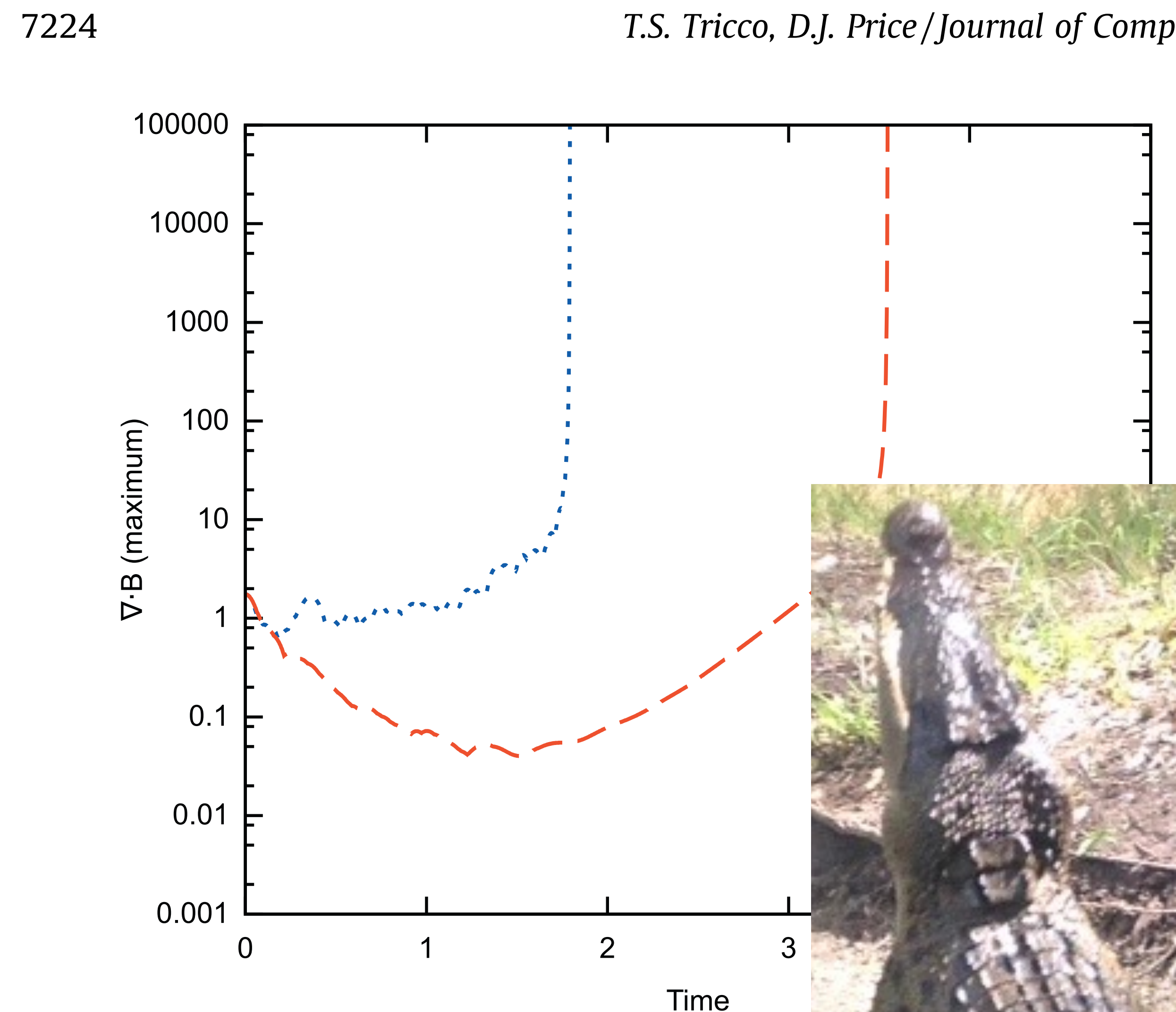
Wavelength of critical damping

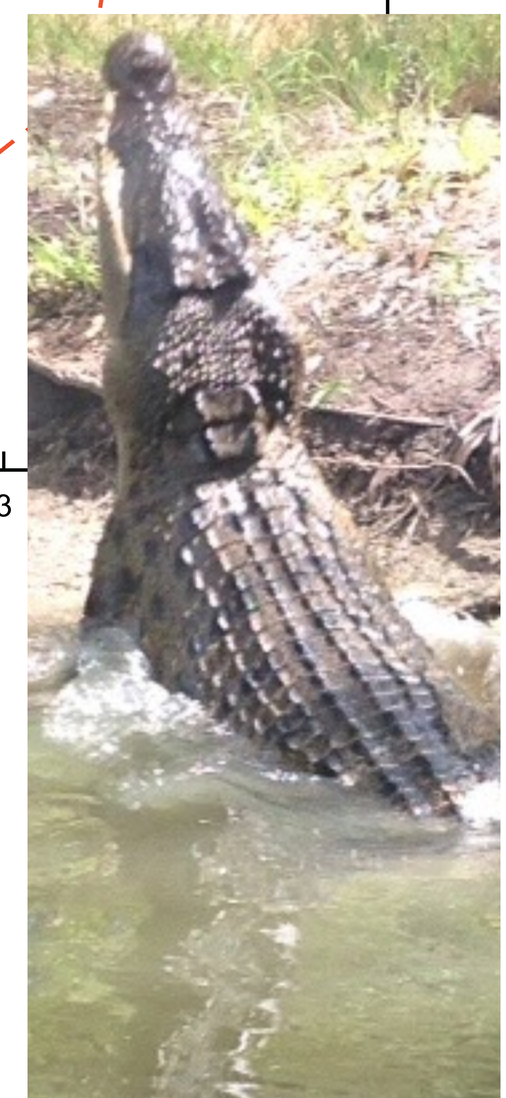


Hyperbolic term only

WHEN CLEANING ATTACKS







Divergence advection test (Dedner et al. 2002) with 10:1 jump in density

"CONSTRAINED" HYPERBOLIC/PARABOLIC DIVERGENCE CLEANING

Tricco & Price (2012); Tricco, Price & Bate (2016), submitted to JCP

- Define energy associated with cleaning field

$$E = \int \left[\frac{1}{2}\frac{B^2}{\mu_0} + \frac{1}{2}\frac{\psi^2}{\mu_0 c_h^2} \right] \mathrm{dV}$$

- Enforce energy conservation in hyperbolic terms

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \int \left[\frac{\mathbf{B}}{\mu_0} \cdot \left(\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t} \right)_{\psi} + \frac{\psi}{\mu_0 c_{\mathrm{h}}^2}\frac{\mathrm{d}\psi}{\mathrm{d}t} - \frac{\psi^2}{2\mu_0 \rho c_{\mathrm{h}}^2}\frac{\mathrm{d}\rho}{\mathrm{d}t} - \frac{\psi^2}{\mu_0 c_{\mathrm{h}}^3}\frac{\mathrm{d}c_{\mathrm{h}}}{\mathrm{d}t} \right] \mathrm{d}V = 0$$

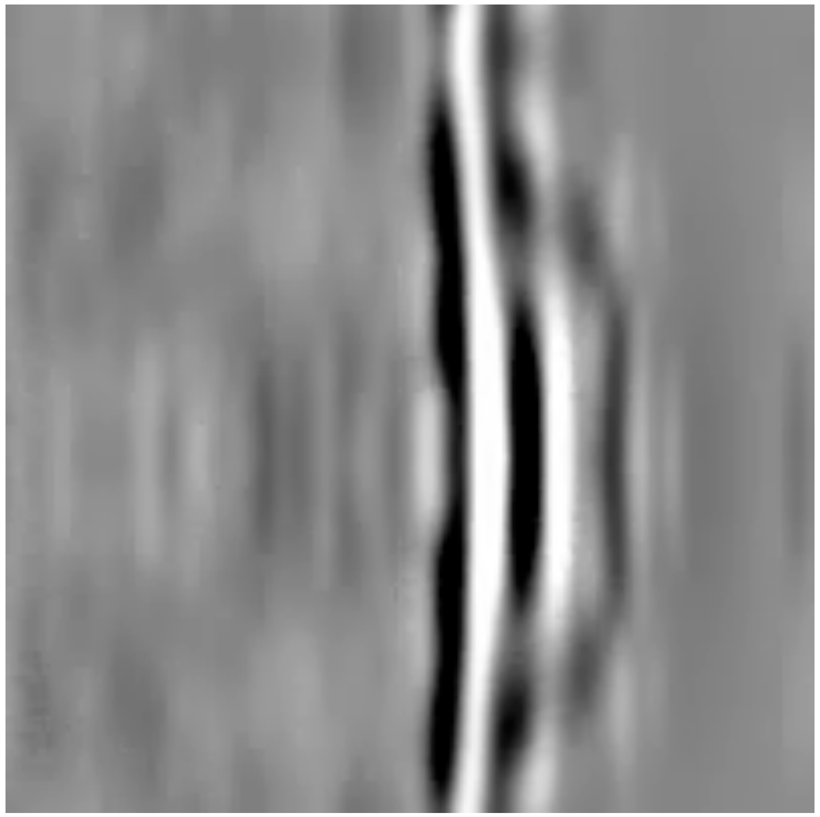
$$\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t} = -\nabla\psi$$

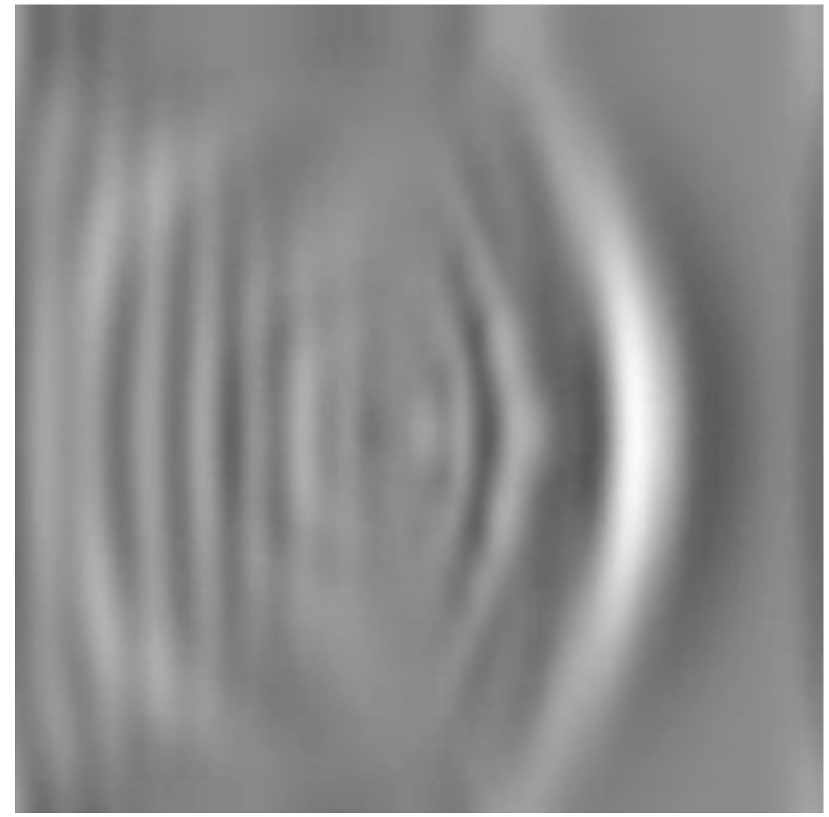
$$\frac{\mathrm{d}\psi}{\mathrm{d}t} = -c_h^2(\nabla\cdot\mathbf{B}) - \frac{\sigma c_h}{h}\psi - \frac{1}{2}\psi(\nabla\cdot\mathbf{v})$$

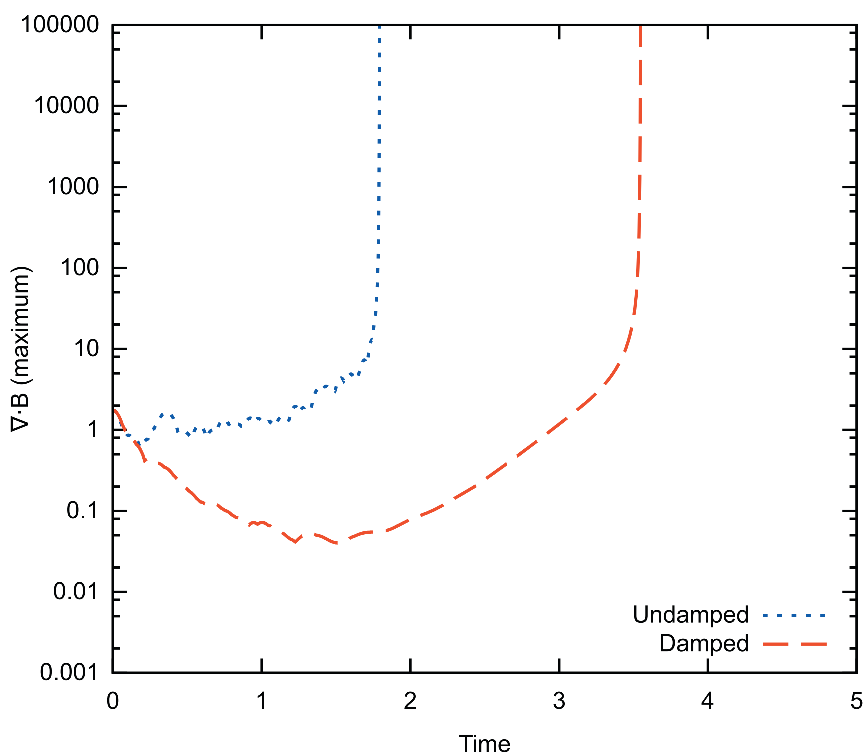
Requires particular choice of operators here

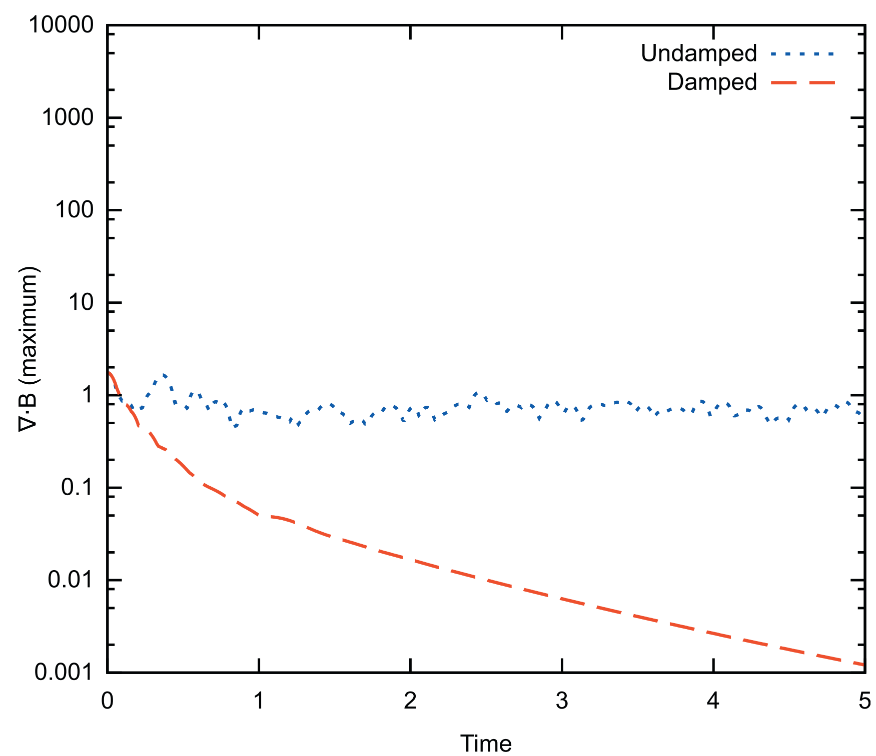
- Can enforce exact energy conservation in SPH discretisation

CONSTRAINED HYPERBOLIC/PARABOLIC CLEANING









Parabolic term is negative definite!

WHAT IF THE CLEANING SPEED VARIES?

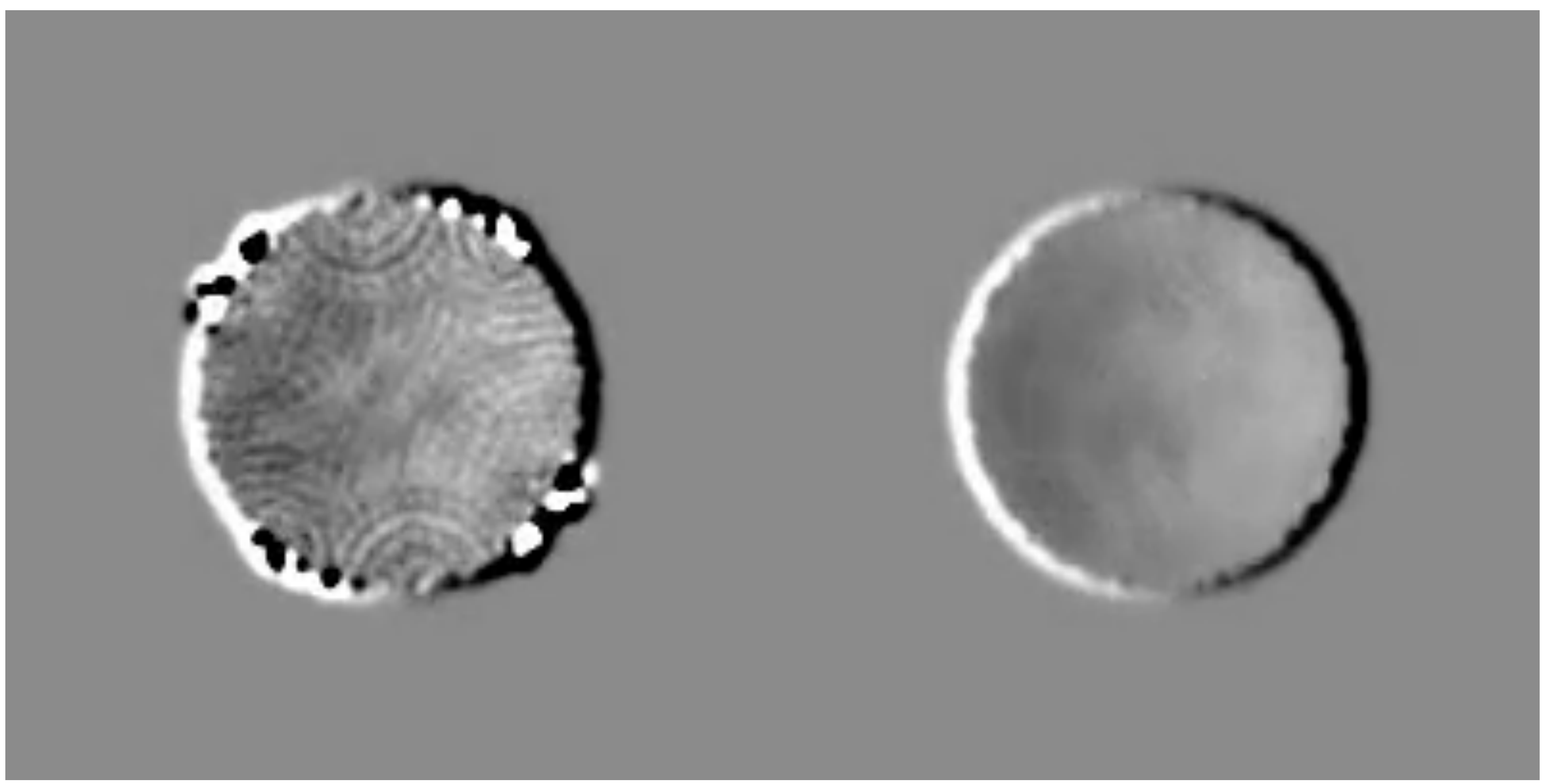
Tricco, Price & Bate (2016), submitted to JCP

Thanks to Gábor Tóth for discussion at last years ASTRONUM!

$$\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t} = -\nabla\psi$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\psi}{c_h}\right) = -c_h(\nabla\cdot\mathbf{B}) - \frac{\psi}{2c_h}(\nabla\cdot\mathbf{v}) - \frac{\sigma}{h}\frac{\psi}{c_h}$$

Hyperbolic terms conserve energy even with variable wave speed!

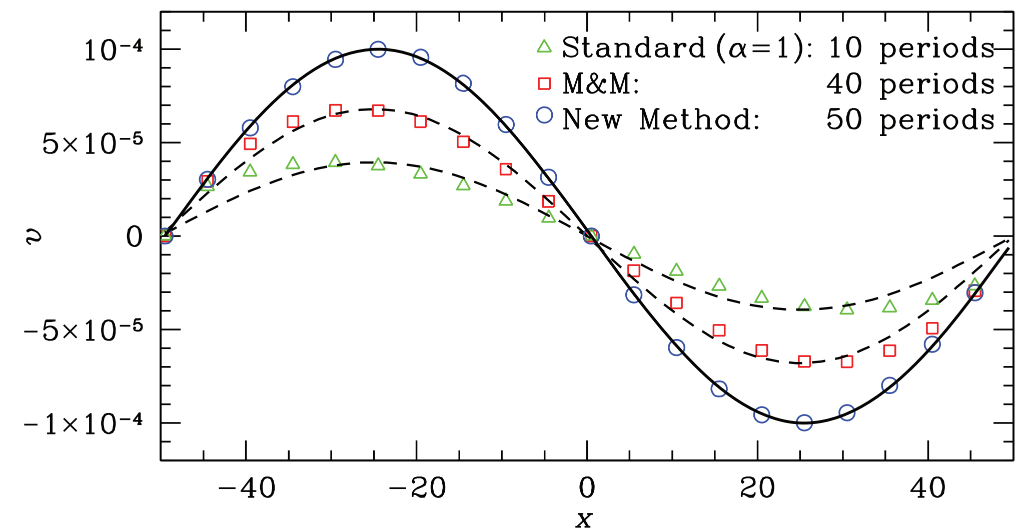


Non-conservative method *Conservative method*

SHOCK DISSIPATION SWITCHES

- Cullen & Dehnen (2010) switch for shock viscosity

$$A = \max\left[-\frac{\mathrm{d}}{\mathrm{d}t}(\nabla\cdot\mathbf{v}), 0\right] \quad \alpha_{loc} = \min\left(\frac{10h^2 A}{c_s^2 + h^2 A}, 1\right)$$



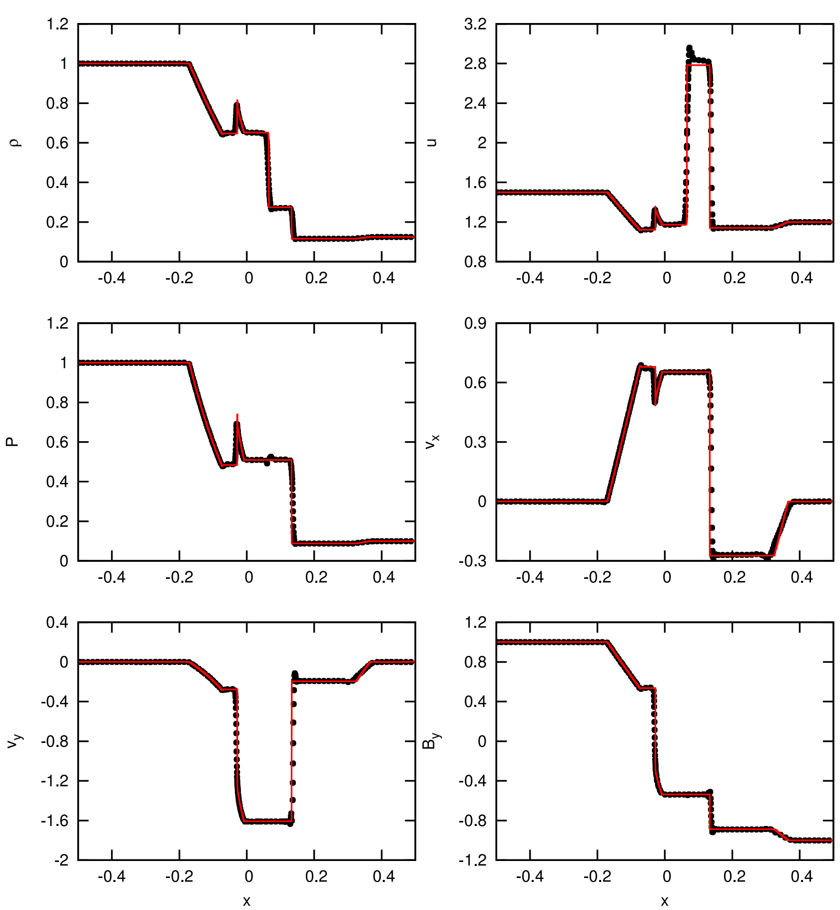
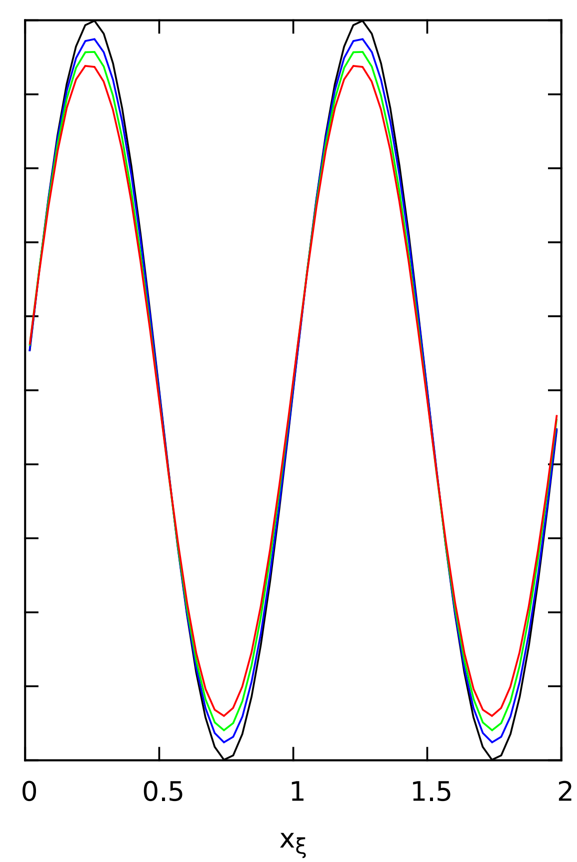


Figure 3. Shocktube test 5A from RJ95 performed in 2D with left state $(\rho, P, v_x, v_y, B_y) = (1, 1, 0, 0, 1)$ and right state $(\rho, P, v_x, v_y, B_y) = (0.125, 0.1, 0, 0, -1)$ with $B_x = 0.75$ at $t = 0.1$. Black circles represent the particles and the red line represents the solution obtained with the ATHENA code using 10^4 grid cells.



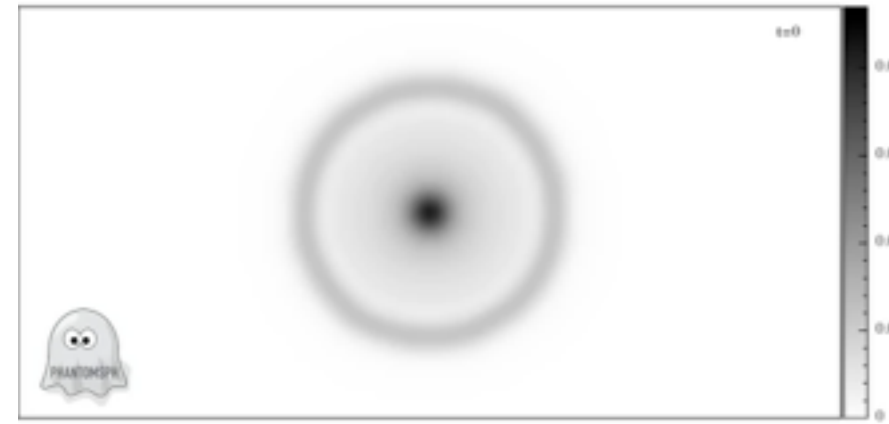
- Tricco & Price (2013) switch for resistivity

$$\alpha^B = \min\left(\frac{h|\nabla\mathbf{B}|}{|\mathbf{B}|}, 1\right)$$

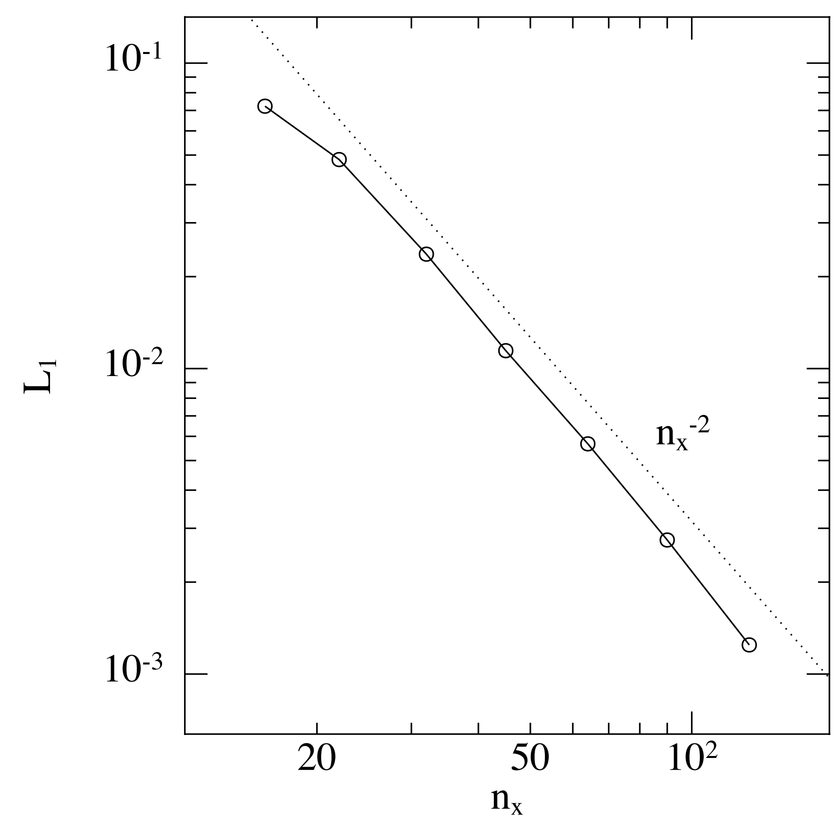
- Revised further in Phantom - 2nd order artificial resistivity, vanishes when v=const

PHANTOM SPMHD CODE

Price et al. (2016) in prep.



Advection of current loop (Gardiner & Stone 2005, 2008)

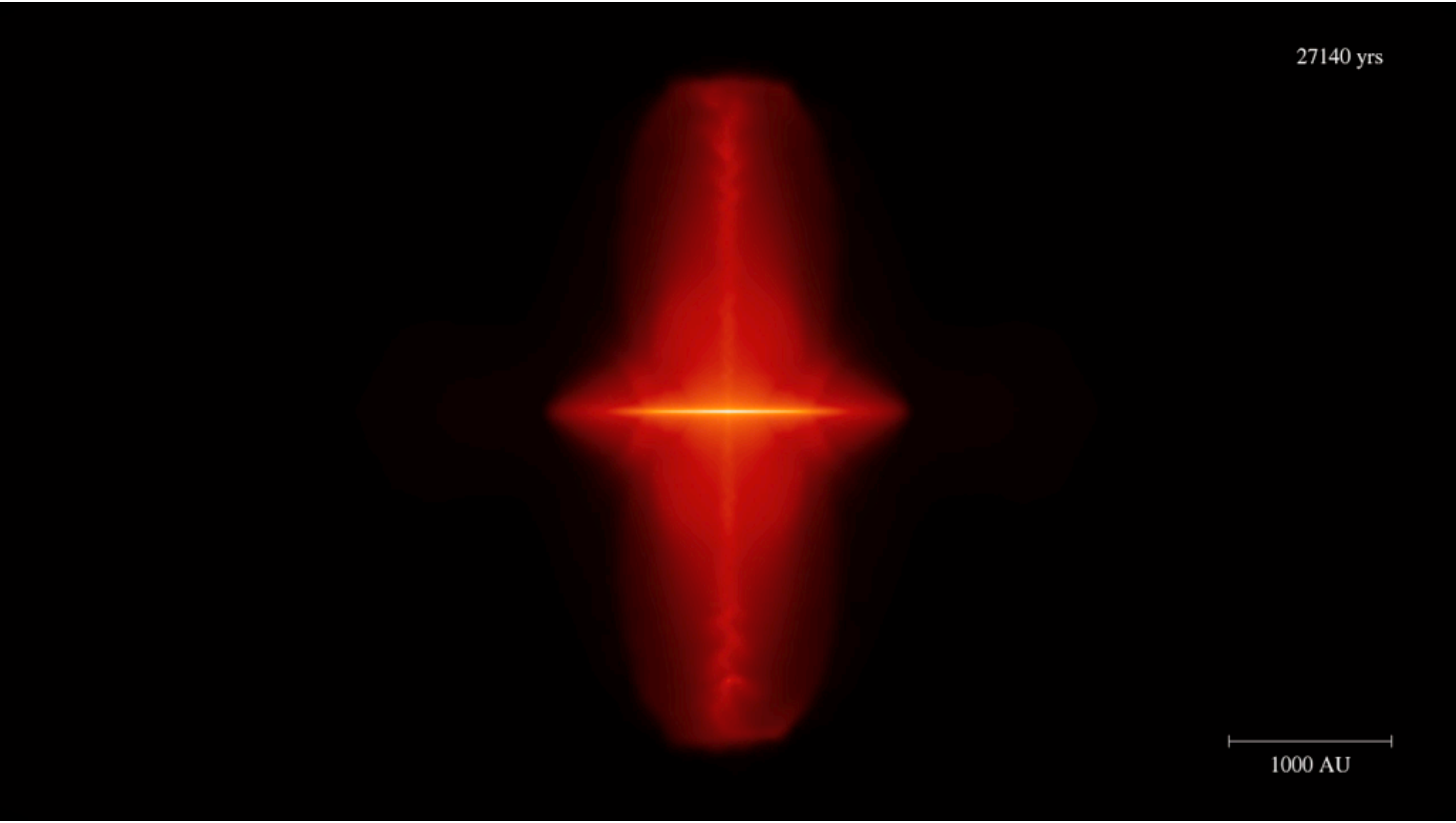


Convergence on circularly polarised Alfvén wave with ALL dissipation turned on

Performed with all dissipation, shock capturing and divergence cleaning turned on

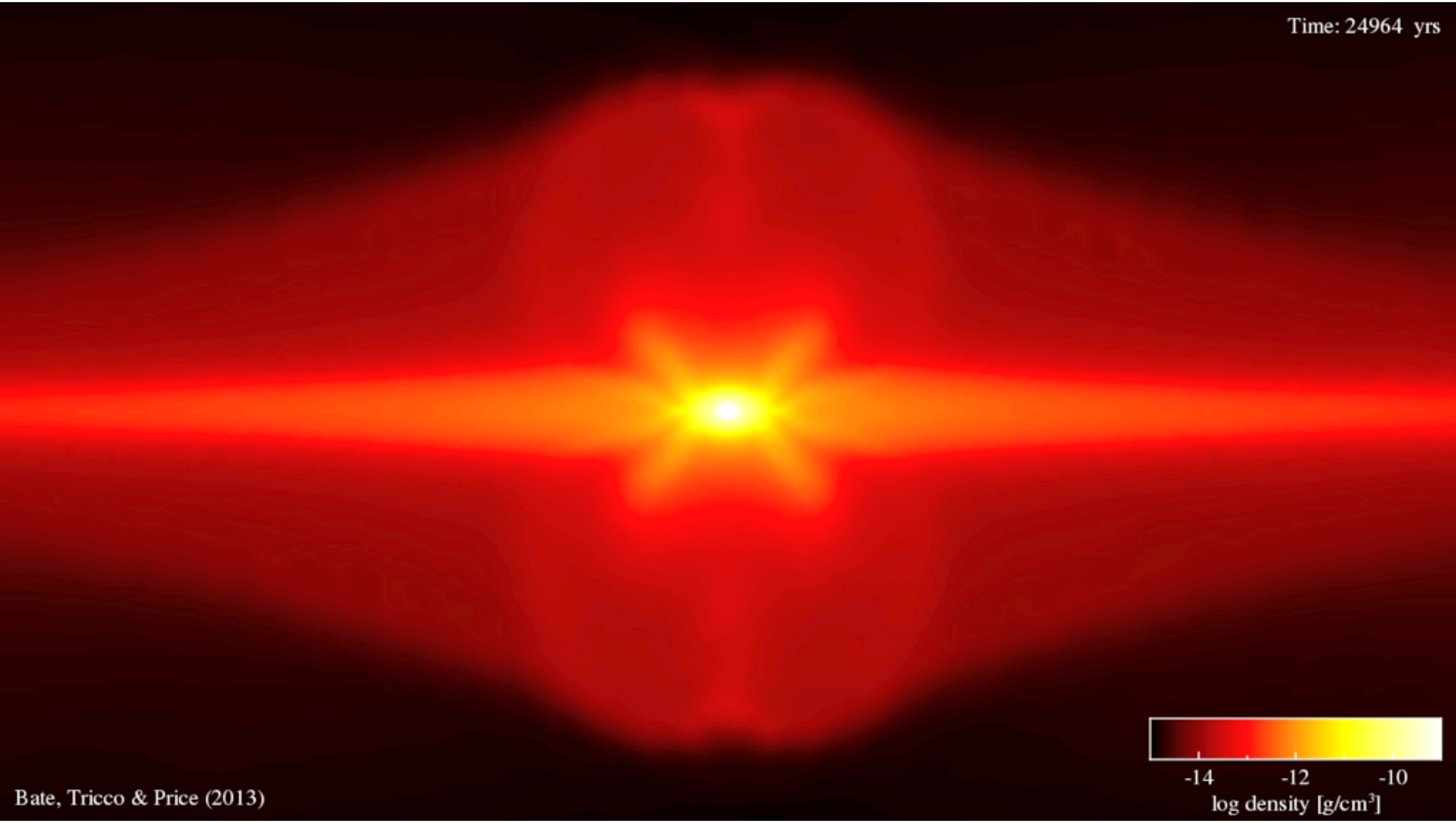
JETS FROM THE FIRST CORE

Price, Tricco & Bate (2012); see also Machida et al. (2008)



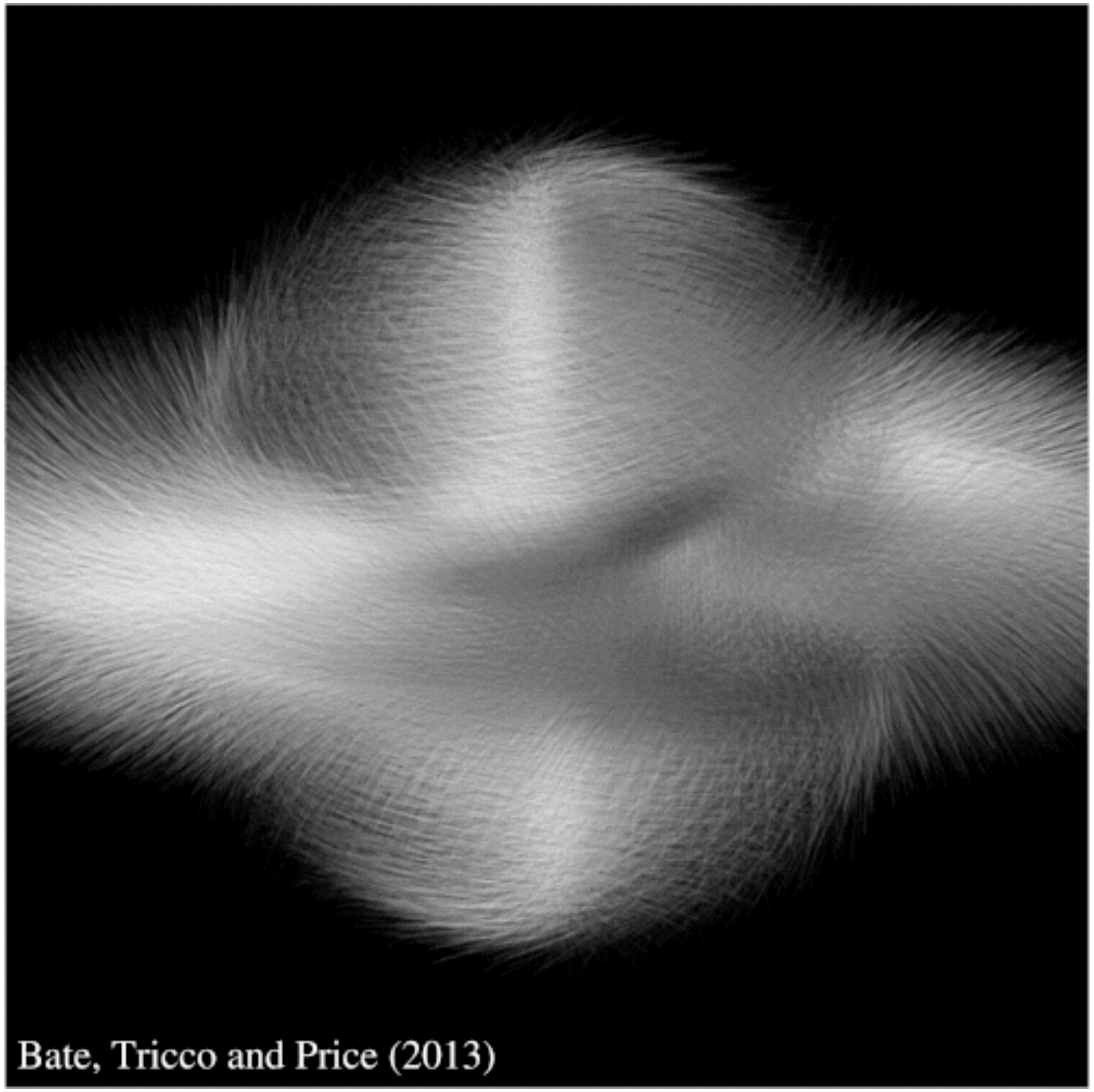
PROTOSTELLAR JETS: SECOND COLLAPSE

Bate, Tricco & Price (2014)

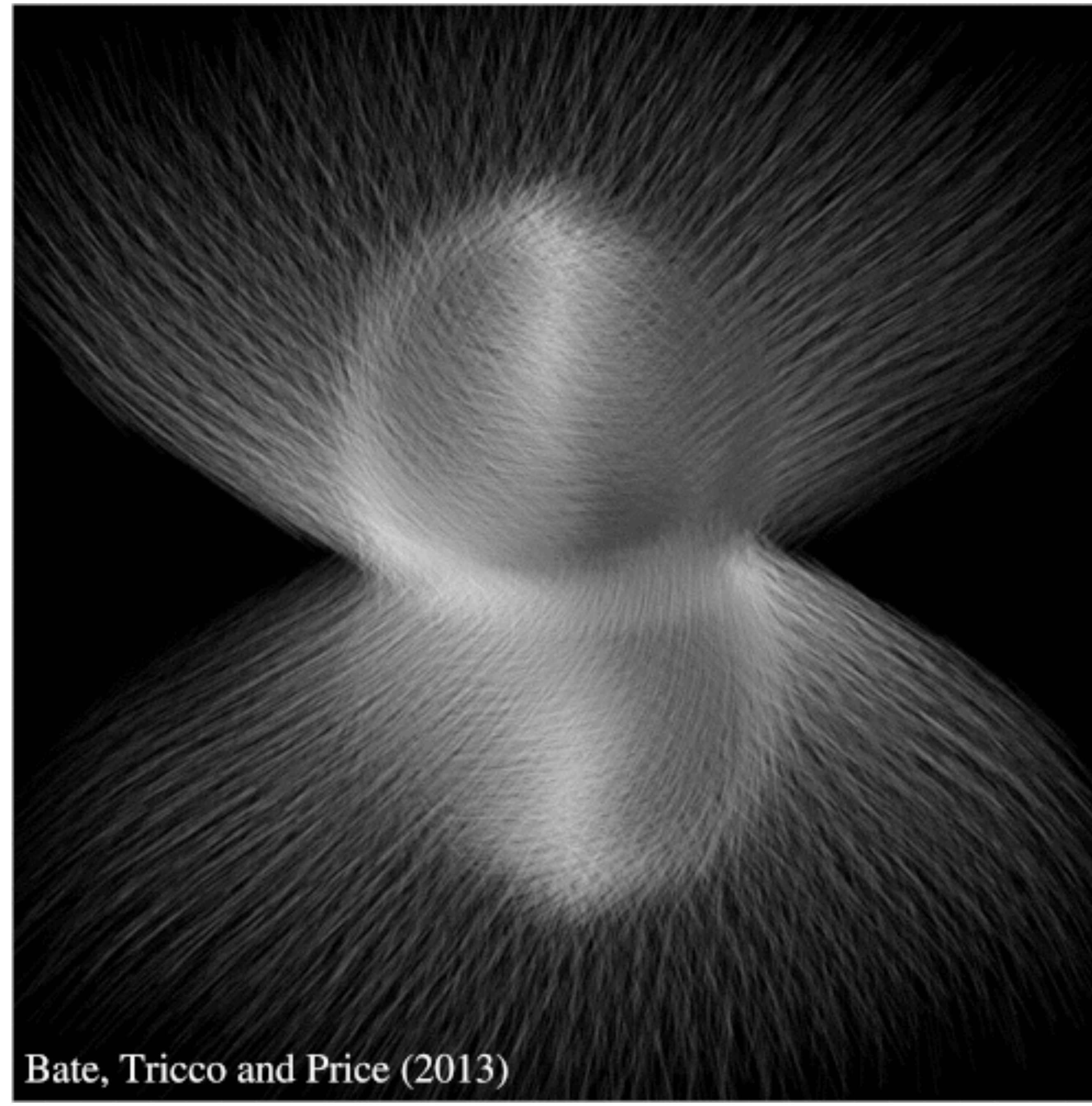


Performed with radiation magnetohydrodynamics (grey FLD: Whitehouse & Bate 2004a,b; Whitehouse, Bate & Monaghan 2006)

MAGNETICALLY LAUNCHED OUTFLOWS



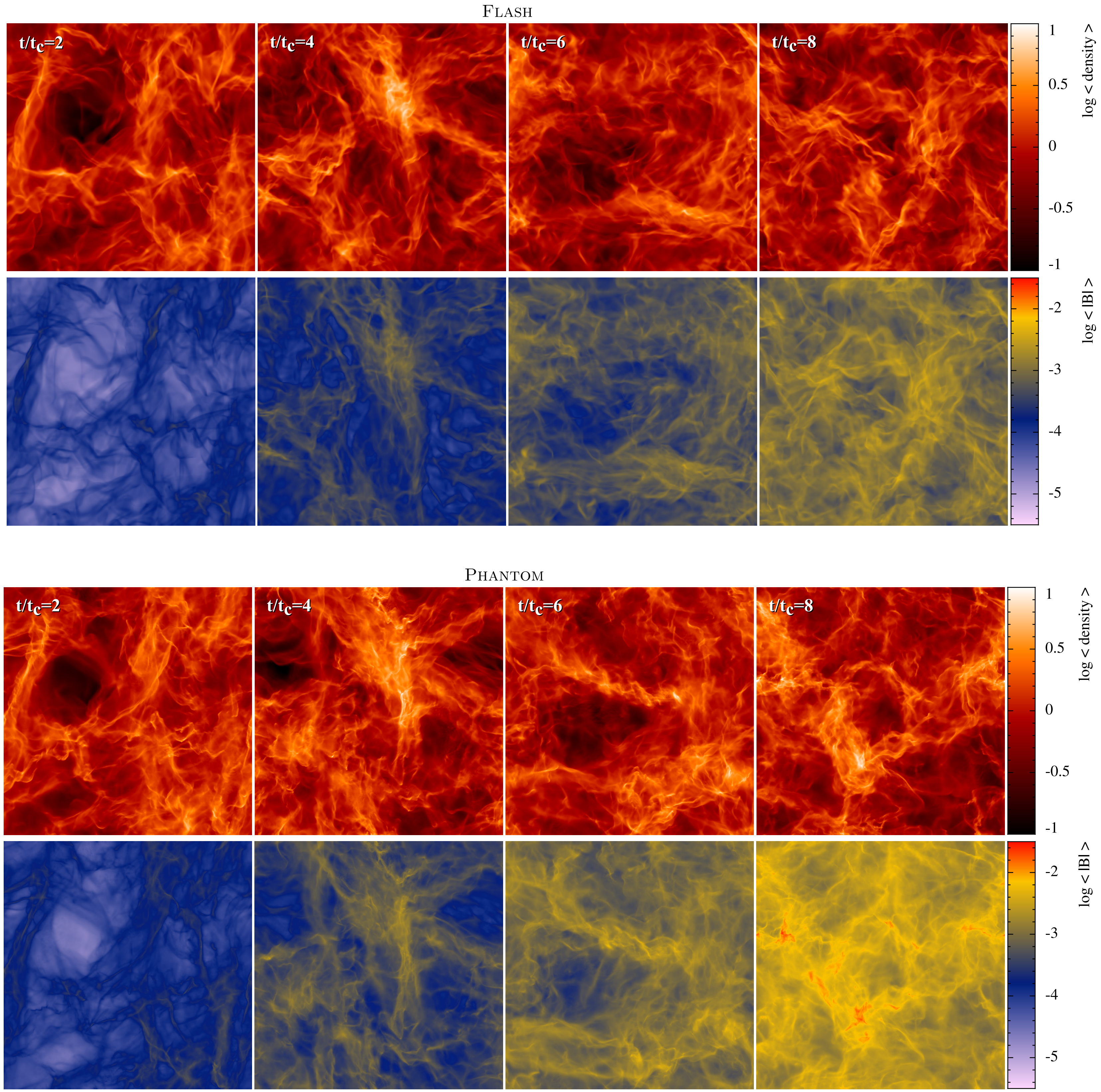
First core (100 x 100 au)



Second (protostellar) core (10 x 10 au)

SMALL SCALE DYNAMO: FLASH VS PHANTOM

Tricco, Price & Federrath (2016)



NON-IDEAL SPMHD

Wurster, Price & Ayliffe (2014), Wurster, Price & Bate (2016)

Strong coupling approximation: $\rho \approx \rho_n; \;\; \rho_i \ll \rho$

$$\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t} = -\mathbf{B}(\nabla\cdot\mathbf{v}) + (\mathbf{B}\cdot\nabla)\mathbf{v} - \nabla\times\left[\eta_O\mathbf{J} + \eta_H\mathbf{J}\times\hat{\mathbf{B}} - \eta_A(\mathbf{J}\times\hat{\mathbf{B}})\times\hat{\mathbf{B}}\right]$$

Ohmic | Hall | Ambipolar

- Spatial discretisation exactly conserves energy
- Guaranteed positive definite contribution to entropy
- RKC super-timestepping for ambipolar/Ohmic terms (Alexiades et al. 1996; O'Sullivan & Downes 2006)

Tests:
Mac-Low et al. (1995)
O'Sullivan & Downes (2006)
Choi et al. (2009)
Falle (2003)

Whistler/Ion-cyclotron modes

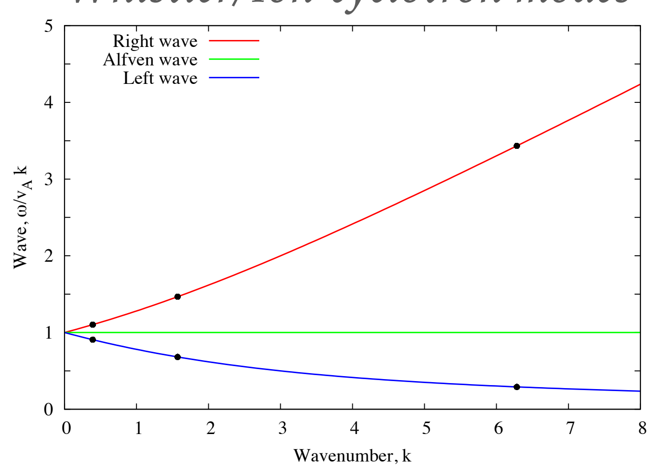
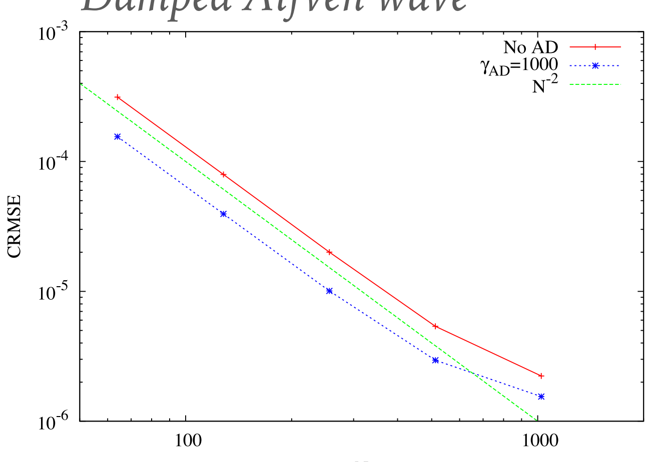
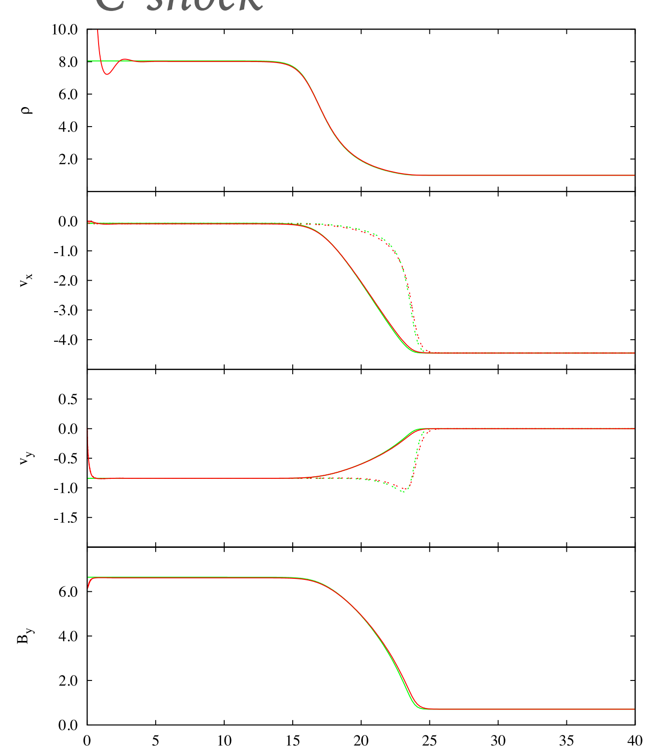


Figure C1. Dispersion relation for the left- and right-circularly polarised wave, corresponding to $\eta_{\rm HE} < 0$ and > 0, respectively. The solid circles are the numerically calculated phase velocities.

Damped Alfvén wave



C-shock



Standing C-shock

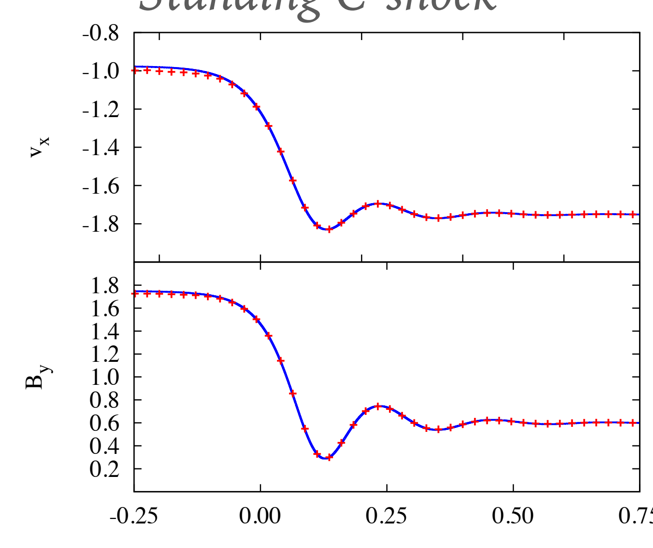
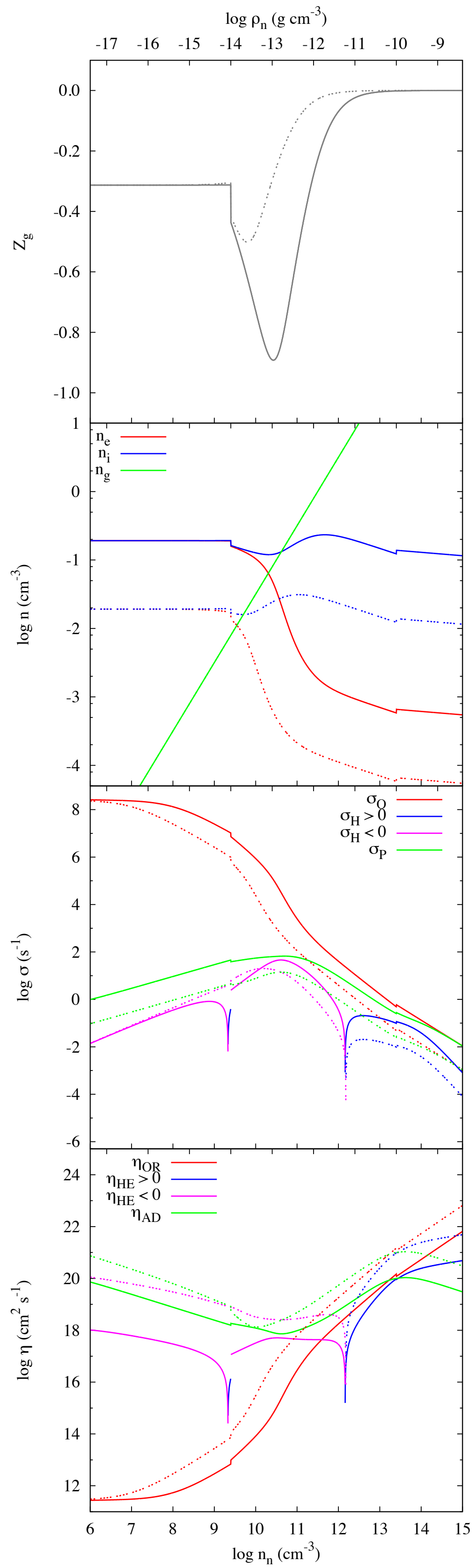
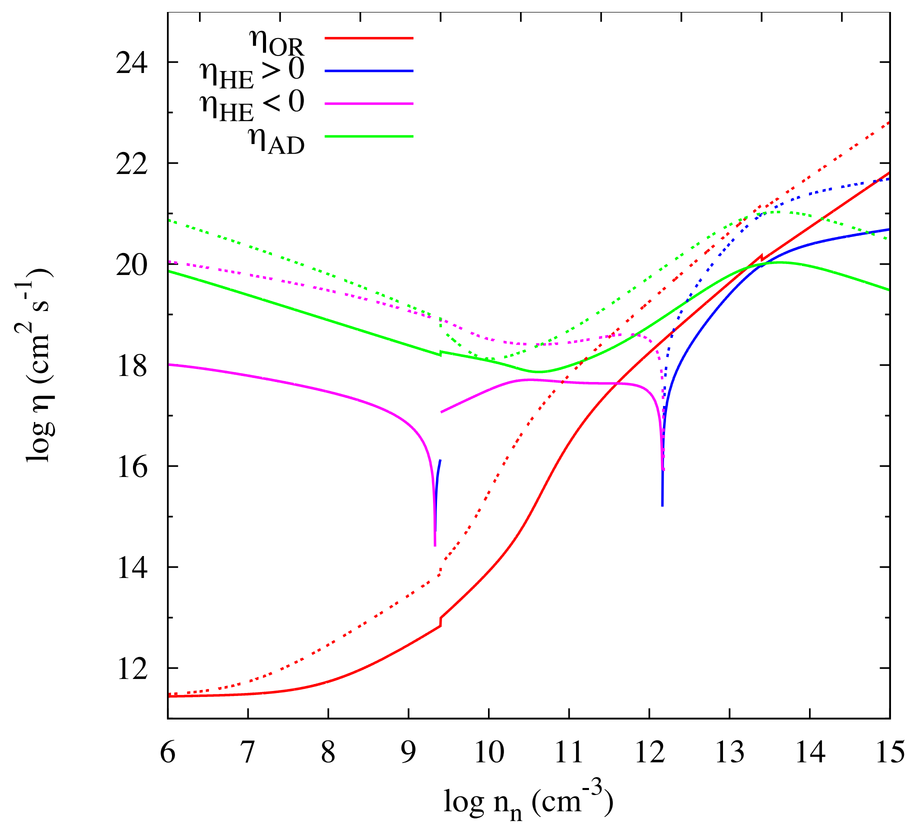


Figure C2. The analytical (solid line) and numerical (crosses) results for the isothermal standing shock. The initial conditions are given in the text. At any given position, the analytical and numerical solutions agree within 3 per cent.

CONDUCTIVITIES

Umebayashi & Nakano (1980), Wardle & Ng (1999), Fujii et al. (2011), Keith & Wardle (2014), Wurster et al. (2016); Marchand et al. (2016)



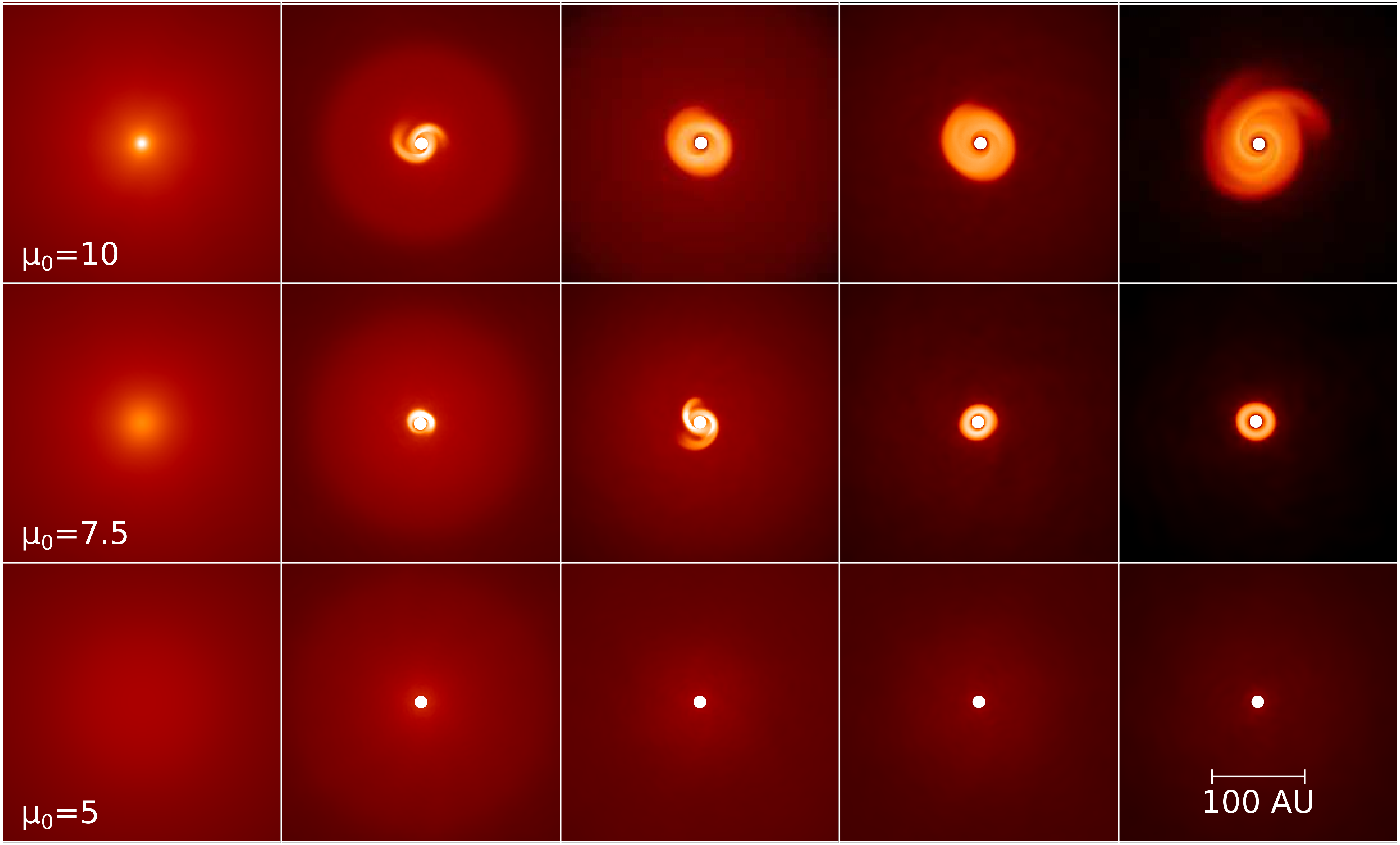


- Solve cosmic ray ionisation/recombination chemical network for grains, ions and neutrals
- Currently assume single grain species 0.1μm
- Gives number density of electrons, ions and grains
- Compute Ohmic, Ambipolar and Hall coefficients at given density, temperature

NICIL Code: Wurster (2016), submitted to PASA

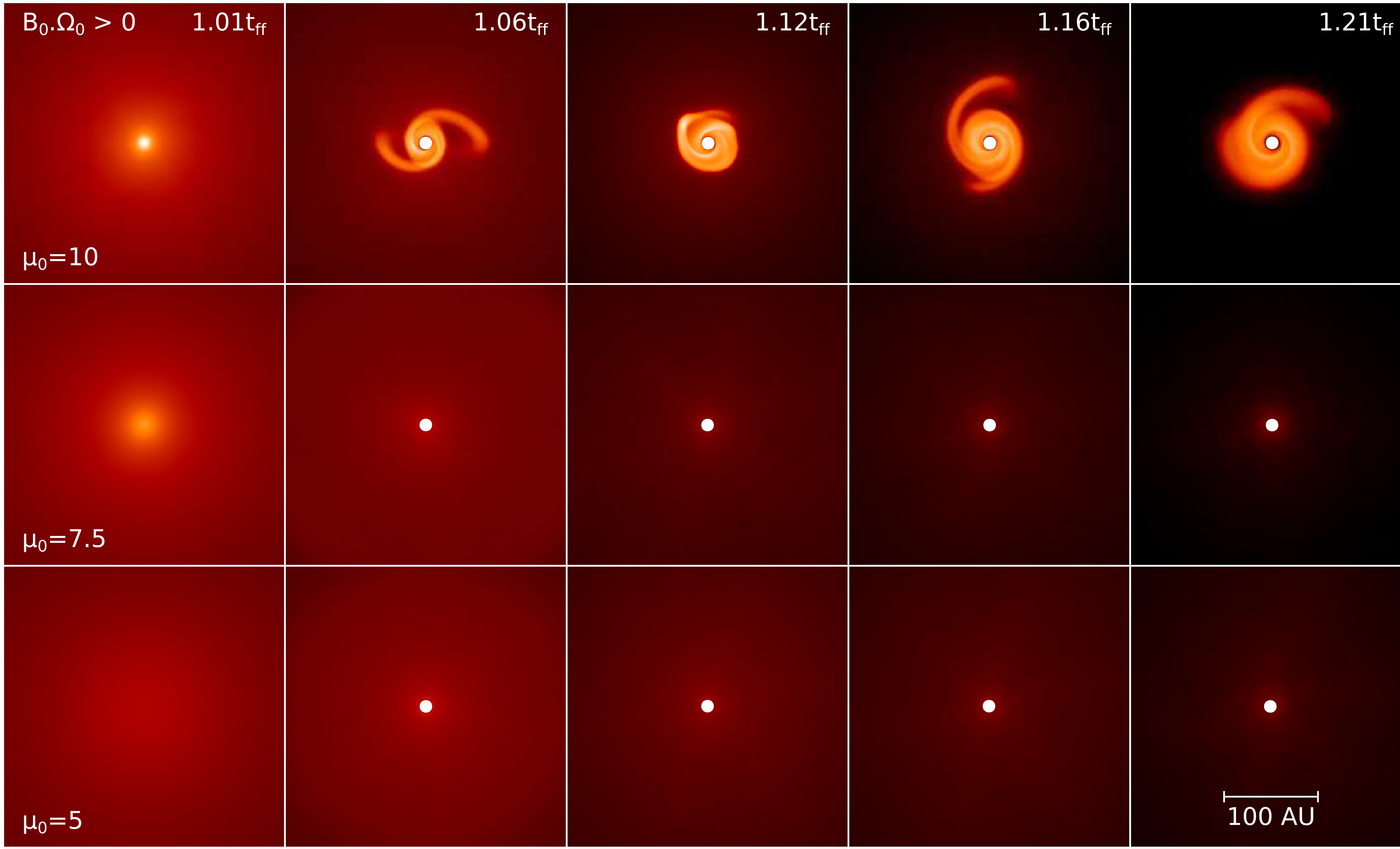
IDEAL MHD: MAGNETIC BRAKING CATASTROPHE

Wurster, Price & Bate (2016)



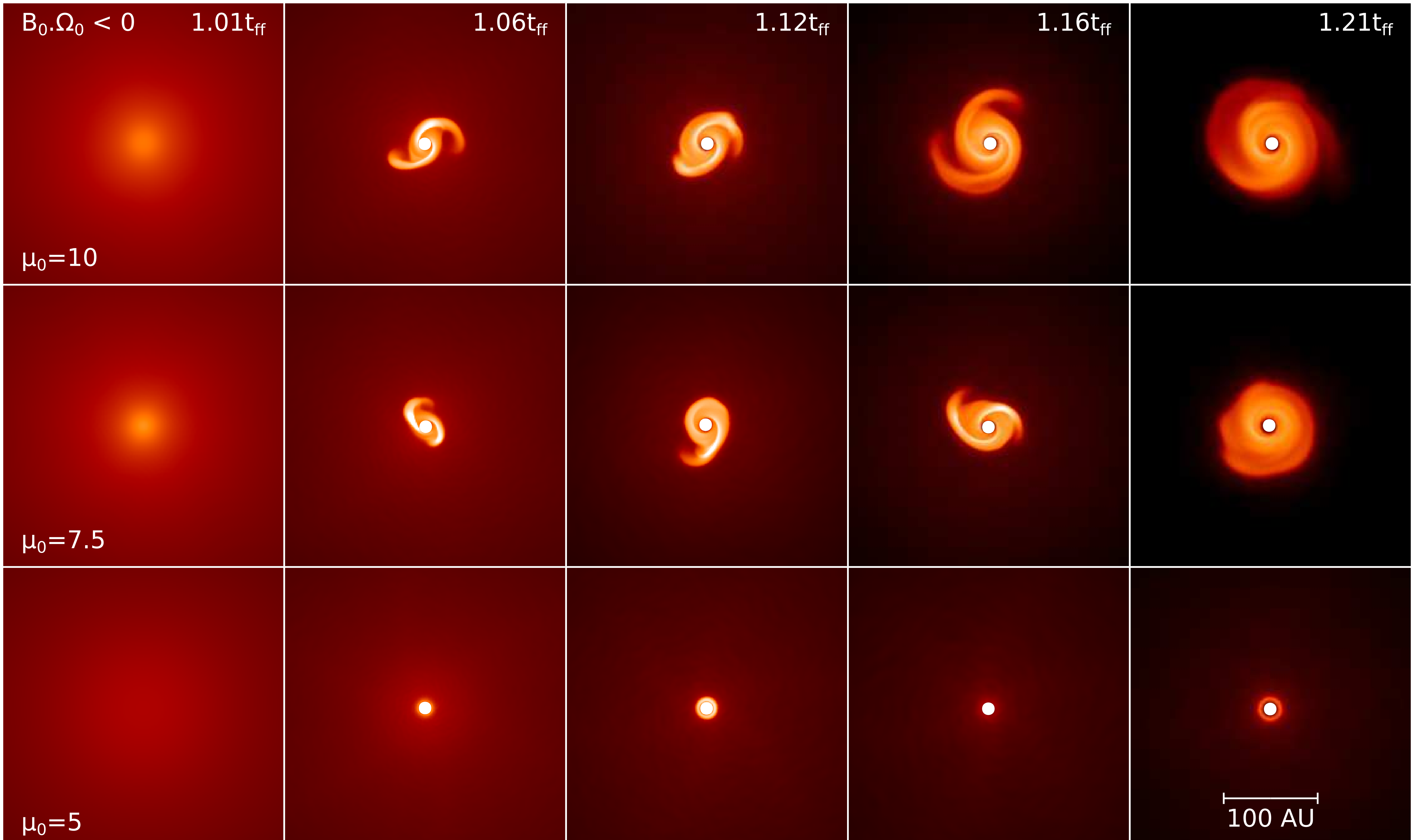
NON-IDEAL MHD: ALIGNED INITIAL FIELD

Wurster, Price & Bate (2016)



NON-IDEAL MHD: ANTI-ALIGNED INITIAL FIELD

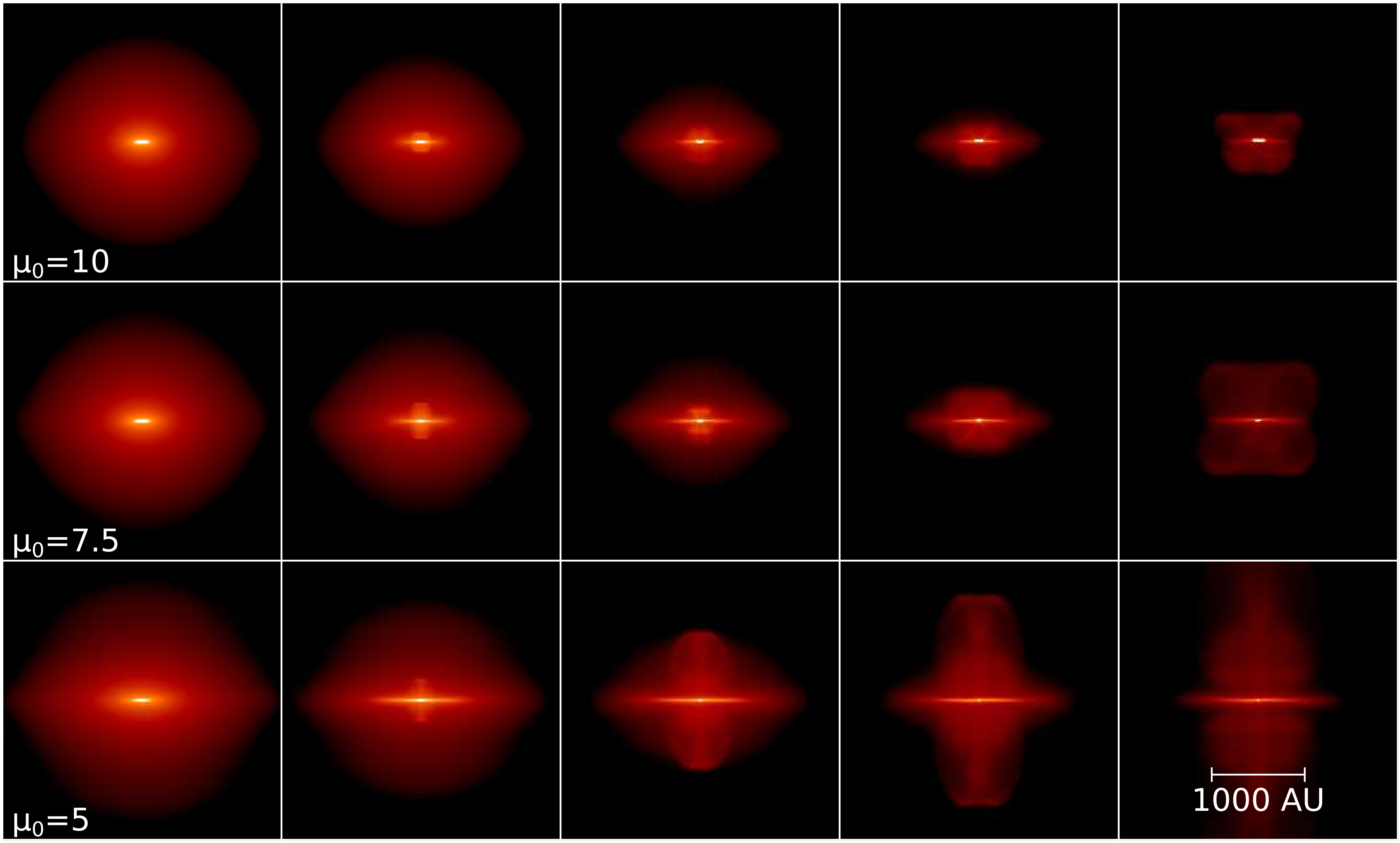
Wurster, Price & Bate (2016)



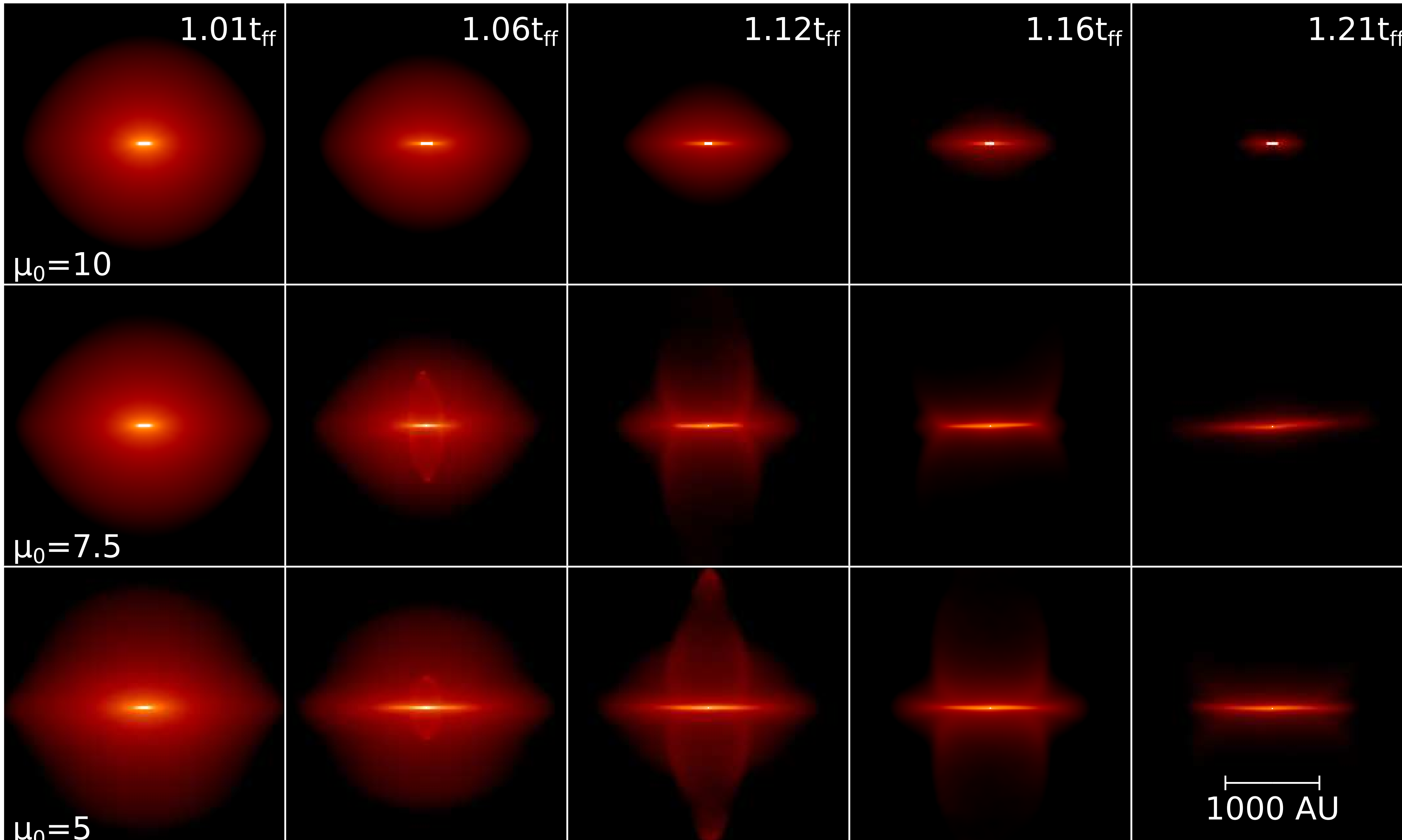
see also Tsukamoto et al. (2015)

OUTFLOWS - IDEAL MHD

Wurster, Price & Bate (2016)

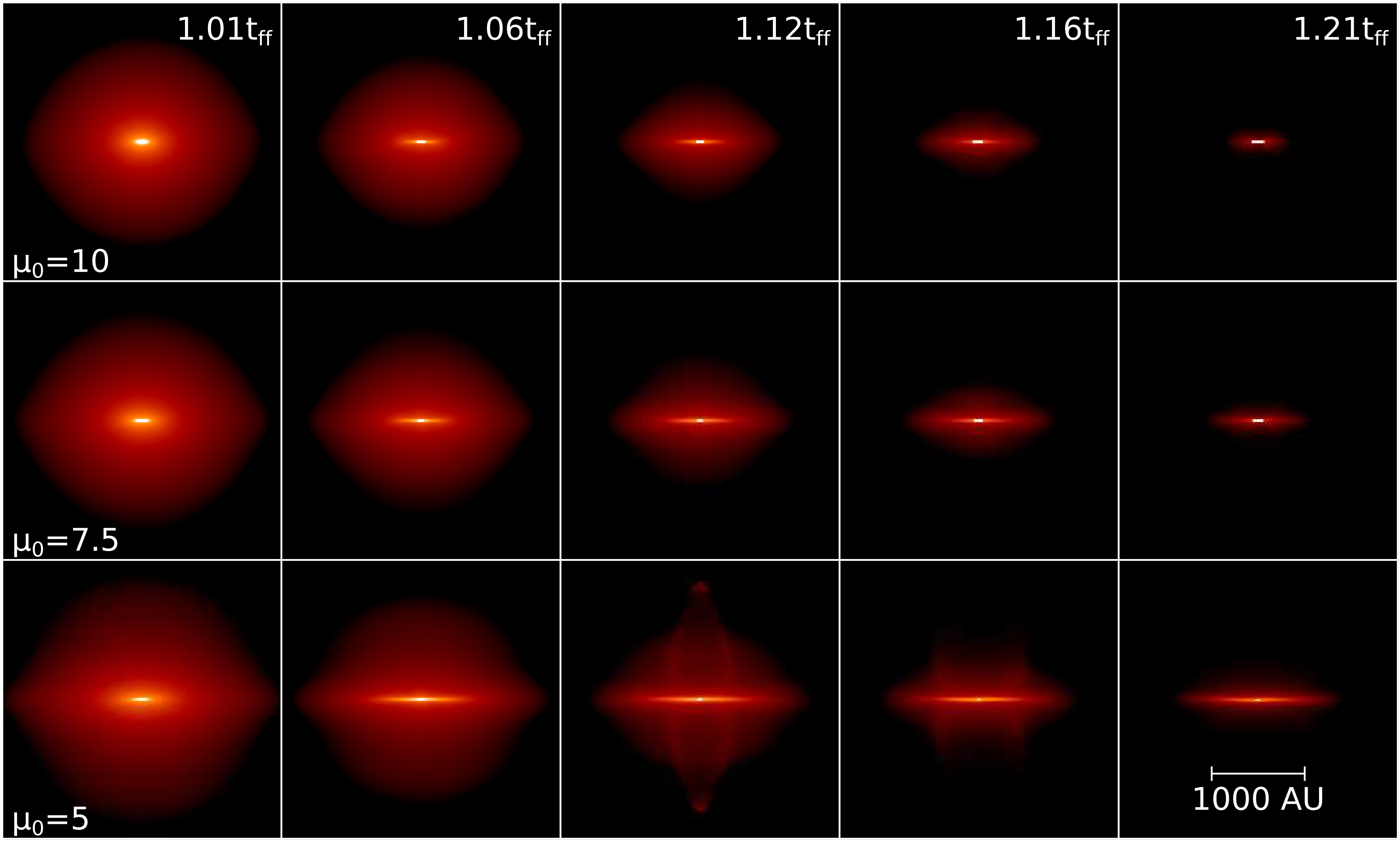


OUTFLOWS: NON-IDEAL MHD / ALIGNED INITIAL FIELD



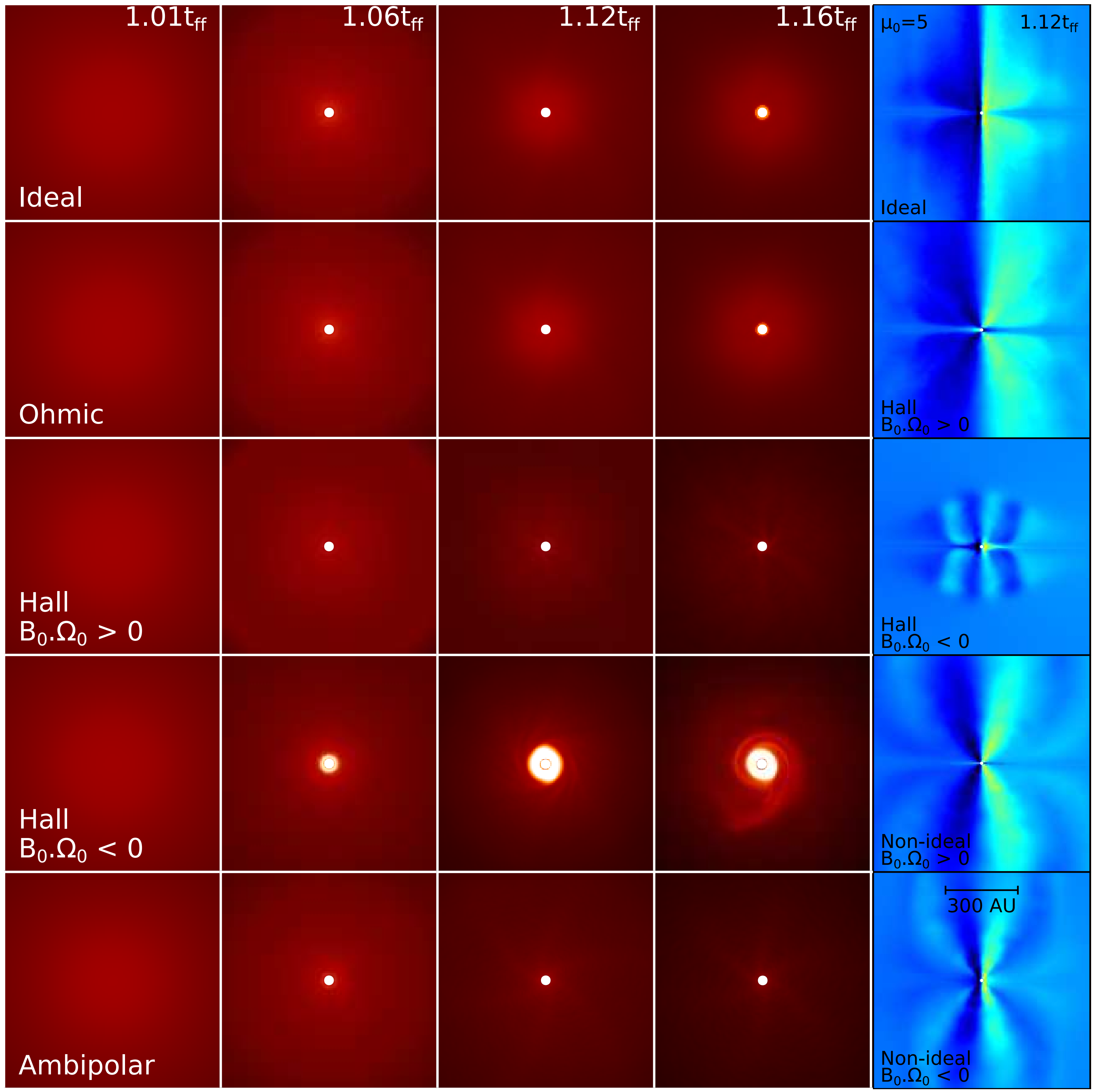
Wurster, Price & Bate (2016)

OUTFLOWS: NON-IDEAL MHD / ANTI-ALIGNED



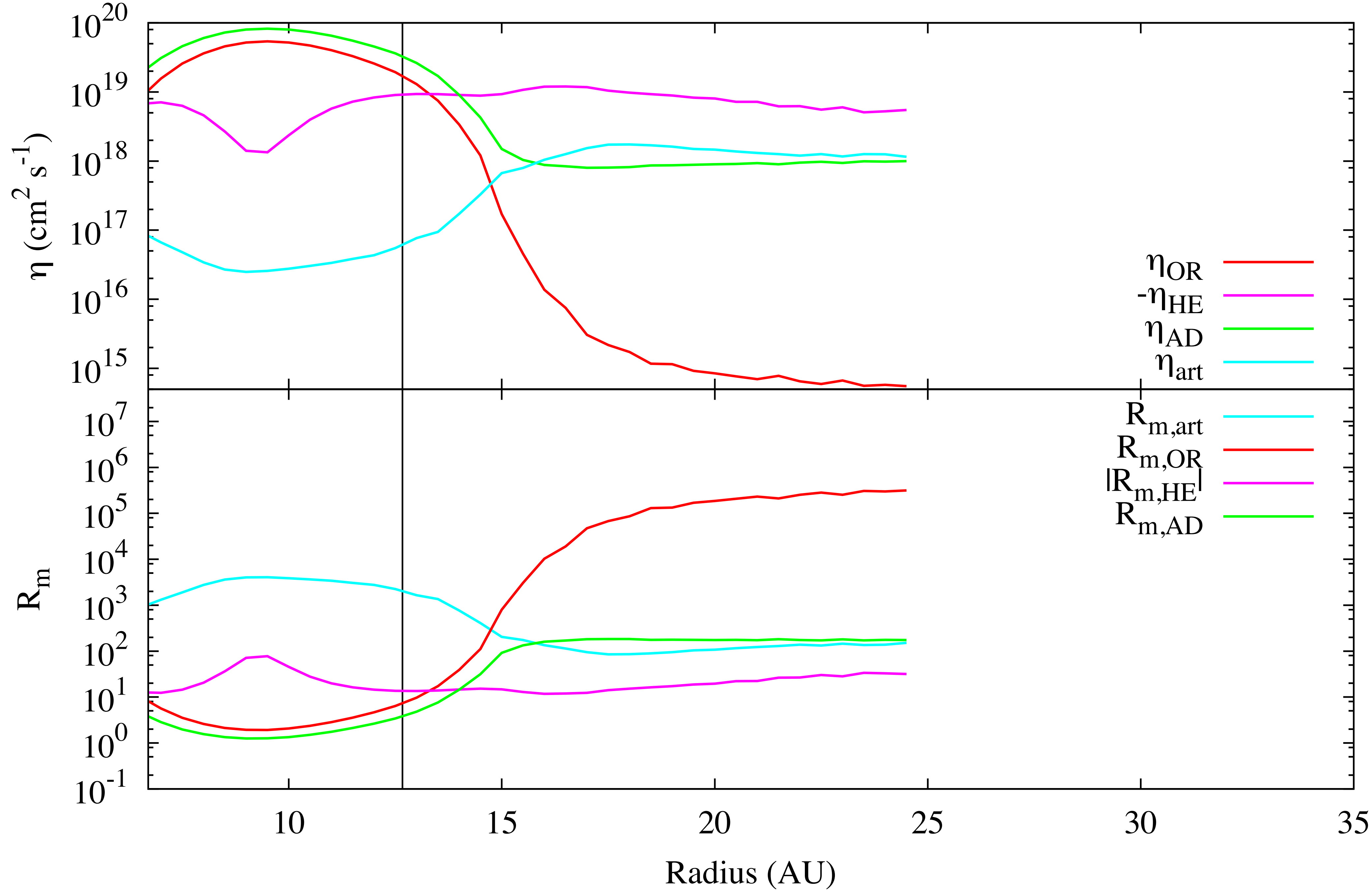
Outflows are anti-correlated with disc formation!

WHICH NON-IDEAL EFFECTS ARE IMPORTANT?



- Hall effect is dominant during disc formation
- Produces counter-rotating envelope when B and rotation are misaligned
- Maybe why half of all stars have planets?

WHICH NON-IDEAL EFFECTS ARE IMPORTANT?



CONCLUSIONS

- New "constrained" hyperbolic/parabolic divergence cleaning
- Can now perform realistic ideal and non-ideal Smoothed Particle Magnetohydrodynamics simulations
- Phantom SPMHD code available on request (public soon)
- Non-ideal MHD, in particular the Hall effect, plays a crucial role in the formation of protostellar discs