Dust

Daniel Price
Monash University
Melbourne, Australia

Guillaume Laibe
Monash University
Now St Andrew's, Scotland


HST/NASA

ASTRONUM 2014, June 23rd-27th Long Beach CA

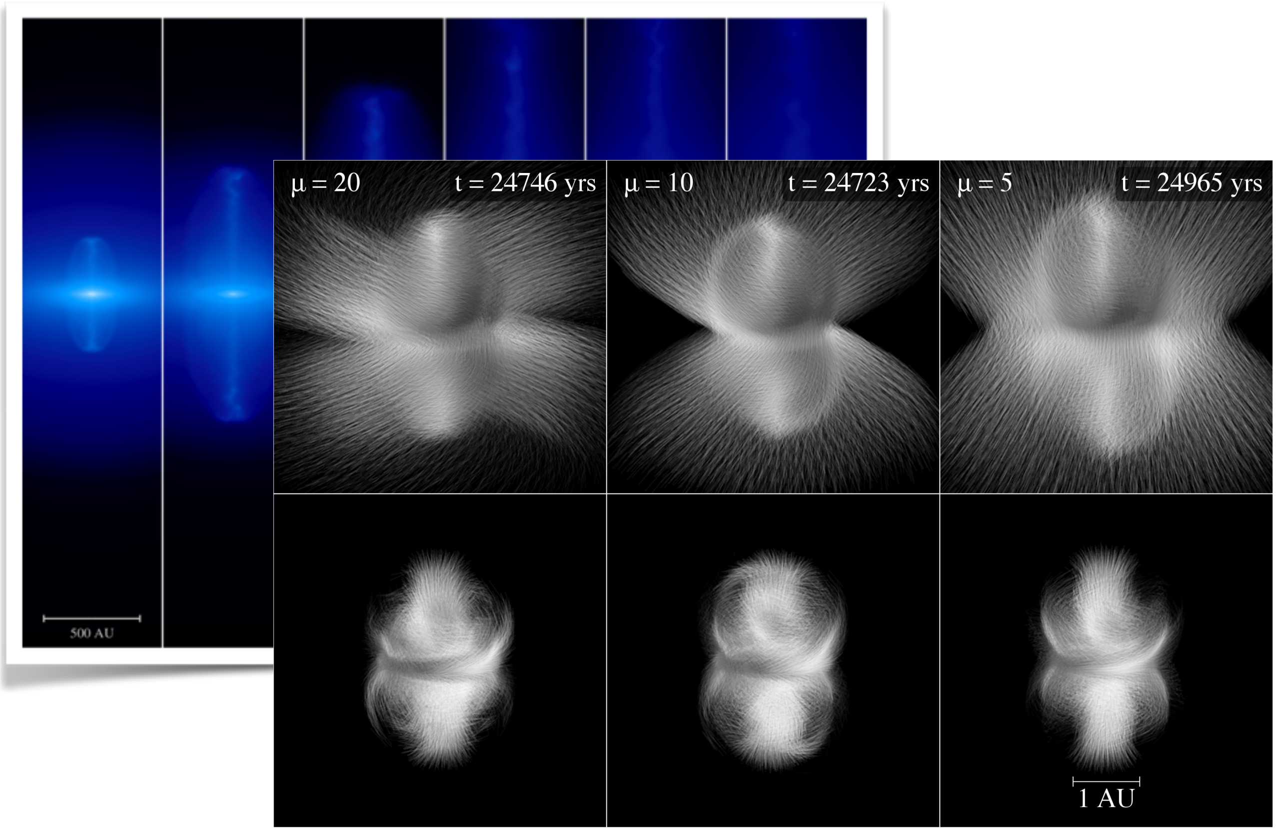
Things I am not going to talk about

Status of Smoothed Particle Magnetohydrodynamics

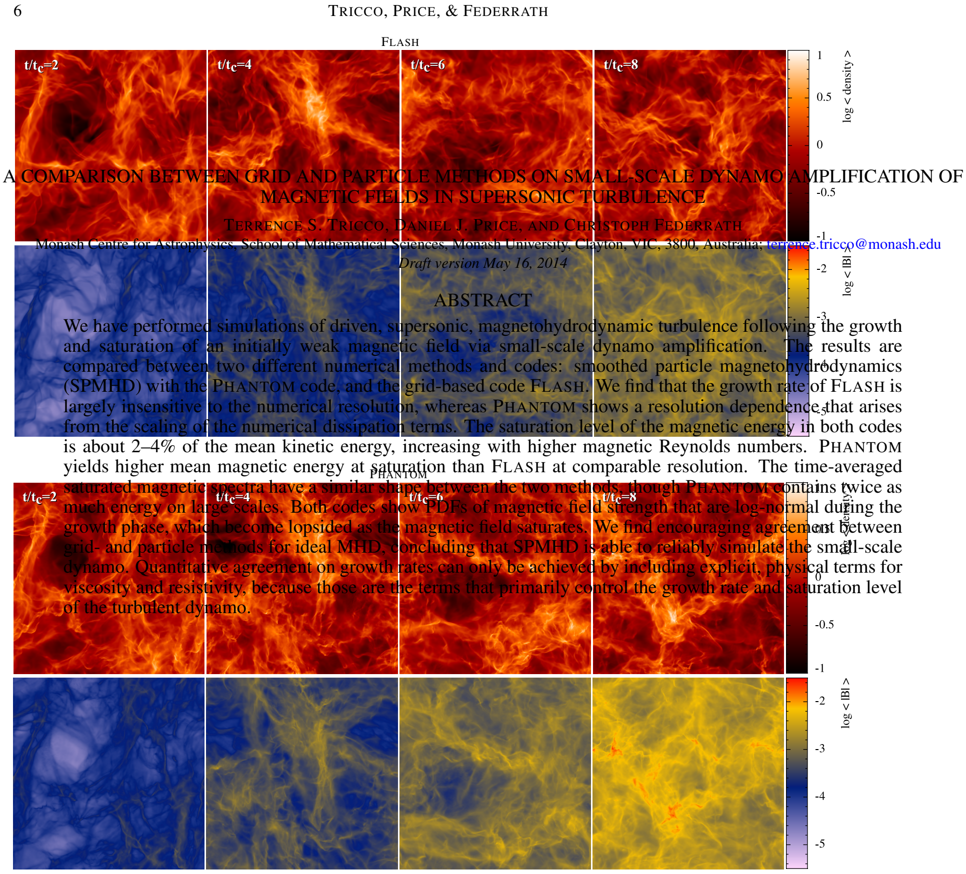
- New "constrained" hyperbolic divergence cleaning (Tricco & Price 2012 JCP 231, 7214)
- Improved switch for artificial resistivity (Tricco & Price 2013, MNRAS 436, 2810)
- Now able to robustly tackle variety of new & interesting applications
- Ambipolar diffusion (Wurster, Price & Ayliffe 2014, submitted to MNRAS)

Protostellar outflows with SPMHD

Price, Tricco & Bate (2012); Bate, Tricco & Price (2014)



Small-scale dynamo in SPMHD


6 TRICCO, PRICE, & FEDERRATH

A COMPARISON BETWEEN GRID AND PARTICLE METHODS ON SMALL-SCALE DYNAMO AMPLIFICATION OF MAGNETIC FIELDS IN SUPERSONIC TURBULENCE

TERRENCE S. TRICCO, DANIEL J. PRICE, AND CHRISTOPH FEDERRATH

Monash Centre for Astrophysics, School of Mathematical Sciences, Monash University, Clayton, VIC, 3800, Australia; terrence.tricco@monash.edu

Draft version May 16, 2014

ABSTRACT

We have performed simulations of driven, supersonic, magnetohydrodynamic turbulence following the growth and saturation of an initially weak magnetic field via small-scale dynamo amplification. The results are compared between two different numerical methods and codes: smoothed particle magnetohydrodynamics (SPMHD) with the PHANTOM code, and the grid-based code FLASH. We find that the growth rate of FLASH is largely insensitive to the numerical resolution, whereas PHANTOM shows a resolution dependence that arises from the scaling of the numerical dissipation terms. The saturation level of the magnetic energy in both codes is about 2–4% of the mean kinetic energy, increasing with higher magnetic Reynolds numbers. PHANTOM yields higher mean magnetic energy at saturation than FLASH at comparable resolution. The time-averaged saturated magnetic spectra have a similar shape between the two methods, though PHANTOM contains twice as much energy on large scales. Both codes show PDFs of magnetic field strength that are log-normal during the growth phase, which become lopsided as the magnetic field saturates. We find encouraging agreement between grid- and particle methods for ideal MHD, concluding that SPMHD is able to reliably simulate the small-scale dynamo. Quantitative agreement on growth rates can only be achieved by including explicit, physical terms for viscosity and resistivity, because those are the terms that primarily control the growth rate and saturation level of the turbulent dynamo.

Grid (FLASH)

vs.

SPMHD (Phantom)

MRI in SPMHD

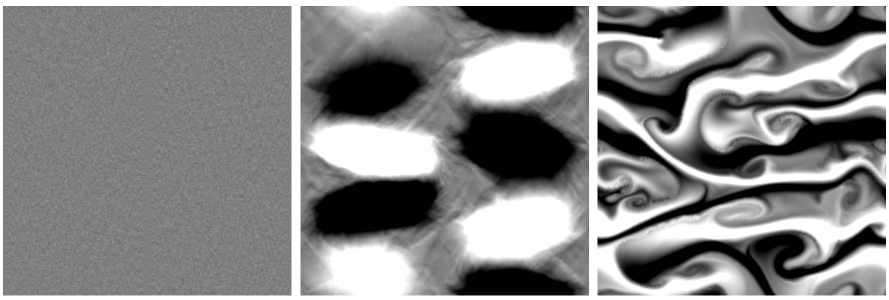
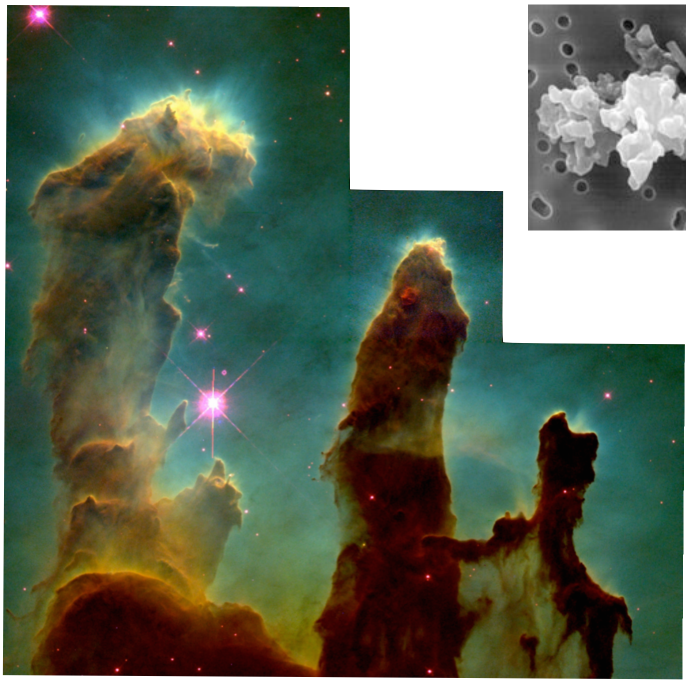


Figure 6.1: Snapshots of B_ϕ at $t = 1$, 20, and 25Ω for the 512^2 2D shearing box MRI test. Random small motions in the velocity lead to perturbations in the magnetic field ($t = 1\Omega$). These coalesce to form large structures ($t = 20\Omega$), which lead to the generation of turbulence ($t = 25\Omega$). Renderings are not all on the same scale.

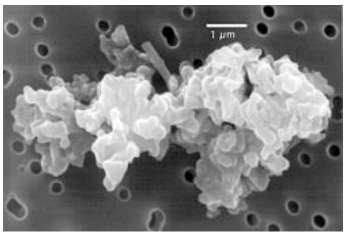
Tricco & Price (in prep.)

A Dusty Wedding

DUST IS KEY TO STAR AND PLANET



Pillars of Creation (the Eagle Nebula)



en.wikipedia.org

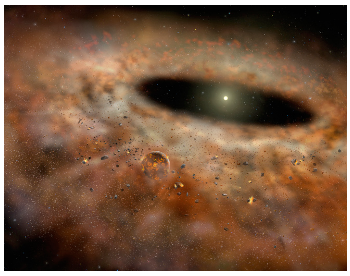
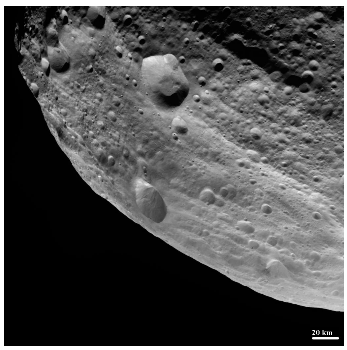


Image: Gemini Observatory/
AURA Artwork by Lynette Cook

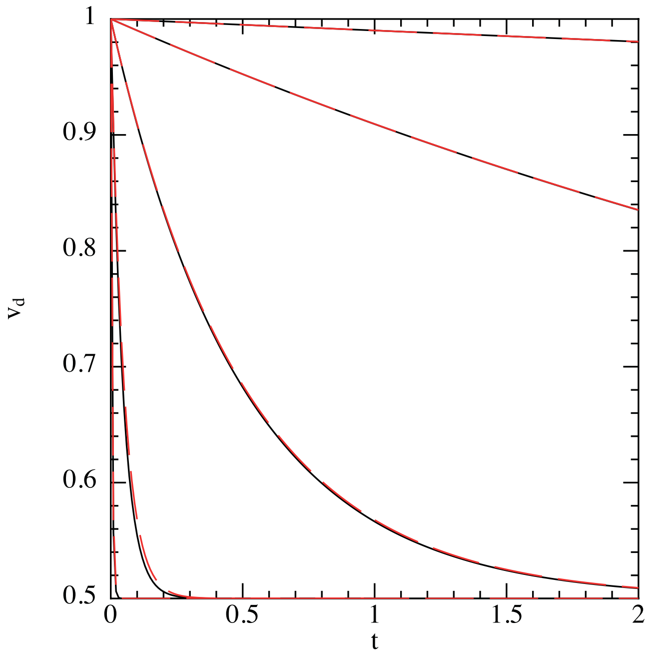


Dust + Gas: A simple example of a two-fluid mixture

- Two fluids coupled by a drag term

$$
\begin{aligned}
\frac{\partial \rho_{\rm g}}{\partial t} + \nabla.\left(\rho_{\rm g}\mathbf{v}_{\rm g}\right) &= 0, \\
\frac{\partial \rho_{\rm d}}{\partial t} + \nabla.\left(\rho_{\rm d}\mathbf{v}_{\rm d}\right) &= 0, \\
\frac{\partial \mathbf{v}_{\rm g}}{\partial t} + \left(\mathbf{v}_{\rm g}.\nabla\right)\mathbf{v}_{\rm g} &= -\frac{\nabla P_{\rm g}}{\rho_{\rm g}} + K(\mathbf{v}_{\rm d} - \mathbf{v}_{\rm g}) + \mathbf{f}, \\
\frac{\partial \mathbf{v}_{\rm d}}{\partial t} + \left(\mathbf{v}_{\rm d}.\nabla\right)\mathbf{v}_{\rm d} &= -K(\mathbf{v}_{\rm d} - \mathbf{v}_{\rm g}) + \mathbf{f},
\end{aligned}
$$

Stopping time



K=0.01

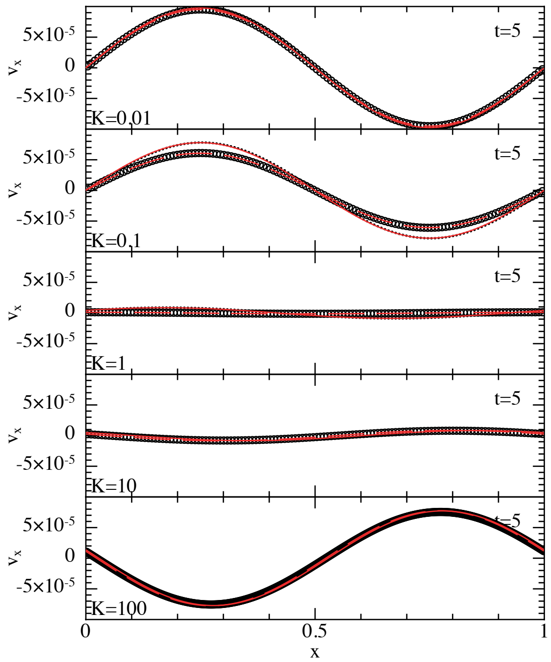
K=100

$$t_{\rm stop} \equiv \frac{\rho_{\rm d}\rho_{\rm g}}{K(\rho_{\rm d} + \rho_{\rm g})}$$

DUSTYWAVE: Waves in a dust-gas

Laibe & Price, 2011, MNRAS 418, 1491

$$\delta v = Ae^{i(kx-\omega t)}$$



Dispersion relation:

$$\omega^3 + iK\left(\frac{1}{\hat{\rho}_{\rm g}} + \frac{1}{\hat{\rho}_{\rm d}}\right)\omega^2 - k^2 c_{\rm s}^2 \omega - iK\frac{k^2 c_{\rm s}^2}{\hat{\rho}_{\rm d}} = 0$$

Limit of strong drag:

$$\omega = \pm k\tilde{c}_{\rm s} - i\frac{\hat{\rho}_{\rm g}\hat{\rho}_{\rm d}}{K\left(\hat{\rho}_{\rm g} + \hat{\rho}_{\rm d}\right)}k^2 c_{\rm s}^2 \left(\frac{1-A^2}{2}\right)$$

Effective sound speed:

$$\tilde{c}_{\rm s} \equiv c_{\rm s} A = c_{\rm s}\left(1 + \frac{\hat{\rho}_{\rm d}}{\hat{\rho}_{\rm g}}\right)^{-\frac{1}{2}}$$

Dustywaves: Analytic solution

Laibe & Price, 2011, MNRAS 418, 1491

```
!-------------------------------
! DUST VELOCITIES
!-------------------------------
vd3r = - (rhogeq*cs**2*k**2*w2r**2*w1r**2*rhogsol + rhogeq**2*w3i**4*k*w1r*vgsol - w3i*cs**2*k**3*rhogeq*Kdrag*vdsol*w2r - w3i*cs**2*k**3*rhogeq*Kdrag*vdsol*w1r +
w3i*cs**2*k**3*rhogeq*Kdrag*vgsol*w2r + w3i*cs**2*k**3*rhogeq*Kdrag*vgsol*w1r -rhogeq**2*cs**2*k**3*w2i**2*w1r*vgsol + rhogeq*cs**4*k**4*w2r*w1r*rhogsol -
rhogeq**2*cs**2*k**3*w2r**2*w1r*vgsol - rhogeq*cs**2*k**3*w2r*w1i*Kdrag*vgsol -rhogeq**2*cs**2*k**3*w2r*w1i**2*vgsol + rhogeq*cs**2*k**3*w2r*w1i*Kdrag*vdsol -
rhogeq**2*cs**2*k**3*w2r*w1r**2*vgsol - rhogeq*w3i**4*cs**2*k**2*rhogsol - rhogeq*cs**4*k**4*w2i*w1i*rhogsol + rhogeq*cs**2*k**2*w1i**2*w2r**2*rhogsol + rhogeq**2*w3i**4*w2r*k*vgsol -
rhogeq*w3i**3*k*Kdrag*vdsol*w1r +  2*rhogeq*w3i**3*k*Kdrag*vgsol*w2r + rhogeq*w2i**2*w1i**2*cs**2*k**2*rhogsol +  rhogeq*w2i**2*w1r**2*cs**2*k**2*rhogsol - rhogeq*w3i**4*w2r*w1r*rhogsol +
rhogeq*w3i**4*w1i*rhogsol*w2i +  w3i*cs**4*k**4*rhogeq*rhogsol*w2i + w3i*cs**4*k**4*rhogeq*rhogsol*w1i + w3i**2*Kdrag**2*k*vgsol*w1r -w3i**3*Kdrag*rhogsol*k**2*cs**2 -
w3i**3*Kdrag*rhogsol*w2r*w1r + w3i**3*Kdrag*rhogsol*w2i*w1i - w3i*cs**2*k**2*rhogeq*rhogsol*w2r**2*w1i - w3i*cs**2*k**2*rhogeq*rhogsol*w1i*w2i**2 +  rhogeq*w3i**3*rhogsol*k**2*cs**2*w2i +
rhogeq*w3i**3*rhogsol*k**2*cs**2*w1i - rhogeq*w3i**3*rhogsol*w2r**2*w1i  - rhogeq*w3i**3*rhogsol*w1i*w2i**2 - rhogeq*w3i**3*rhogsol*w2i*w1r**2 -  w3i*cs**2*k**2*rhogeq*rhogsol*w2i*w1r**2 -
w3i*cs**2*k**2*rhogeq*rhogsol*w2i*w1i**2 - rhogeq*w3i**3*rhogsol*w2i*w1i**2 - rhogeq*w3i**3*k*Kdrag*vdsol*w2r + 2*rhogeq*w3i**3*k*Kdrag*vgsol*w1r + w3r*rhogeq**2*cs**2*k**3*w2i**2*vgsol +
w3r*rhogeq*w2r*w3i**2*cs**2*k**2*rhogsol - w3r*rhogeq**2*w3i**2*k*w1r**2*vgsol + w3r*rhogeq*w3i**2*cs**2*k**2*rhogsol*w1r +  2*w3r*rhogeq*cs**2*k**3*w3i*Kdrag*vdsol -
w3r*rhogeq*cs**4*k**4*w2r*rhogsol +  w3r*rhogeq**2*cs**2*k**3*w2r**2*vgsol + w3r**4*rhogeq**2*w2r*vgsol*k - w3r**4*rhogeq*w2r*w1r*rhogsol +  w3r**4*rhogeq**2*k*w1r*vgsol -
w3r**4*rhogeq*rhogsol*k**2*cs**2 + w3r**4*rhogeq*w2i*rhogsol*w1i +2*w3r*rhogeq*w2i*w3i*w1i*k*Kdrag*vgsol + 2*w3r*rhogeq**2*w2i*k*vgsol*w3i*w1r**2 +
2*w3r*rhogeq*w2r*w3i*w1r*k*Kdrag*vdsol - 2*w3r*rhogeq*w2i*w3i**2*k*Kdrag*vgsol - w3r*w3i**2*Kdrag**2*k*vgsol + w3r*w3i**2*Kdrag**2*k*vdsol - w3r*rhogeq**2*w1i**2*k*vgsol*w3i**2 -
2*w3r*rhogeq**2*w3i**2*k*w2i*w1i*vgsol + 2*w3r*rhogeq**2*w2i*w1i**2*w3i*k*vgsol + w3r*w3i**2*Kdrag*rhogsol*w1i*w2r + w3r*w3i**2*Kdrag*rhogsol*w2i*w1r -
w3r*Kdrag*rhogsol*k**2*cs**2*w1i*w2r - w3r*Kdrag*rhogsol*k**2*cs**2*w2i*w1r +  w3r*Kdrag*k*rhogeq*vgsol*w2i*w1r**2 + w3r*Kdrag*k*rhogeq*vgsol*w2i*w1i**2 + w3r*Kdrag**2*k*vdsol*w2r*w1r
-w3r*Kdrag**2*k*vdsol*w2i*w1i - w3r*Kdrag**2*k*vgsol*w2r*w1r + w3r*Kdrag**2*k*vgsol*w2i*w1i + w3r*Kdrag*k*rhogeq*vgsol*w2r**2*w1i + w3r*Kdrag*k*rhogeq*vgsol*w1i*w2i**2 -
w3r*rhogeq**2*w2r**2*w3i**2*k*vgsol  - 2*w3r*rhogeq*w2r*w3i*w1r*k*Kdrag*vgsol + w3r*rhogeq*w2i*w3i**2*k*Kdrag*vdsol - 2*w3r*rhogeq**2*w2r*w3i**2*k*w1r*vgsol +
w3r*rhogeq*w2r**2*w3i**2*w1r*rhogsol -  2*w3r*rhogeq*w1i*w3i**2*k*Kdrag*vgsol + w3r*rhogeq*w1i*w3i**2*k*Kdrag*vdsol + 2*w3r*rhogeq**2*w3i*w2r**2*w1i*k*vgsol +
w3r*rhogeq*w1i**2*w3i**2*w2r*rhogsol - w3r*rhogeq**2*w3i**2*w2i**2*k*vgsol - w3r*rhogeq*cs**2*k**2*w1i**2*w2r*rhogsol -   w3r*rhogeq*cs**2*k**2*w2r*w1r**2*rhogsol -
w3r*rhogeq*cs**2*k**2*w2r**2*w1r*rhogsol + w3r*rhogeq*w2i**2*w3i**2*w1r*rhogsol + w3r*rhogeq*w2r*w3i**2*w1r**2*rhogsol - 2*w3r*rhogeq**2*cs**2*k**3*w3i*w2i*vgsol -
w3r*rhogeq*w2i**2*w1r*cs**2*k**2*rhogsol +  2*w3r*rhogeq**2*w2i**2*w3i*w1i*k*vgsol + w3r*rhogeq*cs**2*k**3*w1i*Kdrag*vgsol - w3r*rhogeq*cs**2*k**3*w2i*Kdrag*vdsol +
w3r*rhogeq*cs**2*k**3*w2i*Kdrag*vgsol - w3r*rhogeq*cs**2*k**3*w1i*Kdrag*vdsol + w3r*rhogeq**2*cs**2*k**3*w1i**2*vgsol -  2*w3r*rhogeq**2*cs**2*k**3*w3i*w1i*vgsol +
w3r*rhogeq**2*cs**2*k**3*w1r**2*vgsol - w3r*rhogeq*cs**4*k**4*w1r*rhogsol - 2*w3r*rhogeq*cs**2*k**3*w3i*Kdrag*vgsol + 2*w3r*rhogeq**2*cs**2*k**3*w2r*w1r*vgsol -
2*w3r*rhogeq*w2i*w3i*w1i*k*Kdrag*vdsol + 2*w3r*rhogeq**2*cs**2*k**3*w2i*w1i*vgsol + w3r**2*w3i*rhogeq*rhogsol*k**2*cs**2*w1i - w3r**2*w3i*rhogeq*rhogsol*w2r**2*w1i -
w3r**2*w3i*rhogeq*rhogsol*w1i*w2i**2 - w3r**2*w3i*rhogeq*rhogsol*w2i*w1r**2 - w3r**2*w3i*rhogeq*rhogsol*w2i*w1i**2 - w3r**2*w3i*rhogeq*k*Kdrag*vdsol*w2r -
w3r**2*Kdrag*rhogsol*w3i*w2r*w1r + w3r**2*Kdrag*rhogsol*w2i*w3i*w1i - w3r**2*w3i*rhogeq*k*Kdrag*vdsol*w1r + w3r**2*w3i*rhogeq*rhogsol*k**2*cs**2*w2i -
w3r**2*Kdrag*rhogsol*k**2*cs**2*w3i + w3r**2*Kdrag*rhogsol*k**2*cs**2*w2i + w3r**2*Kdrag*rhogsol*k**2*cs**2*w1i + 2*w3r**2*Kdrag*k*rhogeq*vgsol*w2r*w3i +
2*w3r**2*Kdrag*k*rhogeq*vgsol*w3i*w1r + 2*w3r**2*rhogeq**2*w2r*k*vgsol*w3i**2 - w3r**2*rhogeq**2*cs**2*k**3*w2r*vgsol + w3r**2*rhogeq*w2r*w1i*k*Kdrag*vgsol +
w3r**2*rhogeq**2*w2r*w1i**2*k*vgsol - w3r**2*rhogeq*w2r*w1i*k*Kdrag*vdsol + w3r**2*rhogeq**2*w2r*w1r**2*k*vgsol - w3r**2*rhogeq**2*cs**2*k**3*w1r*vgsol - w3r**2*Kdrag**2*k*vdsol*w2r -
w3r**2*rhogeq*w2i**2*w1i**2*rhogsol - w3r**2*rhogeq*w1r**2*w2i**2*rhogsol - w3r**2*rhogeq*w2r**2*w1r**2*rhogsol + w3r**2*rhogeq*cs**4*k**4*rhogsol - w3r**2*rhogeq*w1i**2*w2r**2*rhogsol -
w3r**2*Kdrag*rhogsol*w1i*w2i**2 - w3r**2*Kdrag*rhogsol*w2i*w1r**2 - w3r**2*Kdrag*rhogsol*w2i*w1i**2 - w3r**2*Kdrag*rhogsol*w2r**2*w1i - w3r**2*Kdrag**2*k*vdsol*w1r +
w3r**2*Kdrag**2*k*vgsol*w2r + w3r**2*Kdrag**2*k*vgsol*w1r + w3r**3*Kdrag*rhogsol*w2i*w1r + w3r**3*rhogeq*w2i*k*Kdrag*vdsol - w3r**3*rhogeq**2*w2i**2*k*vgsol +
w3r**2*rhogeq**2*w2i**2*w1r*k*vgsol - 2*w3r**2*rhogeq*w2r*w3i**2*w1r*rhogsol + 2*w3r**2*rhogeq**2*w3i**2*k*w1r*vgsol + 2*w3r**2*rhogeq*w1i*w3i**2*rhogsol*w2i -
w3r**2*rhogeq*w2i*w1r*k*Kdrag*vdsol + w3r**2*rhogeq*w2i*w1r*k*Kdrag*vgsol - w3r**3*rhogeq**2*w1i**2*k*vgsol - 2*w3r**3*rhogeq**2*w2r*k*w1r*vgsol + w3r**3*rhogeq*w2r**2*w1r*rhogsol +
w3r**3*rhogeq*w2r*cs**2*k**2*rhogsol - 2*w3r**3*rhogeq*w2i*k*Kdrag*vgsol - w3r**3*rhogeq**2*w1r**2*k*vgsol + w3r**3*rhogeq*w1r*cs**2*k**2*rhogsol - w3r**3*rhogeq**2*w2r**2*k*vgsol +
w3r**3*rhogeq*w1i*k*Kdrag*vdsol - 2*w3r**3*rhogeq**2*k*w2i*w1i*vgsol + w3r**3*rhogeq*w2r*w1r**2*rhogsol + w3r**3*rhogeq*w1i**2*w2r*rhogsol + w3r**3*rhogeq*w2i**2*w1r*rhogsol -
2*w3r**3*rhogeq*w1i*k*Kdrag*vgsol + w3r**3*Kdrag*rhogsol*w1i*w2r + w3r**2*rhogeq**2*w2r**2*k*w1r*vgsol - 2*w3r**2*rhogeq*w3i**2*cs**2*k**2*rhogsol + w3r**3*Kdrag**2*k*vdsol +
w3i**2*rhogeq**2*cs**2*k**3*w2r*vgsol - w3r**3*Kdrag**2*k*vgsol + w3i**2*rhogeq*w2i*w1r*k*Kdrag*vdsol - w3i**2*Kdrag**2*k*vdsol*w2r + w3i**2*Kdrag**2*k*vgsol*w2r -
w3i**2*Kdrag*rhogsol*w2r**2*w1i - w3i**2*Kdrag*rhogsol*w1i*w2i**2 - w3i**2*Kdrag*rhogsol*w2i*w1r**2 - w3i**2*Kdrag*rhogsol*w2i*w1i**2 + w3i**2*Kdrag*rhogsol*k**2*cs**2*w2i +
w3i**2*Kdrag*rhogsol*k**2*cs**2*w1i - Kdrag*k*rhogeq*vgsol*w3i*w1r*w2r**2 - Kdrag*k*rhogeq*vgsol*w3i*w2r*w1r**2 - Kdrag*k*rhogeq*vgsol*w3i*w2r*w1i**2 -
Kdrag*k*rhogeq*vgsol*w3i*w1r*w2i**2 + Kdrag*rhogsol*k**2*cs**2*w3i*w2r*w1r - Kdrag*rhogsol*k**2*cs**2*w2i*w3i*w1i + Kdrag*rhogsol*w2r**2*w1i**2*w3i + Kdrag*rhogsol*w3i*w1r**2*w2r**2 +
Kdrag*rhogsol*w1i**2*w2i**2*w3i + Kdrag*rhogsol*w2i**2*w1r**2*w3i + Kdrag**2*k*vdsol*w2r*w1i*w3i + Kdrag**2*k*vdsol*w2i*w3i*w1r - Kdrag**2*k*vgsol*w2r*w1i*w3i -
Kdrag**2*k*vgsol*w2i*w3i*w1r - w3i**2*Kdrag**2*k*vdsol*w1r + rhogeq*cs**2*k**3*w2i*w1r*Kdrag*vdsol - rhogeq*cs**2*k**3*w2i*w1r*Kdrag*vgsol - w3i**2*rhogeq**2*w2i**2*w1r*k*vgsol -
w3i**2*rhogeq**2*w2r**2*k*w1r*vgsol - w3i**2*rhogeq*w2i*w1r*k*Kdrag*vgsol - w3i**2*rhogeq*w2r*w1i*k*Kdrag*vgsol - w3i**2*rhogeq**2*w2r*w1i**2*k*vgsol +
w3i**2*rhogeq*w2r*w1i*k*Kdrag*vdsol - w3i**2*rhogeq**2*w2r*w1r**2*k*vgsol + w3i**2*rhogeq**2*cs**2*k**3*w1r*vgsol + w3i**2*rhogeq*w2i**2*w1i**2*rhogsol +
w3i**2*rhogeq*w1r**2*w2i**2*rhogsol + w3i**2*rhogeq*w2r**2*w1r**2*rhogsol - w3i**2*rhogeq*cs**4*k**4*rhogsol + w3i**2*rhogeq*w1i**2*w2r**2*rhogsol)/(w1i**2 - 2*w3i*w1i + w3r**2 + w1r**2 +
w3i**2 - 2*w3r*w1r)/(w2r**2 - 2*w3r*w2r + w2i**2 + w3i**2 - 2*w2i*w3i + w3r**2)/k/rhogeq/Kdrag
```

Resolution study

Laibe & Price, 2012, MNRAS 420, 2345

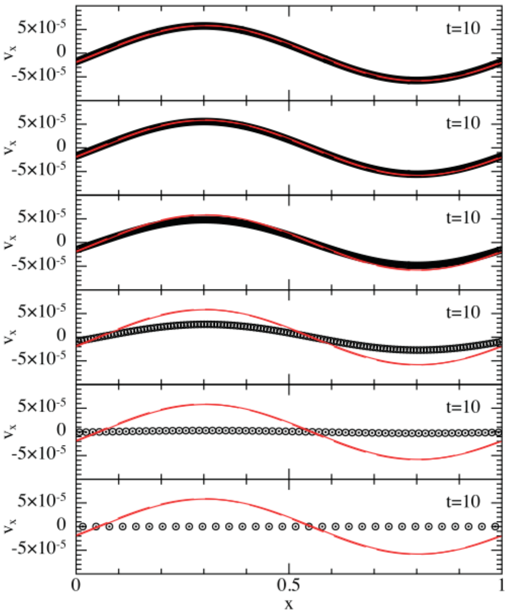
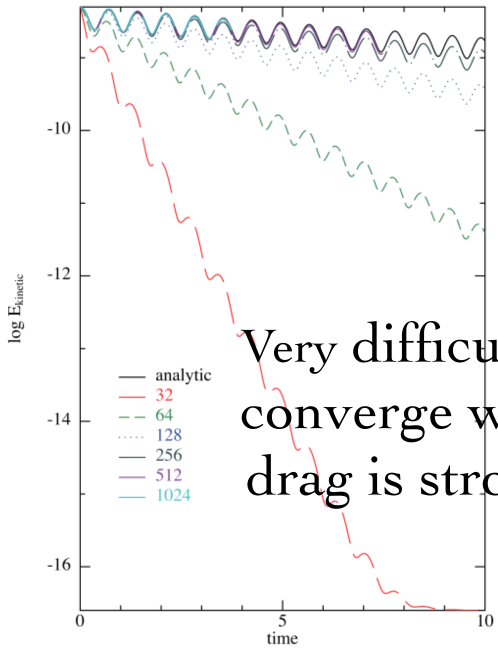


Figure 8. Resolution study for the DUSTYWAVE test in one dimension using a high drag coefficient ($K = 100$) and a dust-to-gas ratio of unity using 32, 64, 128, 256, 512 and 1024 particles from bottom to top. At large drag, high resolution is required to resolve the small differential motions between the fluids and thus to prevent overdamping of the numerical solution, corresponding to the criterion $h \lesssim c_s t_s$, here implying $\gtrsim$240 particles. See also Fig. 9.



Very difficult to converge when drag is strong!

Figure 9. As in Fig. 8 but showing the kinetic energy as a function of time in the numerical solution at a progressively increasing resolution, compared to the analytic solution given by the solid black line. The kinetic energy decay converges to the analytic solution at ~256–512 particles per wavelength, implying a demanding resolution criterion ($h \lesssim c_s t_s$) for high drag.

Resolution Criterion

Laibe & Price, 2012, MNRAS 420, 2345

Temporal: $\Delta t < t_{\rm stop}$ (can be fixed with implicit timestepping methods)

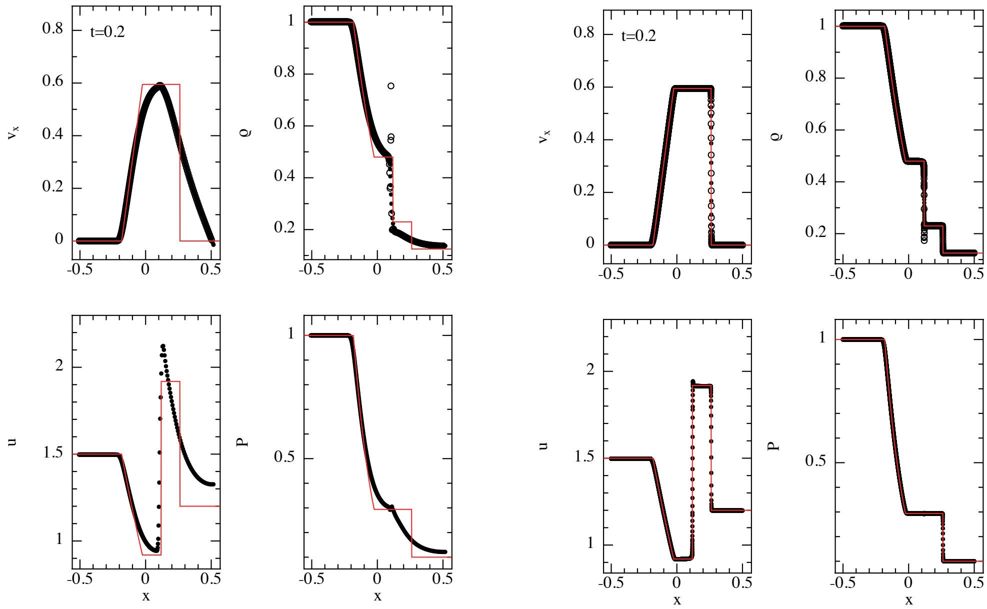
Spatial: $\Delta x \lesssim t_{\rm stop} c_{\rm s}$ (much more difficult to fix)

$t_{\rm stop} \to 0$ $(K \to \infty)$ implies $\Delta t \to 0$, $\Delta x \to 0$

- Require infinite timesteps AND infinite resolution in the obvious limit of perfect coupling!

Dustyshock

Laibe & Price, 2012, MNRAS 420, 2345



sensible resolution ludicrous resolution

Dusty Gas with One Fluid

Laibe & Price (2014a,b, MNRAS 440, 2136-2163

- Reformulate equations on the barycentre of both fluids

$$\mathbf{v} \equiv \frac{\rho_{\rm g}\mathbf{v}_{\rm g} + \rho_{\rm d}\mathbf{v}_{\rm d}}{\rho_{\rm g} + \rho_{\rm d}}$$

- Change of variables, from $\mathbf{v}_{\rm g}, \mathbf{v}_{\rm d}, \rho_{\rm g}, \rho_{\rm d}$

  to $\mathbf{v}, \Delta\mathbf{v}, \rho, \rho_{\rm d}/\rho_{\rm g}$

TWO BECOME ONE

A phoenix from the ashes

- Two fluids coupled by a drag term

$$
\begin{aligned}
\frac{\partial \rho_{\rm g}}{\partial t} + \nabla . \left(\rho_{\rm g} \mathbf{v}_{\rm g}\right) &= 0, \\
\frac{\partial \rho_{\rm d}}{\partial t} + \nabla . \left(\rho_{\rm d} \mathbf{v}_{\rm d}\right) &= 0, \\
\frac{\partial \mathbf{v}_{\rm g}}{\partial t} + \left(\mathbf{v}_{\rm g} . \nabla\right) \mathbf{v}_{\rm g} &= -\frac{\nabla P_{\rm g}}{\rho_{\rm g}} + K(\mathbf{v}_{\rm d} - \mathbf{v}_{\rm g}) + \mathbf{f}, \\
\frac{\partial \mathbf{v}_{\rm d}}{\partial t} + \left(\mathbf{v}_{\rm d} . \nabla\right) \mathbf{v}_{\rm d} &= -K(\mathbf{v}_{\rm d} - \mathbf{v}_{\rm g}) + \mathbf{f},
\end{aligned}
$$

TWO BECOME ONE

A phoenix from the ashes

- One mixture with a differential velocity

$$
\frac{{\rm d}\rho}{{\rm d}t} = -\rho(\nabla \cdot \mathbf{v}),
$$

$$
\frac{{\rm d}\mathbf{v}}{{\rm d}t} = -\frac{\nabla P_{\rm g}}{\rho} - \frac{1}{\rho}\nabla \cdot \left(\frac{\rho_{\rm g}\rho_{\rm d}}{\rho}\Delta\mathbf{v}\Delta\mathbf{v}\right) + \mathbf{f},
$$

$$
\frac{\rm d}{{\rm d}t}\left(\frac{\rho_{\rm d}}{\rho_{\rm g}}\right) = -\frac{1}{\rho}\nabla \cdot \left(\frac{\rho_{\rm g}\rho_{\rm d}}{\rho}\Delta\mathbf{v}\right),
$$

$$
\frac{{\rm d}\Delta\mathbf{v}}{{\rm d}t} = -\frac{\Delta\mathbf{v}}{t_{\rm s}} + \frac{\nabla P_{\rm g}}{\rho_{\rm g}} - (\Delta\mathbf{v} \cdot \nabla)\mathbf{v} + \frac{1}{2}\nabla\left[\frac{\rho_{\rm d} - \rho_{\rm g}}{\rho_{\rm g} + \rho_{\rm d}}\Delta\mathbf{v}^2\right],
$$

No approximations!

Laibe & Price (2014) MNRAS

Eulerian form

$$\frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot \mathbf{F} = \mathbf{S}$$

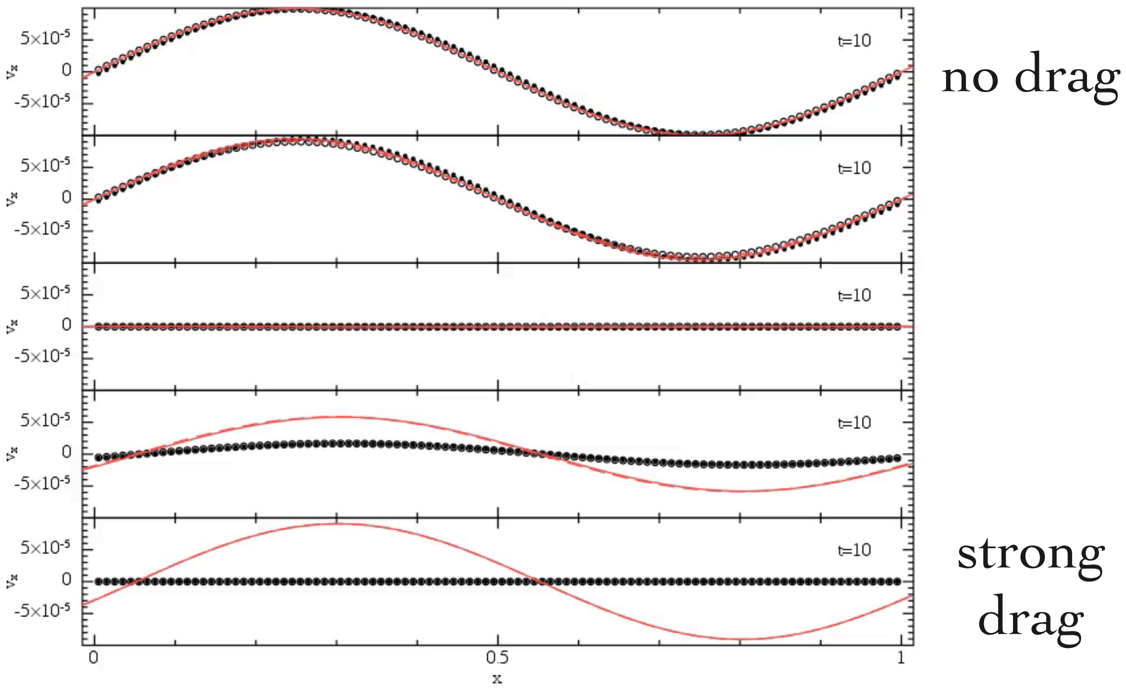
$$\mathbf{u} = \begin{bmatrix} \rho \\ \rho\epsilon \\ \rho\mathbf{v} \\ \rho\epsilon\mathbf{v}_{\rm d} \end{bmatrix} \quad \mathbf{F} = \begin{bmatrix} \rho\mathbf{v} \\ \rho\epsilon\mathbf{v}_{\rm d} \\ \rho\mathbf{v}\mathbf{v} + P\mathbf{I} + \rho\epsilon(1-\epsilon)\Delta\mathbf{v}\Delta\mathbf{v} \\ \rho\epsilon\mathbf{v}_{\rm d}\mathbf{v}_{\rm d} \end{bmatrix} \quad \mathbf{S} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -K\Delta\mathbf{v} \end{bmatrix}$$

$$\mathbf{v}_{\rm d} \equiv \mathbf{v} + (1-\epsilon)\Delta\mathbf{v} \qquad \epsilon \equiv \frac{\rho_{\rm d}}{\rho}$$

Laibe & Price (2014) MNRAS

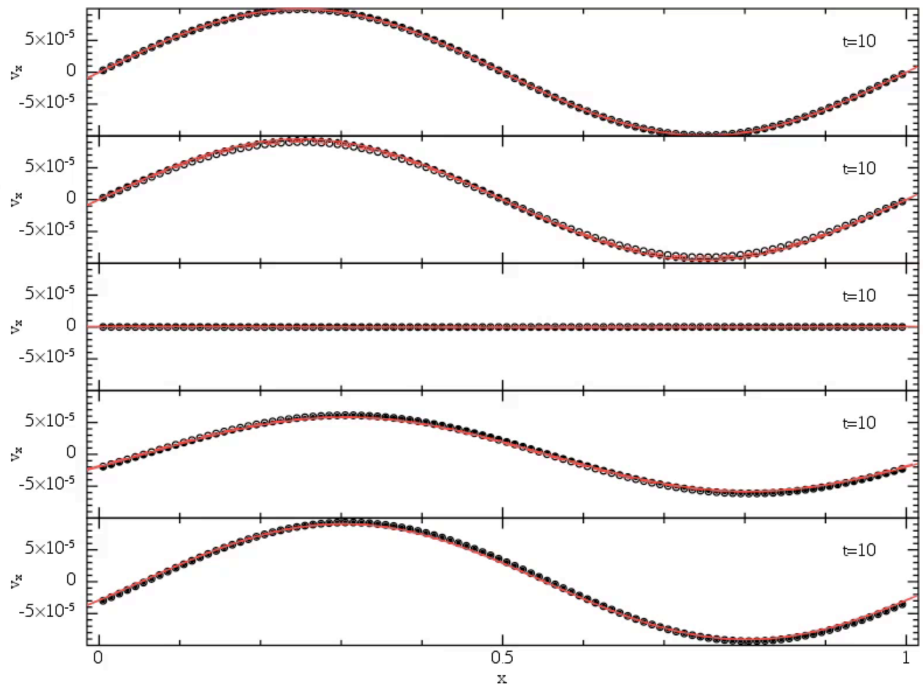
DUSTY WAVES: TWO FLUID

Laibe & Price (2012a)



DUSTY WAVES: ONE FLUID

Laibe & Price (2014b

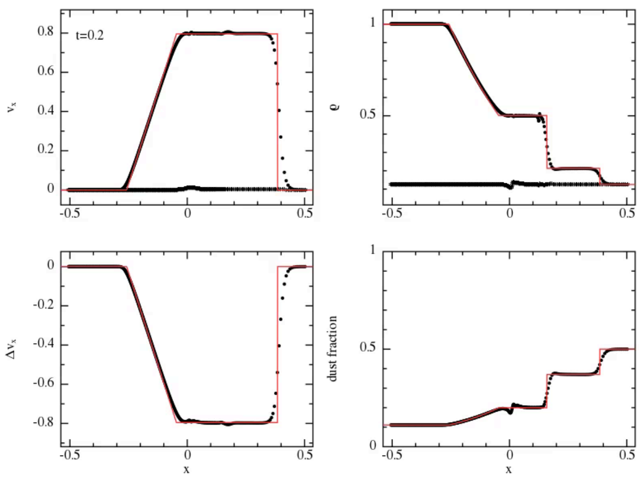


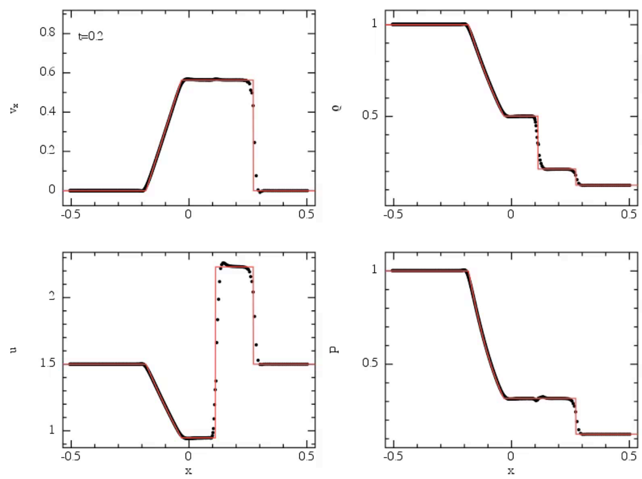
no drag

strong drag

Dustyshock with one fluid

Laibe & Price (2014b





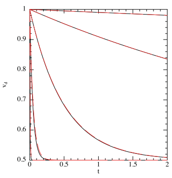
no drag

strong drag

No spatial resolution issue, but
still requires implicit timestepping

Terminal velocity approximation

- Assume $\Delta \mathbf{v} = \frac{\nabla P_{\rm g}}{\rho_{\rm g}} t_{\rm s}$, valid when $t_{\rm stop} < \Delta t$



$$\frac{{\rm d}\rho}{{\rm d}t} = -\rho(\nabla \cdot \mathbf{v})$$

$$\frac{{\rm d}\mathbf{v}}{{\rm d}t} = -\frac{\nabla P}{\rho} + \mathbf{f}$$

$$\frac{{\rm d}\epsilon}{{\rm d}t} = -\frac{1}{\rho}\nabla \cdot (\epsilon t_{\rm s} \nabla P) \qquad \epsilon \equiv \frac{\rho_{\rm d}}{\rho}$$

"Diffusion approximation for dust"

Breaks down when diffusion controls timestep!

Laibe & Price (2014)

See also Youdin & Goodman (2005); Chiang (2008); Barranco 2009, Jacquet et al. 2011

Zeroth order approximation

- Assume $\Delta \mathbf{v} = 0$, valid when $t_{\rm stop} \ll \Delta t$

$$\frac{{\rm d}\rho}{{\rm d}t} = -\rho(\nabla \cdot \mathbf{v})$$

$$\frac{{\rm d}\mathbf{v}}{{\rm d}t} = -\frac{\nabla P}{\rho} + \mathbf{f}$$

$$\frac{{\rm d}\epsilon}{{\rm d}t} = 0$$

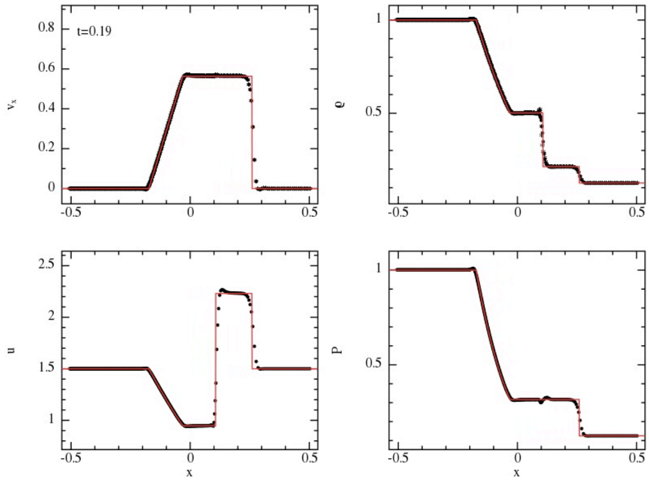
$$P = \tilde{c}_{\rm s}\rho$$

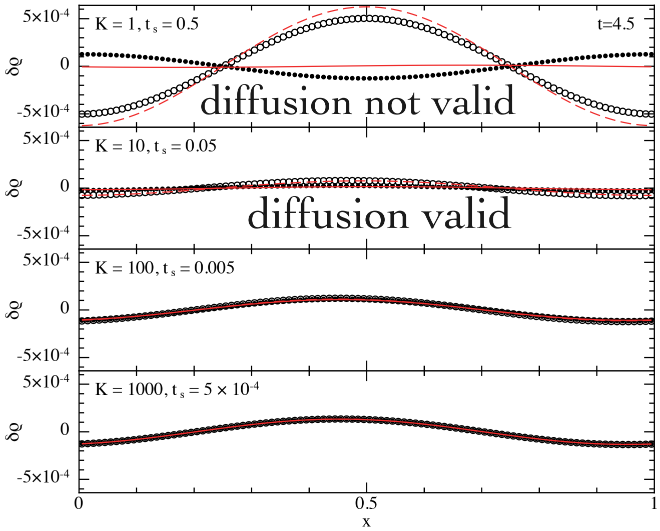
"Heavy fluid"

Laibe & Price (2014

Shock+wave

Price & Laibe (in prep.





Explicit timestepping only!

Relation to multi-fluid MHD?

$$\frac{\partial \rho_j}{\partial t} + \nabla \cdot (\rho_j \boldsymbol{v}_j) = 0, \quad (1)$$

$$\frac{\mathrm{d}\boldsymbol{v}_{\mathrm{e}}}{\mathrm{d}t} = -\frac{\nabla P_{\mathrm{e}}}{\rho_{\mathrm{e}}} - \frac{e}{m_{\mathrm{e}}}\left(\boldsymbol{E} + \frac{\boldsymbol{v}_{\mathrm{e}}}{c} \times \boldsymbol{B}\right) - \sum_{j=\mathrm{i,\ n}} \nu_{\mathrm{e}j}(\boldsymbol{v}_{\mathrm{e}} - \boldsymbol{v}_j), \quad (2)$$

$$\frac{\mathrm{d}\boldsymbol{v}_{\mathrm{i}}}{\mathrm{d}t} = -\frac{\nabla P_{\mathrm{i}}}{\rho_{\mathrm{i}}} + \frac{e}{m_{\mathrm{i}}}\left(\boldsymbol{E} + \frac{\boldsymbol{v}_{\mathrm{i}}}{c} \times \boldsymbol{B}\right) - \sum_{j=\mathrm{e,\ n}} \nu_{\mathrm{i}j}(\boldsymbol{v}_{\mathrm{i}} - \boldsymbol{v}_j), \quad (3)$$

$$\frac{\mathrm{d}\boldsymbol{v}_{\mathrm{n}}}{\mathrm{d}t} = -\frac{\nabla P_{\mathrm{n}}}{\rho_{\mathrm{n}}} + \sum_{j=\mathrm{e,\ i}} \nu_{\mathrm{n}j}(\boldsymbol{v}_j - \boldsymbol{v}_{\mathrm{n}}). \quad (4)$$

e.g. Pandey & Wardle (2008)

Multiple fluid -> single fluid

MHD

The continuity equation for the bulk fluid is obtained by summing up equation (1) for each species:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \, \boldsymbol{v}) = 0. \quad (11)$$

The momentum equation can be derived by adding equations (2)–(4) to obtain

$$\rho \frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} + \nabla \cdot \left(\frac{\rho_{\mathrm{i}} \rho_{\mathrm{n}}}{\rho} \boldsymbol{v}_{\mathrm{D}} \boldsymbol{v}_{\mathrm{D}} \right) = -\nabla P + \frac{\boldsymbol{J} \times \boldsymbol{B}}{c}, \quad (12)$$

where $P = P_{\mathrm{e}} + P_{\mathrm{i}} + P_{\mathrm{n}}$ is the total pressure, $\boldsymbol{v}_{\mathrm{D}} = \boldsymbol{v}_{\mathrm{i}} - \boldsymbol{v}_{\mathrm{n}}$ is the ion–neutral drift velocity and $\boldsymbol{J} = n_{\mathrm{e}} \, e \, (\boldsymbol{v}_{\mathrm{i}} - \boldsymbol{v}_{\mathrm{e}})$ is the current density.

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times \left[(\boldsymbol{v} \times \boldsymbol{B}) - \frac{4\pi\eta}{c} \boldsymbol{J} - \frac{4\pi\eta_{\mathrm{H}}}{c} \boldsymbol{J} \times \hat{\boldsymbol{B}} + \frac{4\pi\eta_{\mathrm{A}}}{c} \left(\boldsymbol{J} \times \hat{\boldsymbol{B}} \right) \times \hat{\boldsymbol{B}} \right],$$

Pandey & Wardle (2008)

SPMHD implementation: Wurster, Price & Ayliffe (2014)

DUST

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0,$$

$$\rho \frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} + \nabla \cdot \left[\frac{\rho_{\mathrm{g}} \rho_{\mathrm{d}}}{\rho} \Delta\mathbf{v} \Delta\mathbf{v} \right] = -\nabla P + \mathbf{f}$$

$$\frac{\mathrm{d}\epsilon}{\mathrm{d}t} = -\frac{1}{\rho} \nabla \cdot \left[\frac{\rho_{\mathrm{g}} \rho_{\mathrm{d}}}{\rho} \Delta\mathbf{v} \right]$$

Laibe & Price (2014)

Summary

- New single fluid formulation for dust-gas mixtures
- Solves both spatial and temporal resolution problems for small grains
- New "Diffusion approximation for dust"
- Similar to existing methods for non-ideal MHD

Corollary

- When diffusion controls the timestep, diffusion is the wrong approximation

Open-source implementation: ______________________________