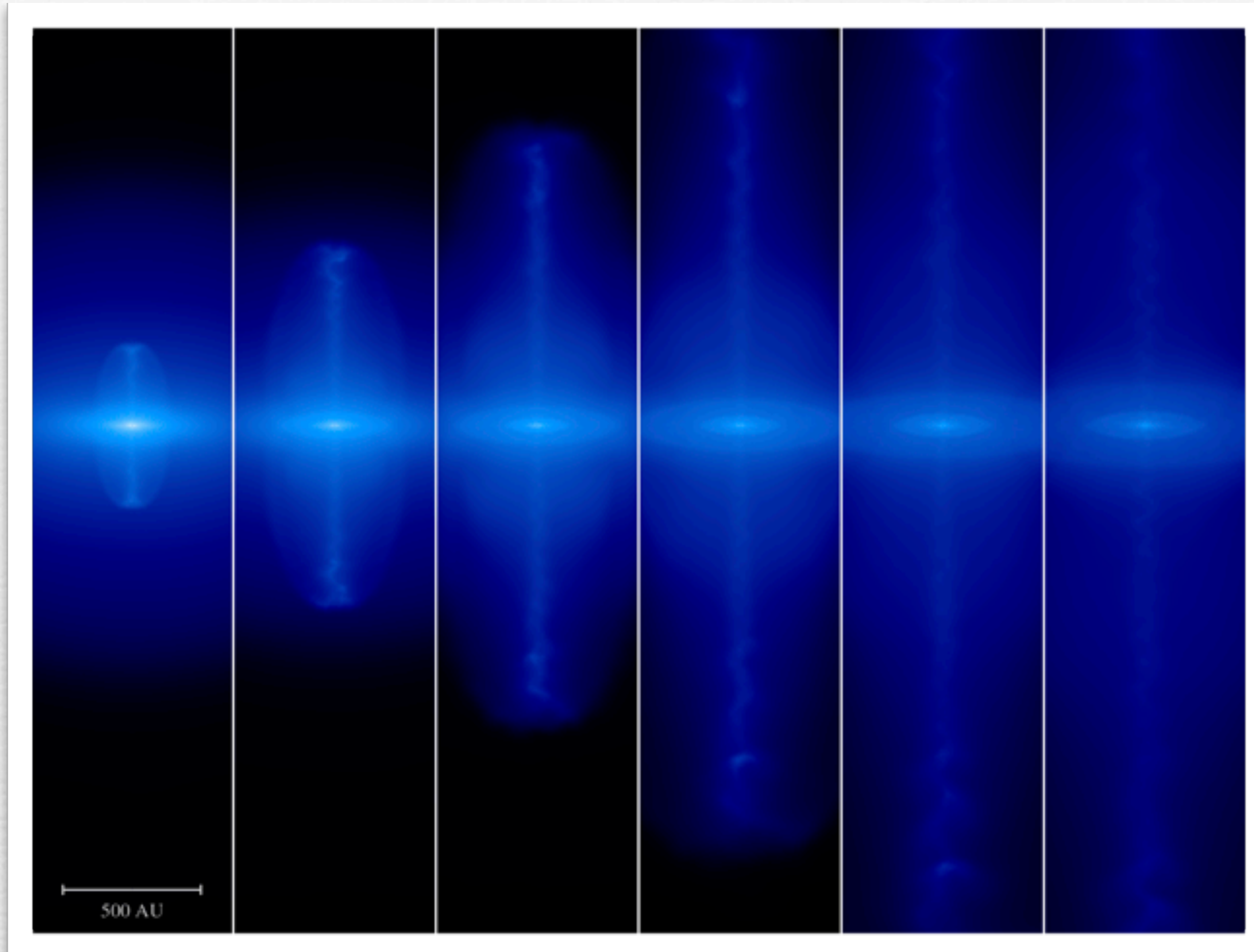


Smoothed Particle Magnetohydrodynamics: The state of the art

Daniel Price (Monash), Terry Tricco (Monash)
Matthew Bate (Exeter), Guillaume Laibe (Monash, now St. Andrews)

MoCA
Monash Centre for Astrophysics

ASTRONUM2013 - July 1st-5th 2013, Biarritz, France

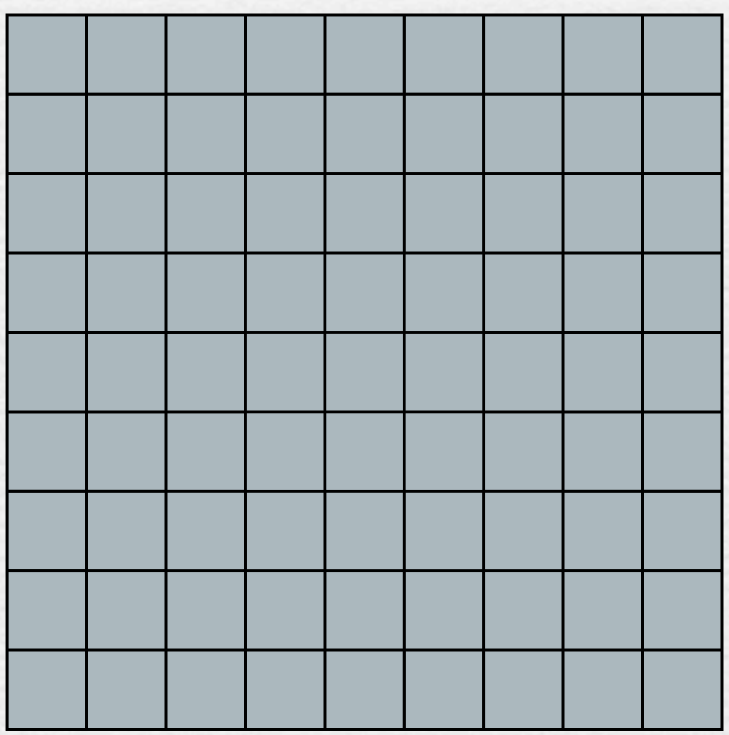
“There is no Solution without Mesh”

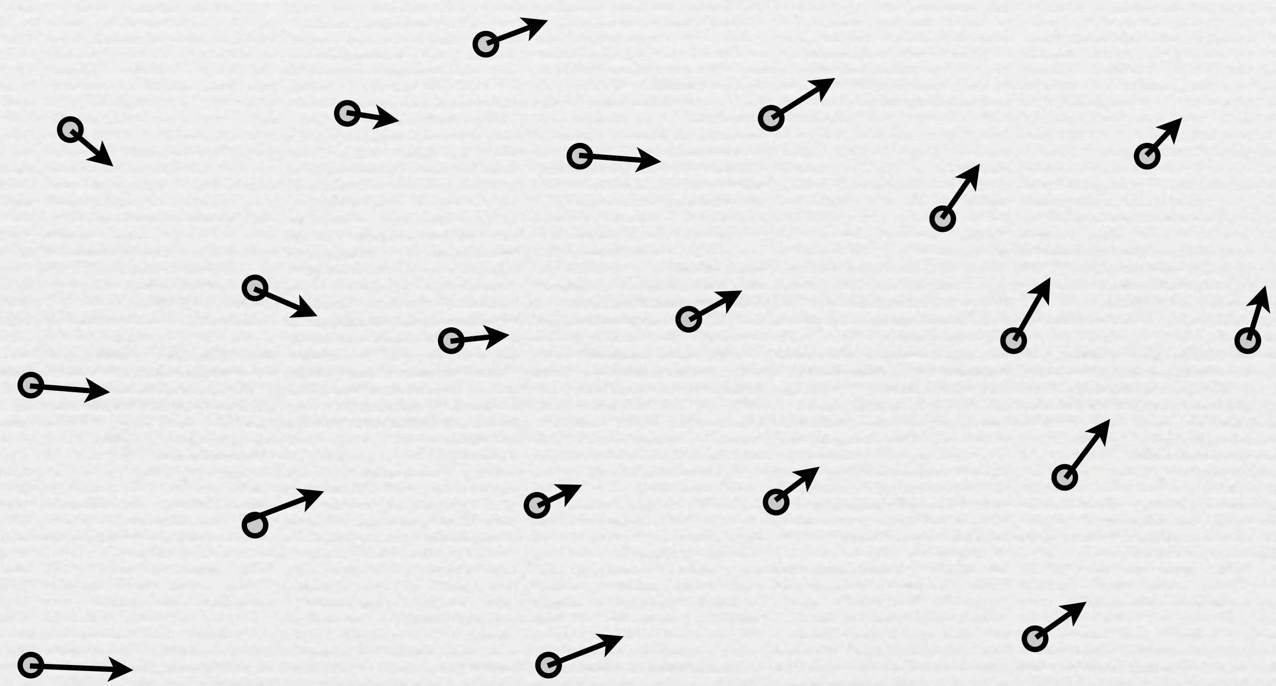
- Tahar Amari, yesterday

Smoothed Particle Hydrodynamics

Grid

SPH





$$\frac{\mathrm{d}\mathbf{x}}{\mathrm{d}t} = \mathbf{v}$$

Smoothed Particle Magnetohydrodynamics

Price & Monaghan 2004a,b,2005, Price 2012

$$L_{sph} = \sum_b m_b \left[\frac{1}{2} v_b^2 - u_b(\rho_b, s_b) - \frac{1}{2\mu_0} \frac{B_b^2}{\rho_b} \right]$$

$$\int \delta L \mathrm{dt} = 0 \qquad \mathrm{d}u = \frac{P}{\rho^2} \mathrm{d}\rho$$

$$\delta \rho_b = \sum_c m_c (\delta \mathbf{r}_b - \delta \mathbf{r}_c) \cdot \nabla_b W_{bc}$$

$$\delta \left(\frac{\mathbf{B}_b}{\rho_b} \right) = - \sum_c m_c (\delta \mathbf{r}_b - \delta \mathbf{r}_c) \frac{\mathbf{B}_b}{\rho_b} \cdot \nabla_b W_{bc}$$

Subtract B∇.B source term to correct tensile instability

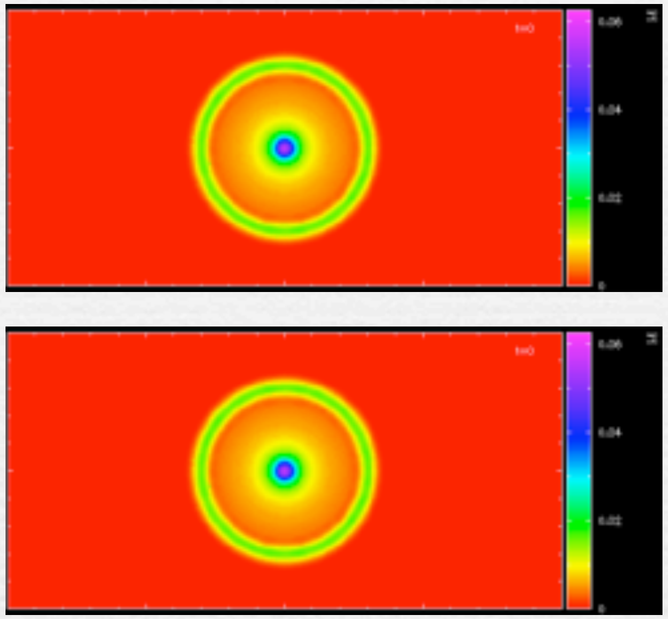
Add dissipation terms (viscosity, resistivity, conductivity) to capture shocks and discontinuities

$$\frac{dv_a^i}{dt} = - \sum_b m_b \left[\left(\frac{S^{ij}}{\rho^2} \right)_a + \left(\frac{S^{ij}}{\rho^2} \right)_b \right] \nabla_a^j W_{ab},$$

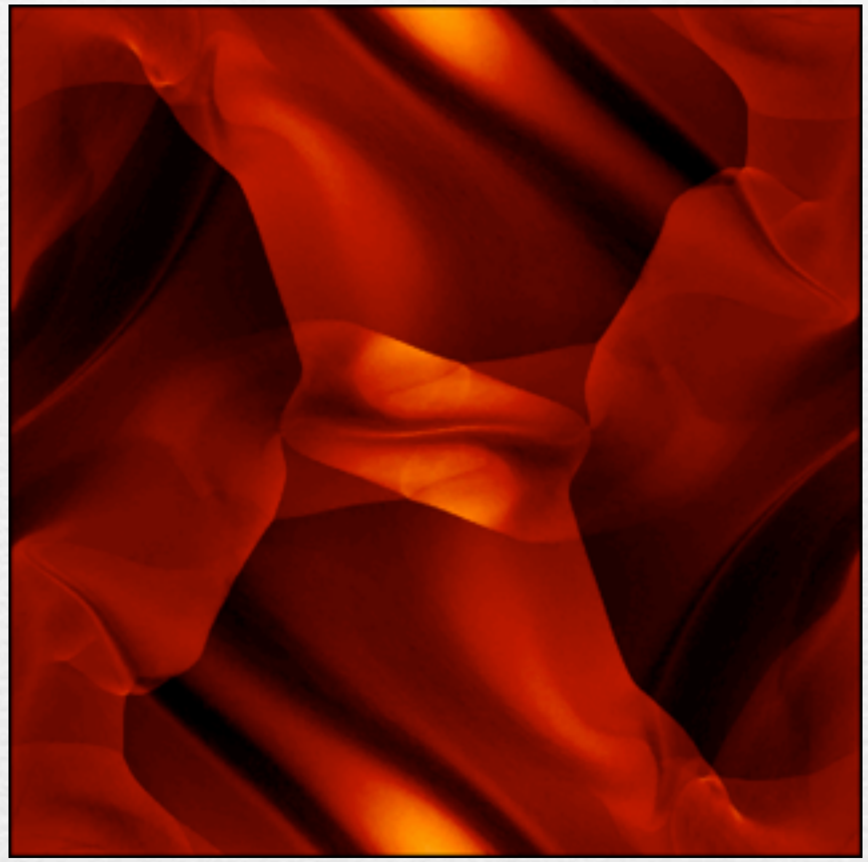
$$S_{ij} = \left(P + \frac{B^2}{2\mu_0} \right) \delta_{ij} - \frac{B_i B_j}{\mu_0}$$

Smoothed Particle Magnetohydrodynamics

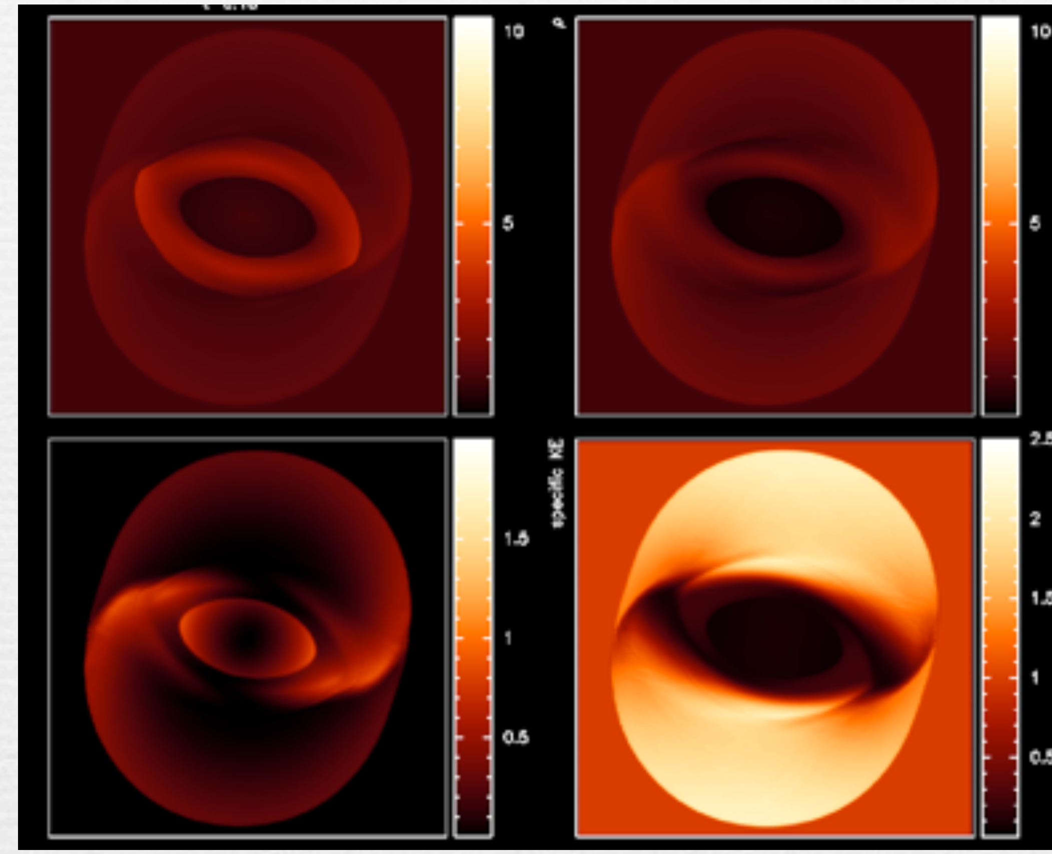
Price & Monaghan (2004a,b,2005), see review by Price 2012, J. Comp. Phys. 231, 759



advection of a current loop (Gardiner & Stone 2006, Rosswog & Price 2007)

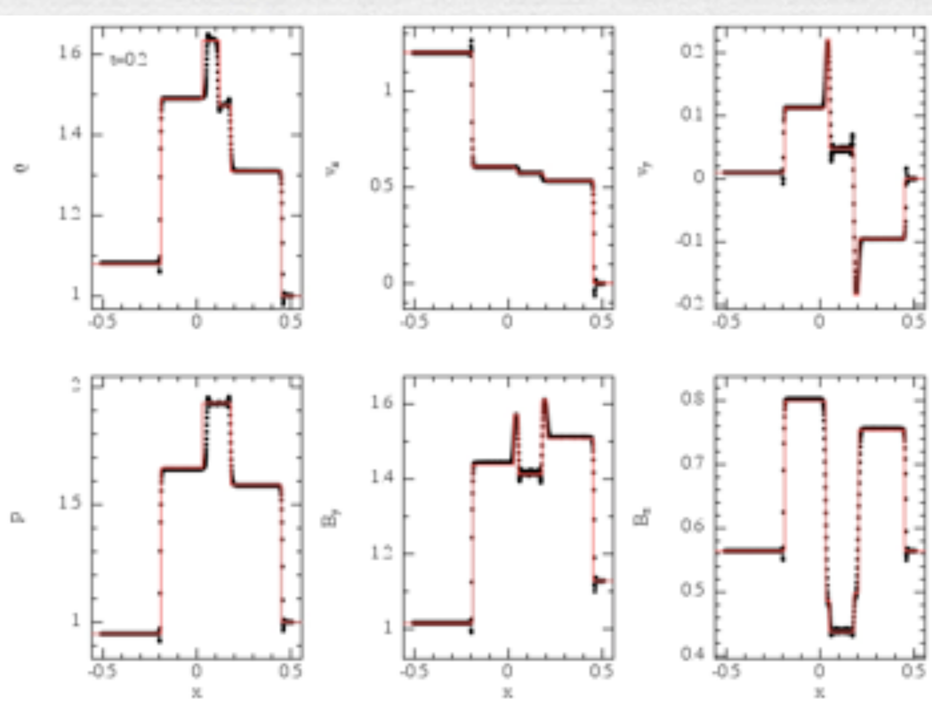


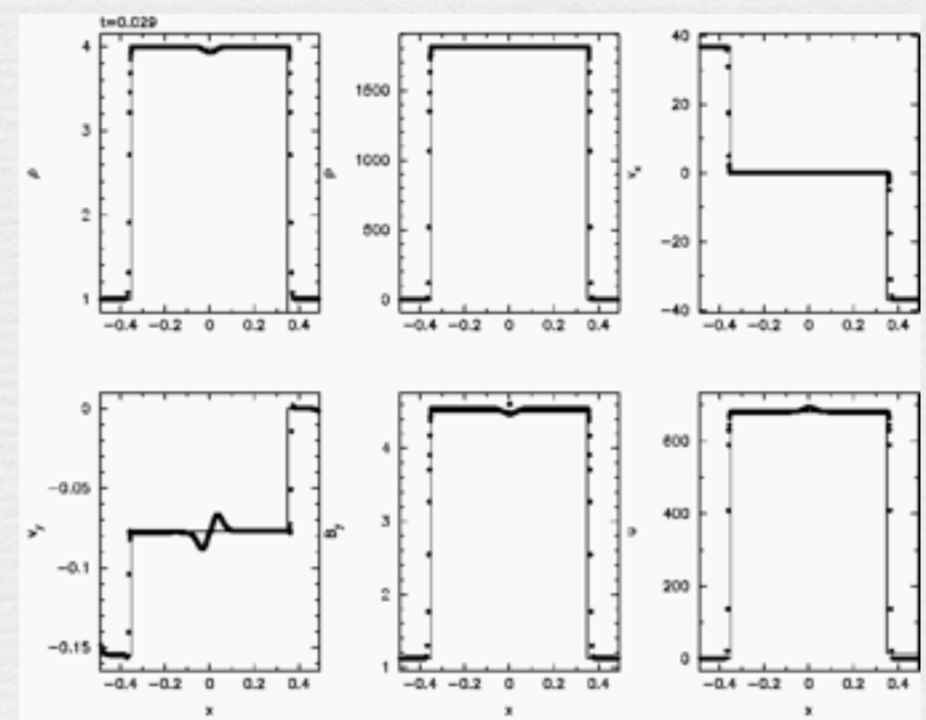
Orszag-Tang vortex problem (Balsara 1998, PM05, Rosswog & Price 2007)



Magnetic rotor problem (Tóth 2000, PM05)

7-wave MHD shock (RJ95)





Mach 25 MHD shock (Balsara 98)

$$\nabla \cdot \mathbf{B} = 0$$

Previous approach: Euler potentials

(Rosswog & Price 2007, Price & Bate 2007, 2008, 2009, Brandenburg 2010)

$$\mathbf{B} = \nabla\alpha \times \nabla\beta$$

$$\frac{d\alpha}{dt} = 0$$

$$\frac{d\beta}{dt} = 0$$

There is no ideal
MHD code
- J. Raeder

- advection of magnetic fields: no change in topology (A.B = 0)
- does not follow wind-up of magnetic fields
- difficult to model resistive effects - reconnection processes not treated correctly

Hyperbolic/parabolic divergence cleaning

Dedner et al. (2002), as adapted by Price & Monaghan (2005)
See also Mignone & Tzeferacos (2010)

$$\left(\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t}\right)_\psi = -\nabla\psi$$

$$\frac{\mathrm{d}\psi}{\mathrm{d}t} = -c_h^2\nabla\cdot\mathbf{B} - \frac{\psi}{\tau}$$

damping time:

$$\tau \equiv \frac{\sigma h}{c_h}$$

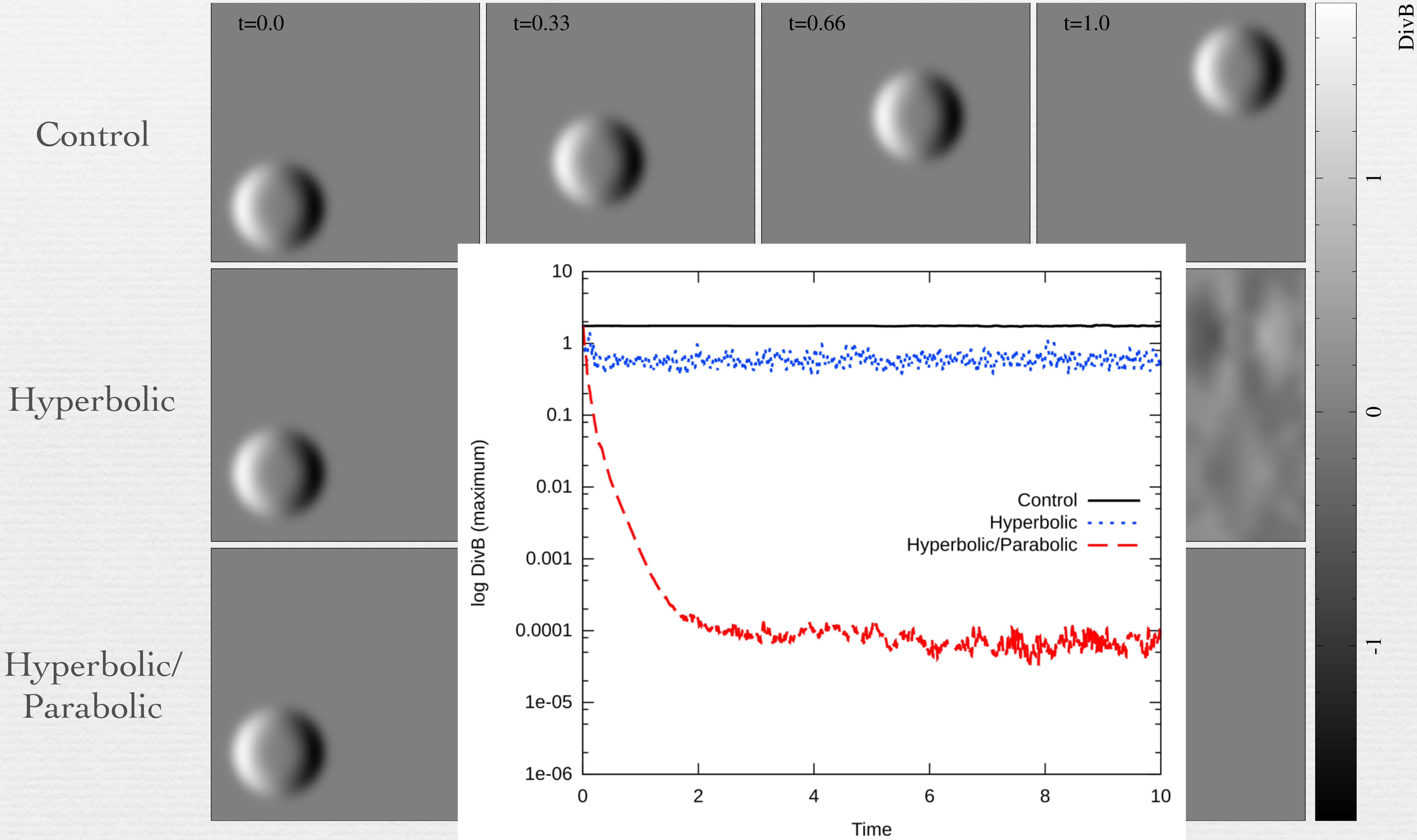
σ = 0.2-0.3 in 2D
σ = 0.8-1.0 in 3D

Combine to produce damped wave equation for div B:

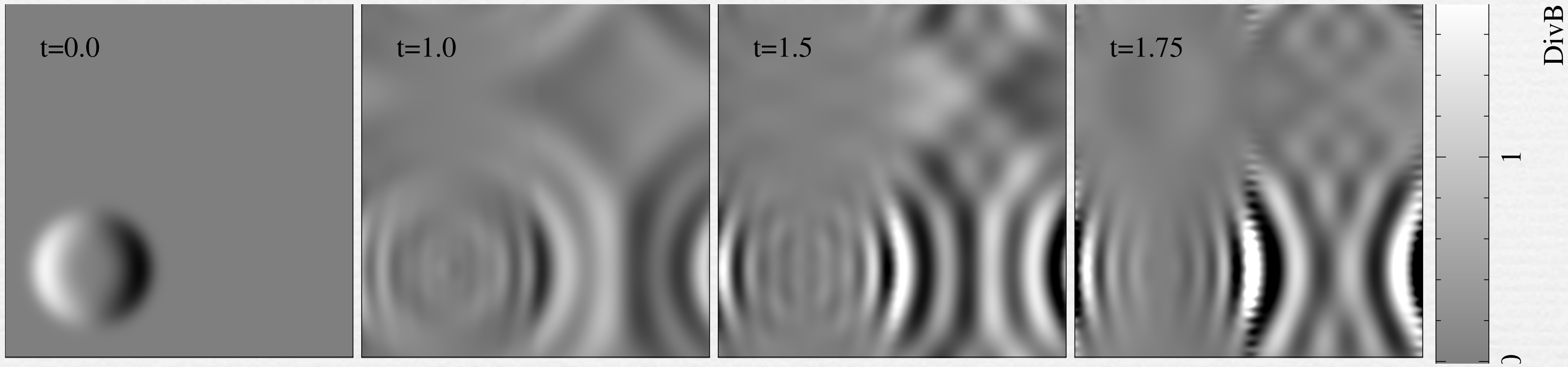
$$\frac{\partial^2(\nabla\cdot\mathbf{B})}{\partial t^2} - c_h^2\nabla^2(\nabla\cdot\mathbf{B}) + \frac{1}{\tau}\frac{\partial(\nabla\cdot\mathbf{B})}{\partial t} = 0.$$

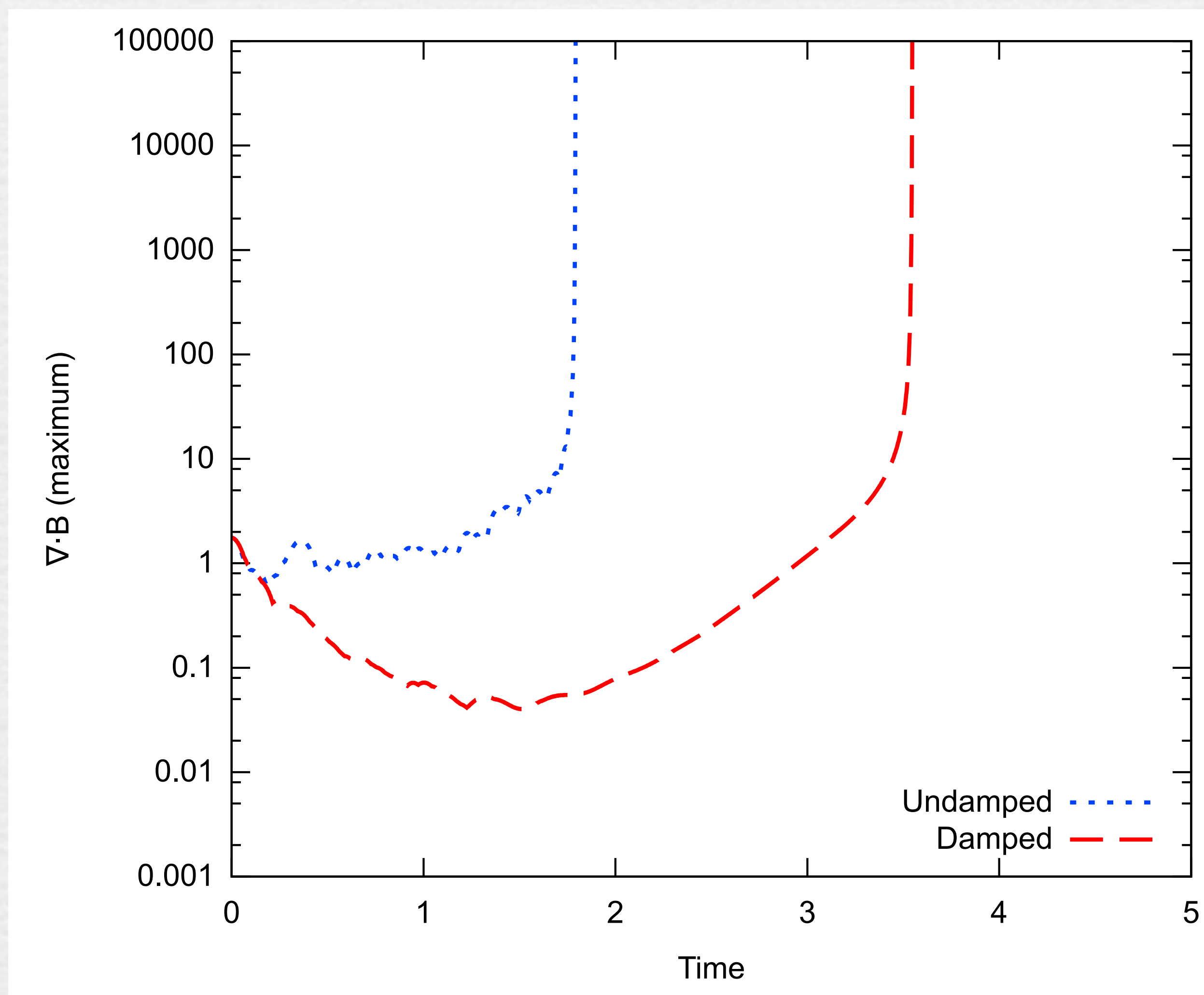
Hyperbolic/parabolic divergence cleaning

Dedner et al. (2002), c.f. Price & Monaghan (2005)



Issues at density jumps + free surfaces





see Tricco & Price (2012)

Constrained hyperbolic/parabolic divergence cleaning

Tricco & Price 2012, J. Comp. Phys. 231, 7214

- Define energy associated with ψ field

$$E = \int \left[\frac{B^2}{2\mu_0 \rho} + e_\psi \right] \rho \mathrm{d}V.$$

Find: $$e_\psi \equiv \frac{\psi^2}{2\mu_0 \rho c_h^2}$$

Also need: $$\frac{\mathrm{d}\psi}{\mathrm{d}t} = -c_h^2 \nabla \cdot \mathbf{B} - \frac{\psi}{\tau} - \tfrac{1}{2}\psi \nabla \cdot \mathbf{v}.$$

Constrained hyperbolic/parabolic divergence cleaning for smoothed particle magnetohydrodynamics

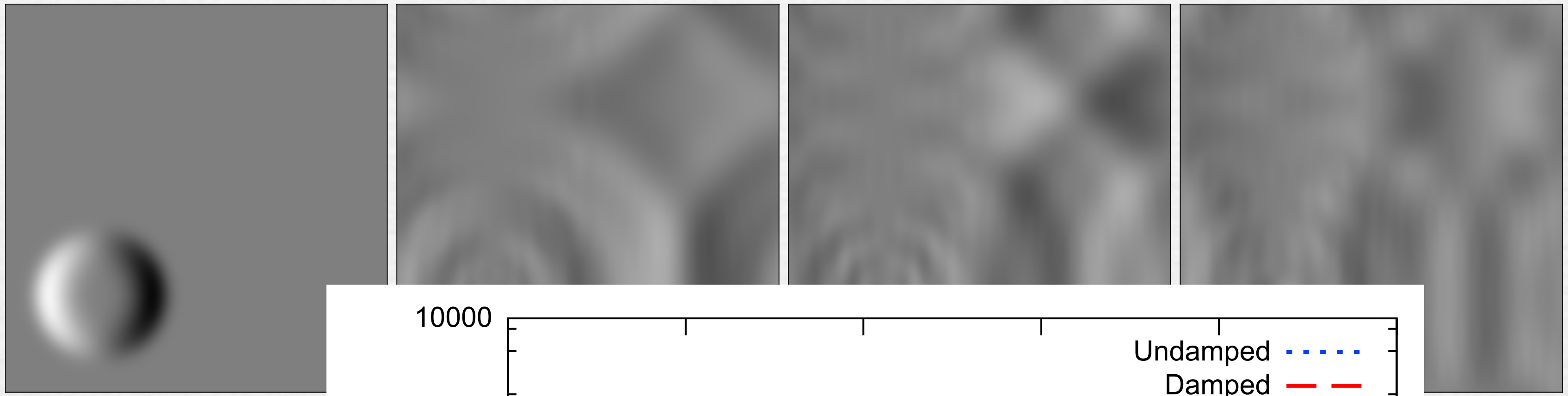
Tricco & Price 2012, J. Comp. Phys. 231, 7214

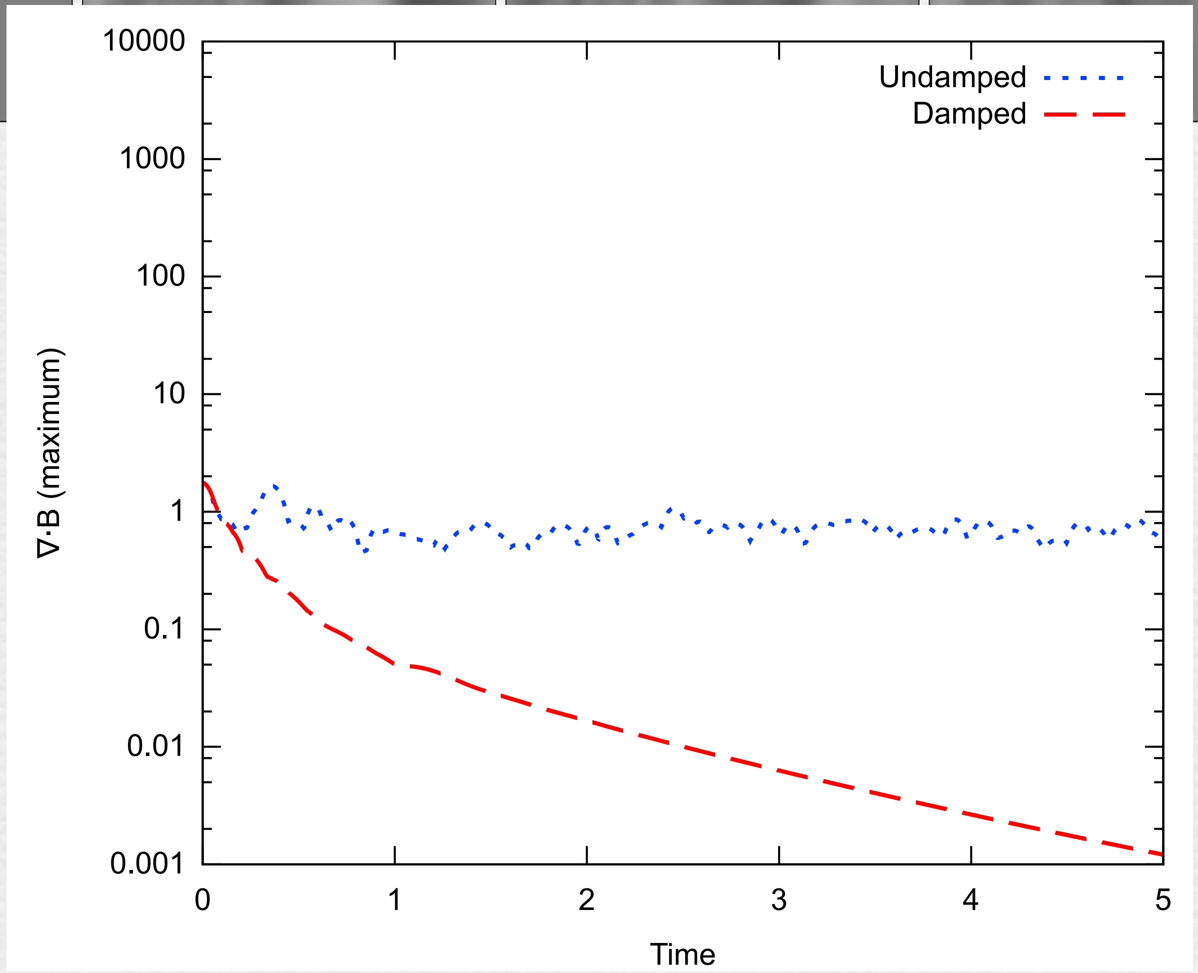
$$E = \sum_a m_a \left[\frac{B_a^2}{\mu_0 \rho_a} + \frac{\psi_a^2}{\mu_0 \rho_a c_h^2} \right].$$

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \sum_a m_a \left[\frac{\mathbf{B}_a}{\mu_0 \rho_a} \cdot \left(\frac{\mathrm{d}\mathbf{B}_a}{\mathrm{d}t} \right)_\psi + \frac{\psi_a}{\mu_0 \rho_a c_h^2} \frac{\mathrm{d}\psi_a}{\mathrm{d}t} \right] = 0.$$

Constrains the numerical operators for $\nabla\psi$ and $\nabla\cdot B$

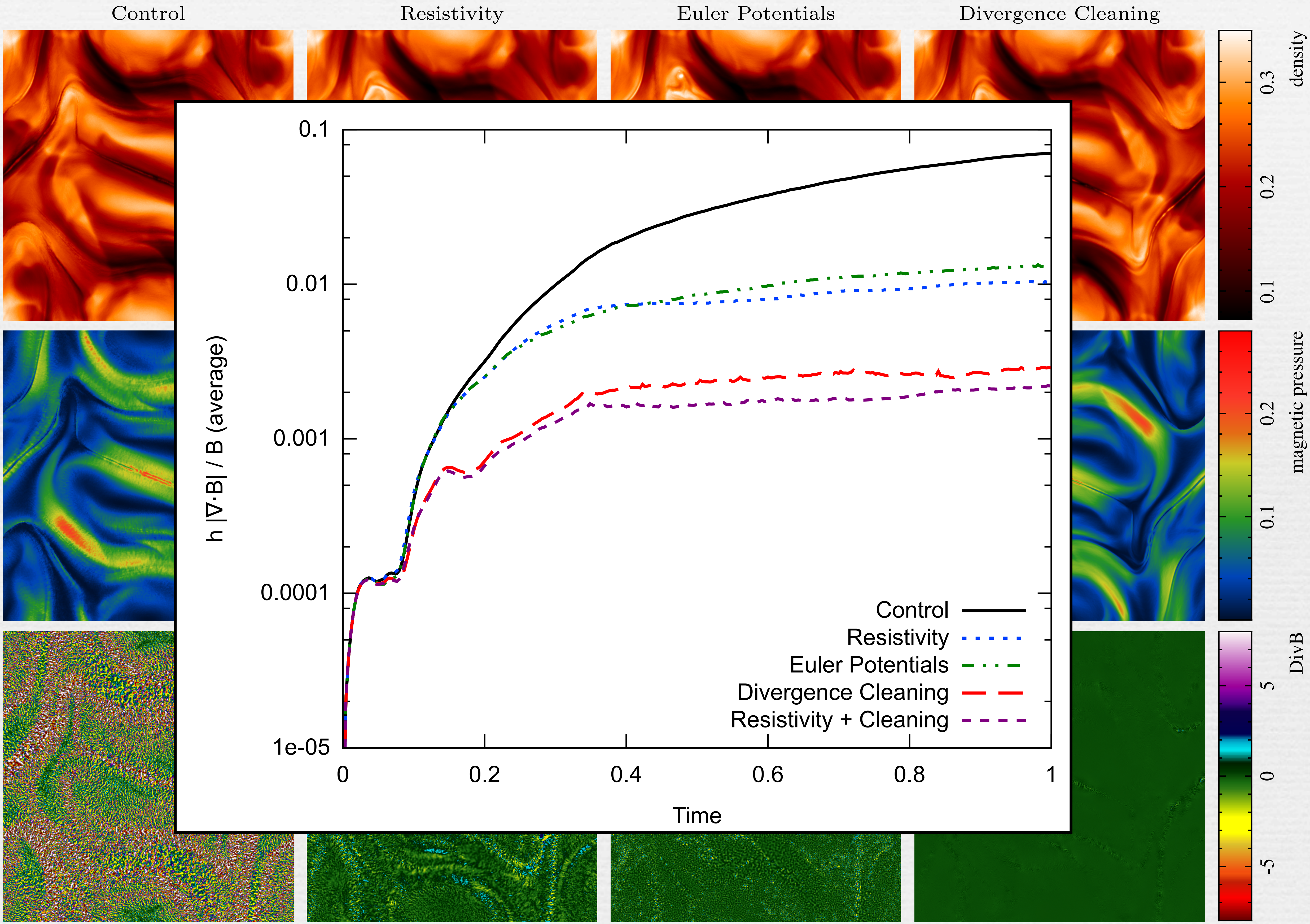
Constrained hyperbolic/parabolic divergence cleaning for smoothed particle magnetohydrodynamics





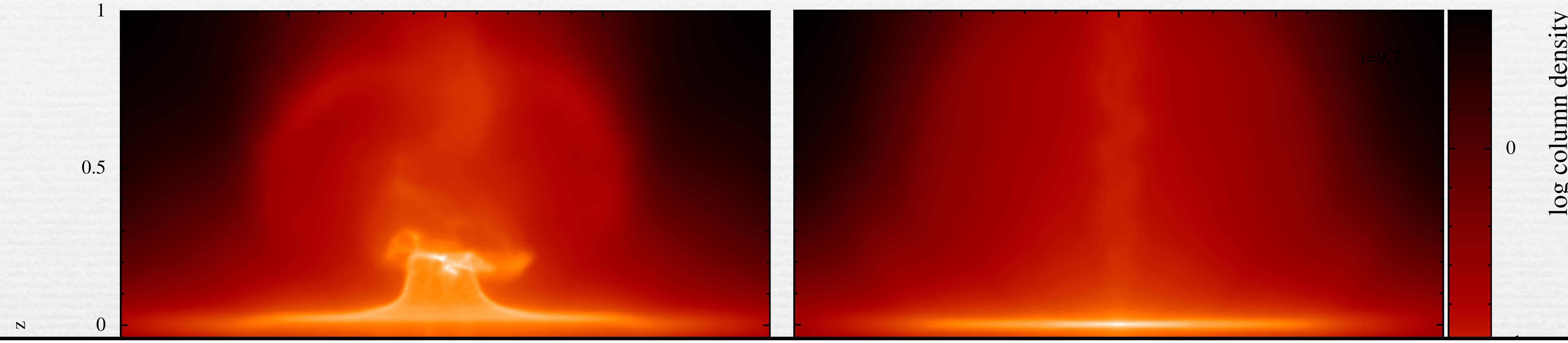
Conservative formulation is stable

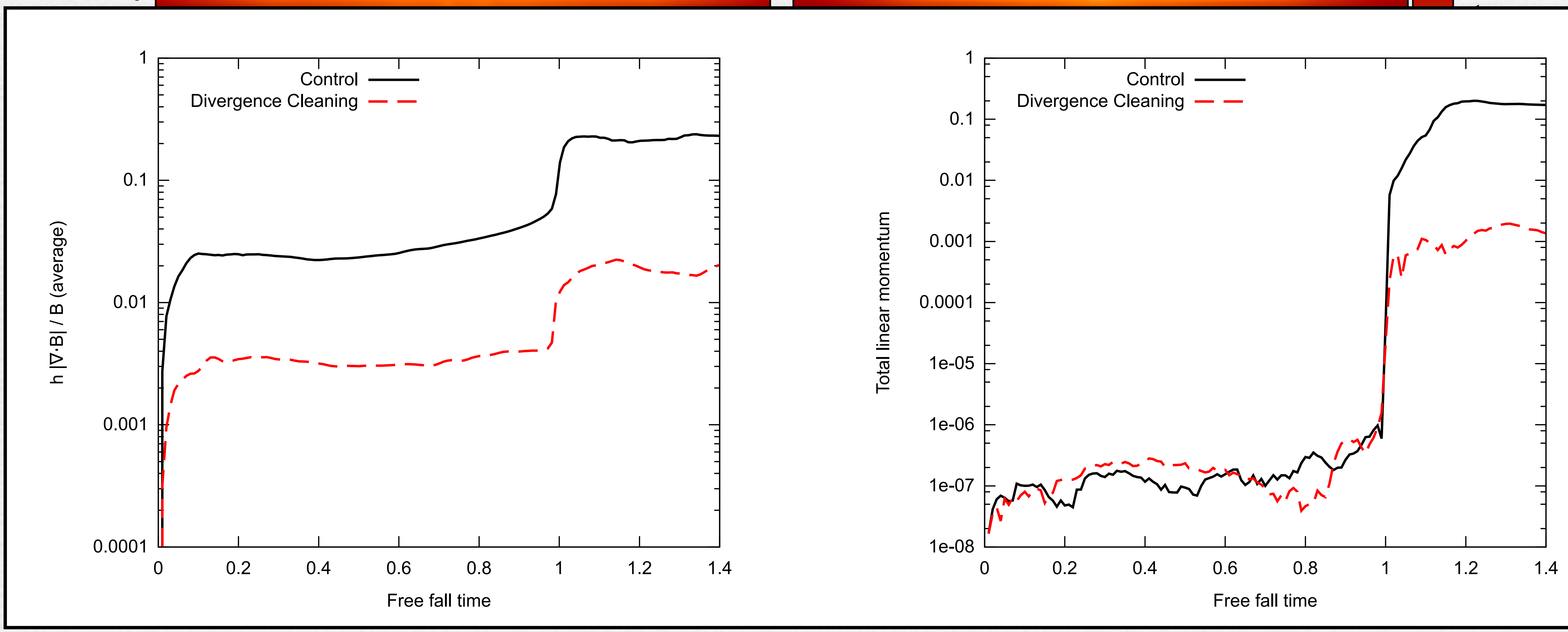
Does it work?

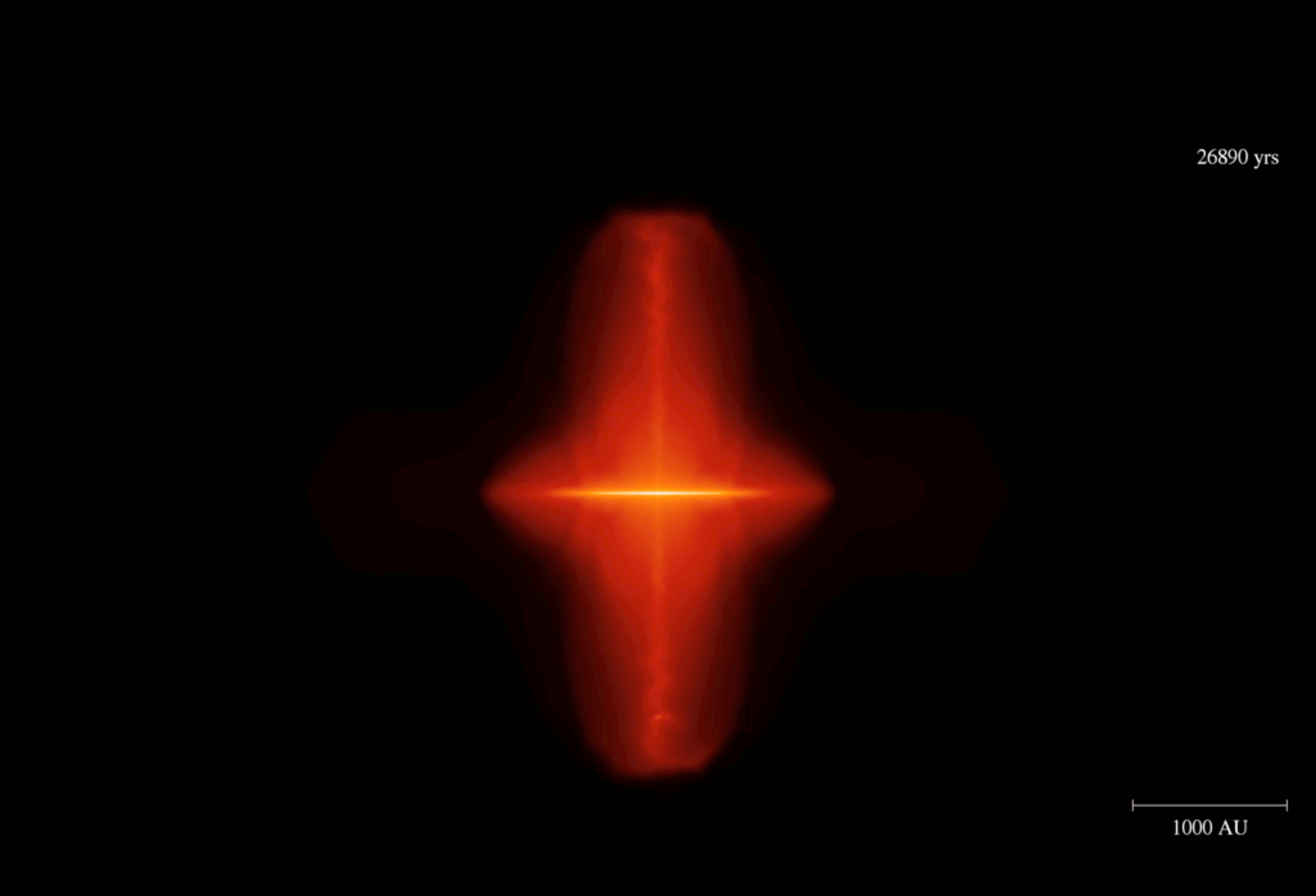


Control
Resistivity
Euler Potentials
Divergence Cleaning
density
0.3
0.2
0.1
magnetic pressure
0.2
0.1
DivB
5
0
-5
0.1
0.01
0.001
0.0001
1e-05
h |∇·B| / B (average)
0
0.2
0.4
0.6
0.8
1
Time
Control
Resistivity
Euler Potentials
Divergence Cleaning
Resistivity + Cleaning

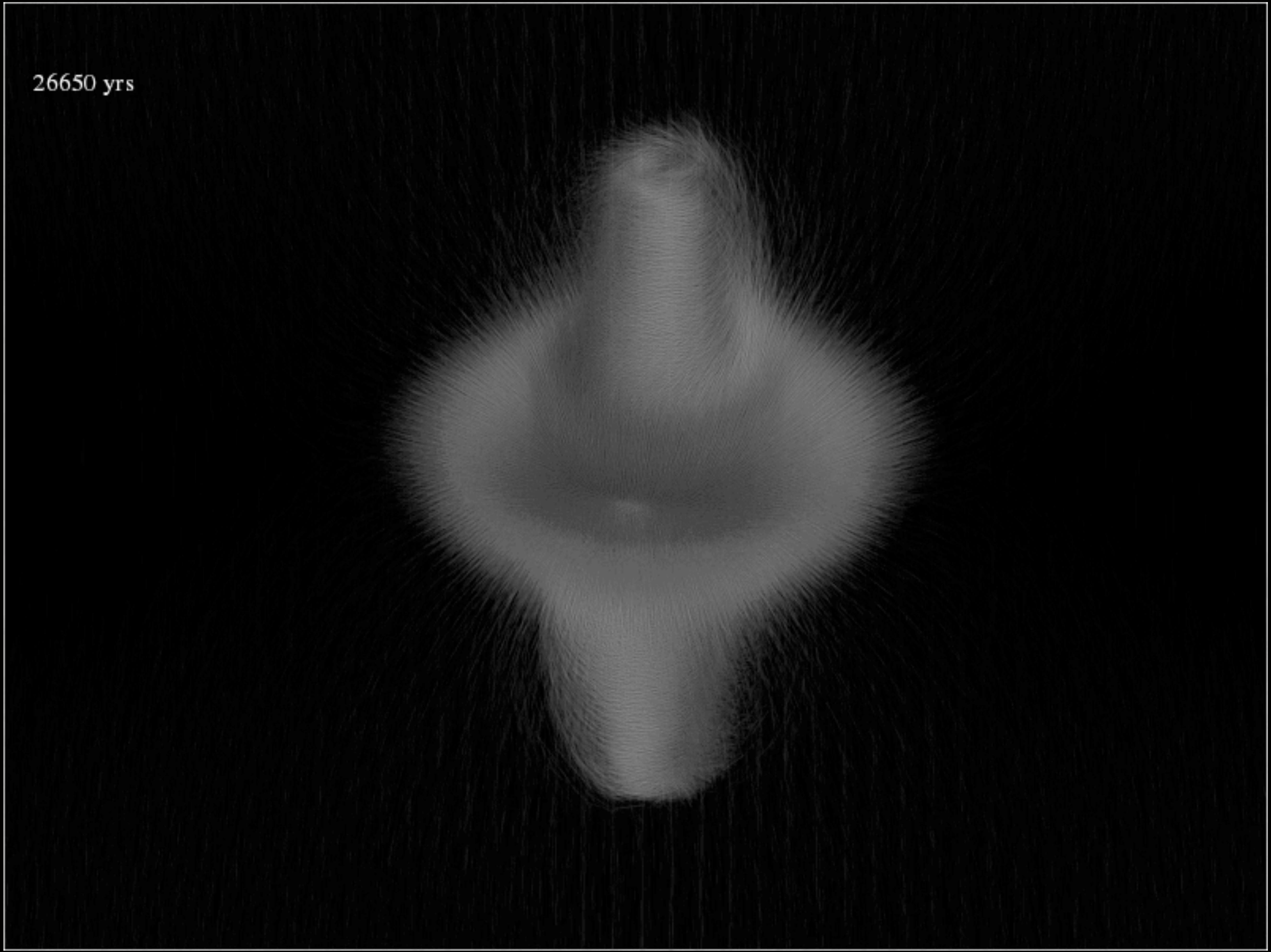
Star formation with divergence cleaning

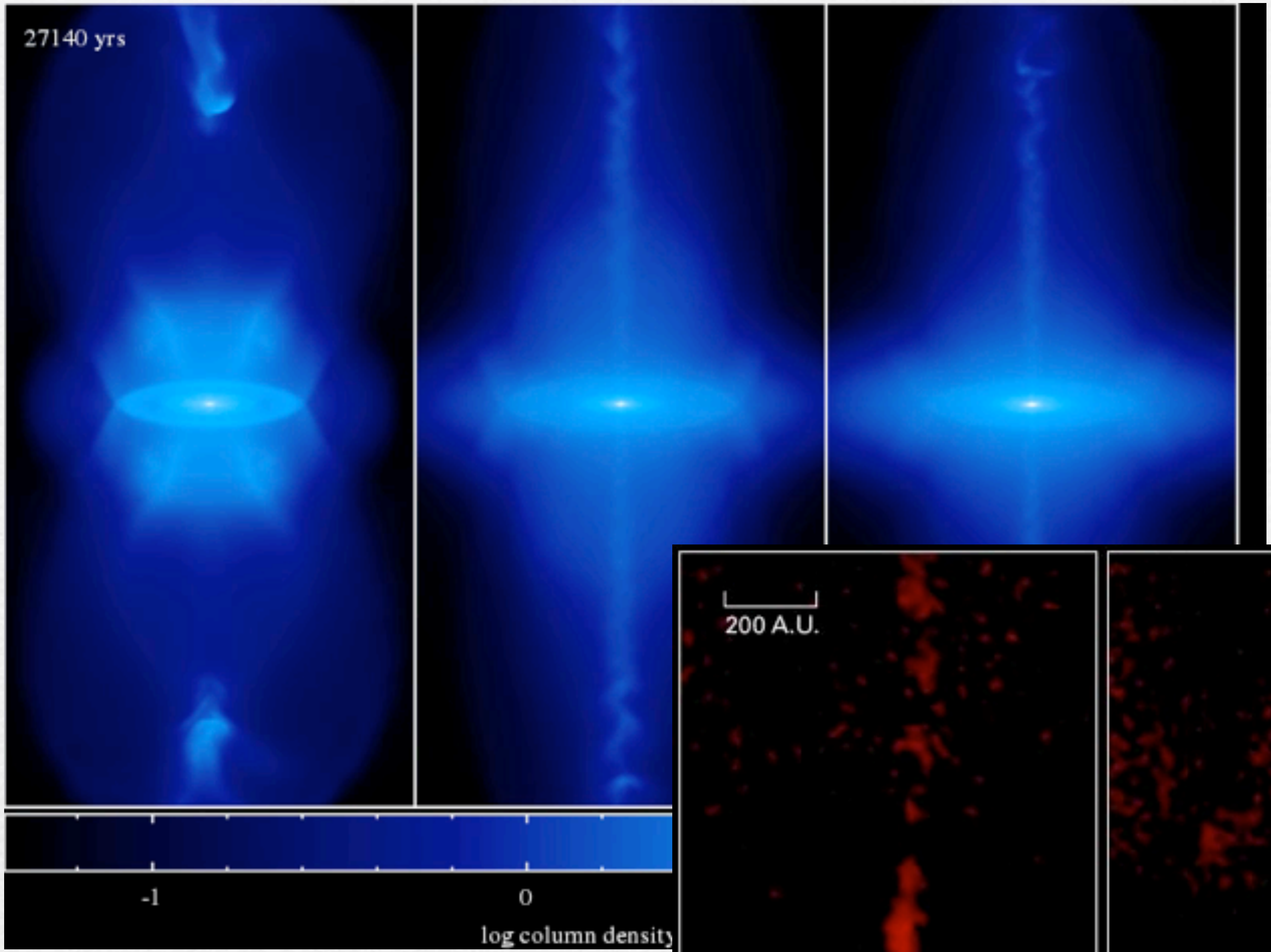


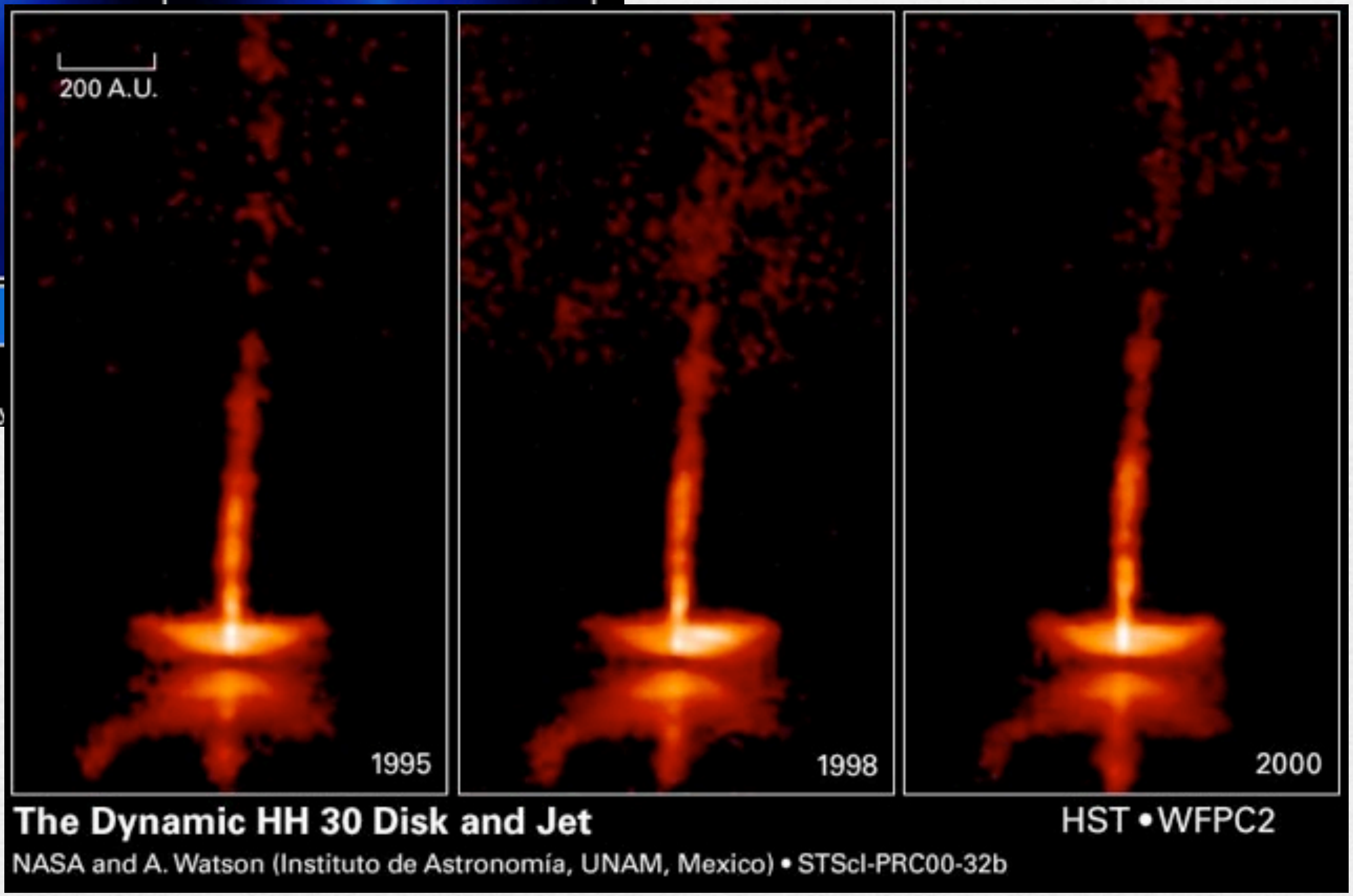




Price, Tricco & Bate (2012)

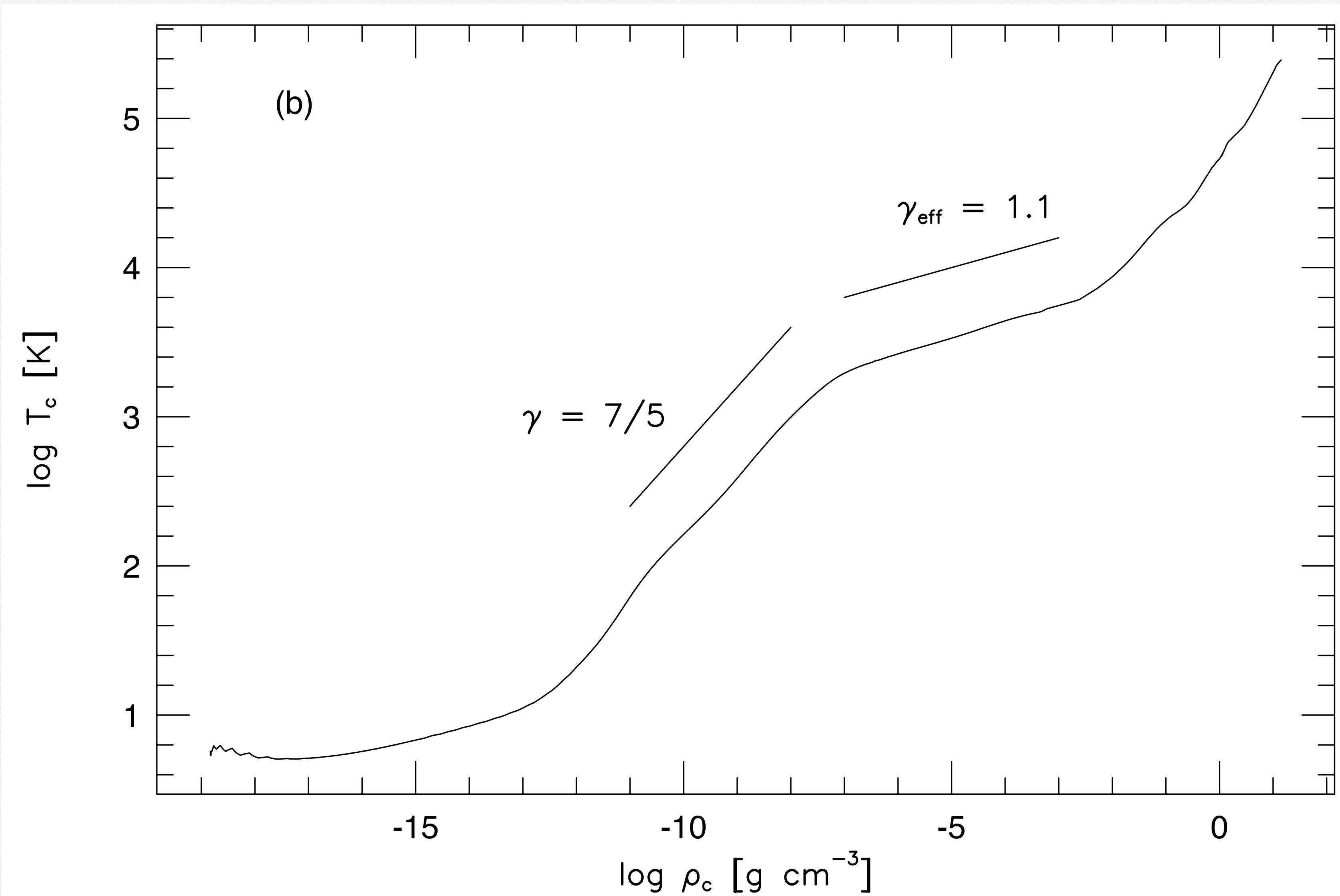

26650 yrs


27140 yrs
-1
0
log column density


200 A.U.
1995
1998
2000
The Dynamic HH 30 Disk and Jet
HST • WFPC2
NASA and A. Watson (Instituto de Astronomía, UNAM, Mexico) • STScI-PRC00-32b

First and second core

(Larson 1969)



Masanaga & Inutsuka (2000)

- Temperature in core constant, then rises above 10^{-13} g/cm^3
- At ~2000K, H_2 dissociates, leading to second isothermal phase and collapse to form the second, protostellar core.
- This core accretes to reach final stellar mass and contracts until fusion sets in

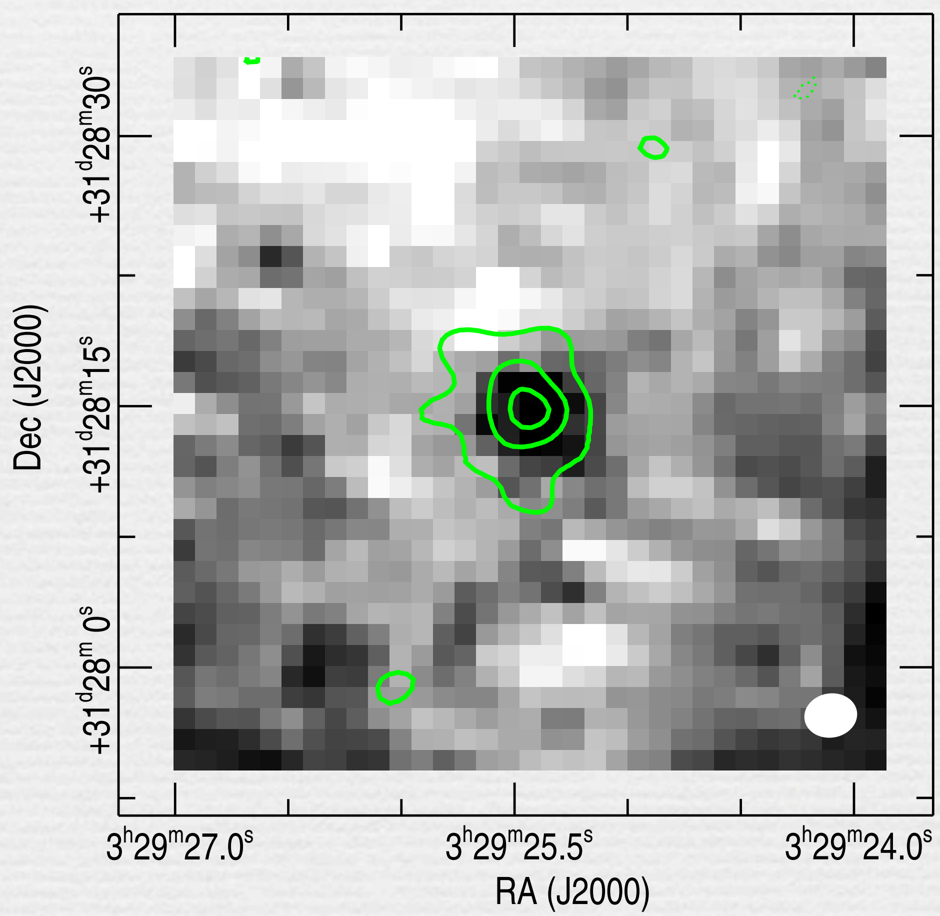
$$P = \begin{cases} c_s^2 \rho, & \rho < \rho_c \\ c_s^2 \rho_c (\rho/\rho_c)^{7/5} & \rho_c \leq \rho < \rho_d \\ c_s^2 \rho_c (\rho_d/\rho_c)^{7/5} \rho_d (\rho/\rho_d)^{1.1} & \rho \geq \rho_d \end{cases}$$

First core is SHORT LIVED (1000 - 10,000 years)

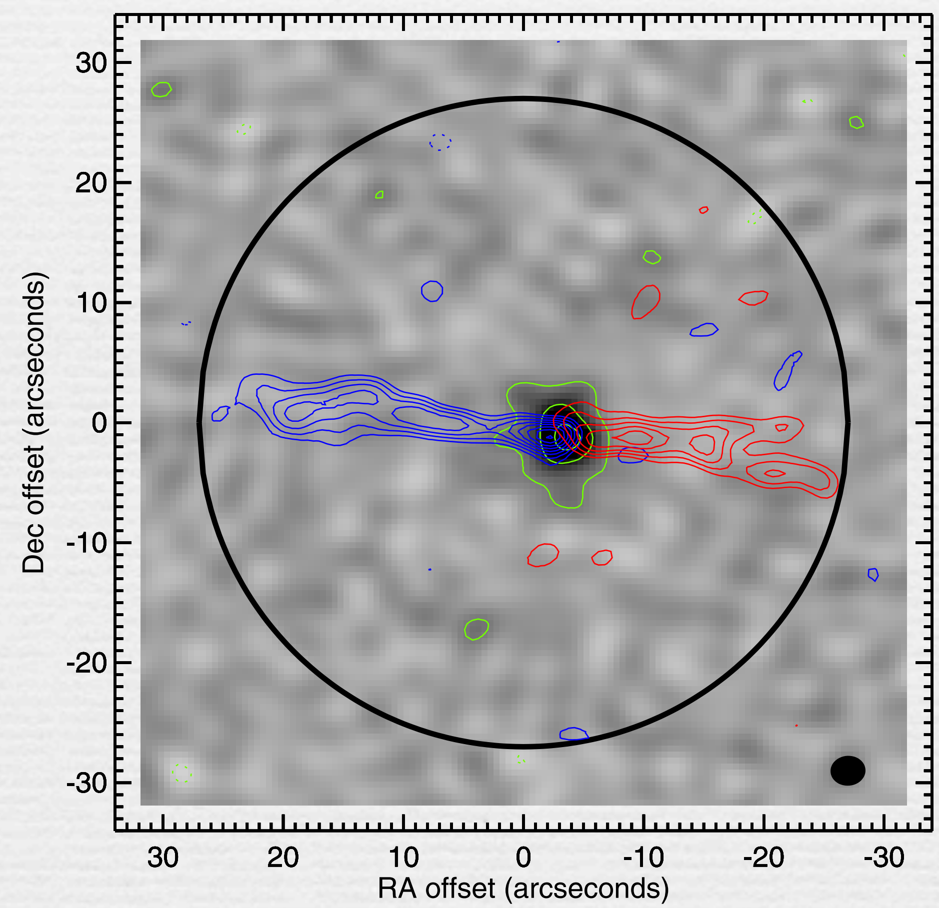
THE ASTROPHYSICAL JOURNAL, 742:1 (11pp), 2011 November 20 doi:10.1088/0004-637X/742/1/1

DETECTION OF A BIPOLAR MOLECULAR OUTFLOW DRIVEN BY A CANDIDATE FIRST HYDROSTATIC CORE

MICHAEL M. DUNHAM[1], XUEPENG CHEN[1], HÉCTOR G. ARCE[1], TYLER L. BOURKE[2], SCOTT SCHNEE[3], AND MELISSA L. ENOCH[4]
[1] Department of Astronomy, Yale University, P.O. Box 208101, New Haven, CT 06520, USA; michael.dunham@yale.edu
[2] Harvard-Smithsonian Center for Astrophysics, 60 Garden Street, Cambridge, MA 02138, USA
[3] National Radio Astronomy Observatory, 520 Edgemont Road, Charlottesville, VA 22903, USA
[4] Department of Astronomy, University of California at Berkeley, 601 Campbell Hall, Berkeley, CA 94720, USA
Received 2011 April 4; accepted 2011 August 5; published 2011 October 28



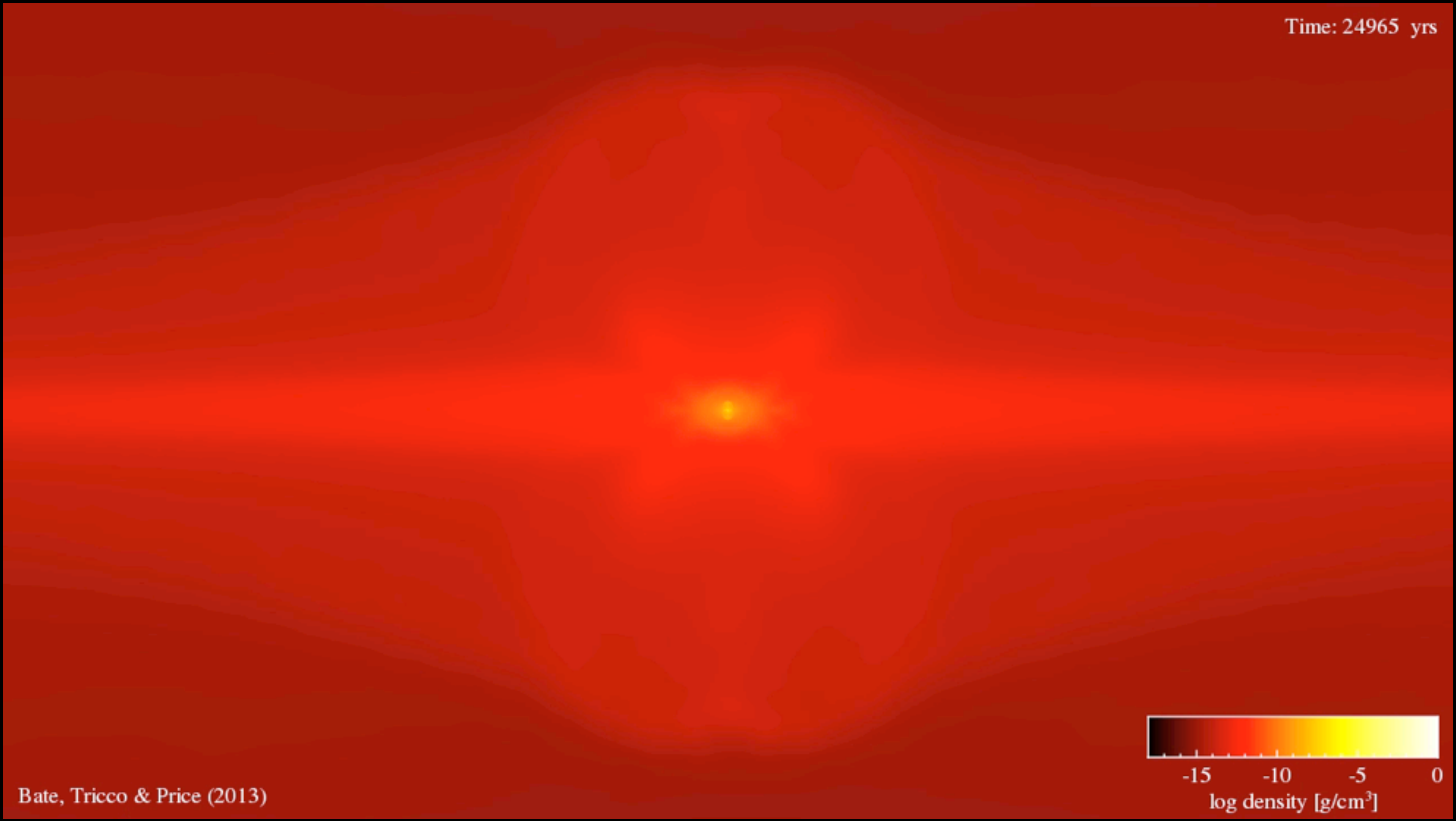
~1 solar mass core

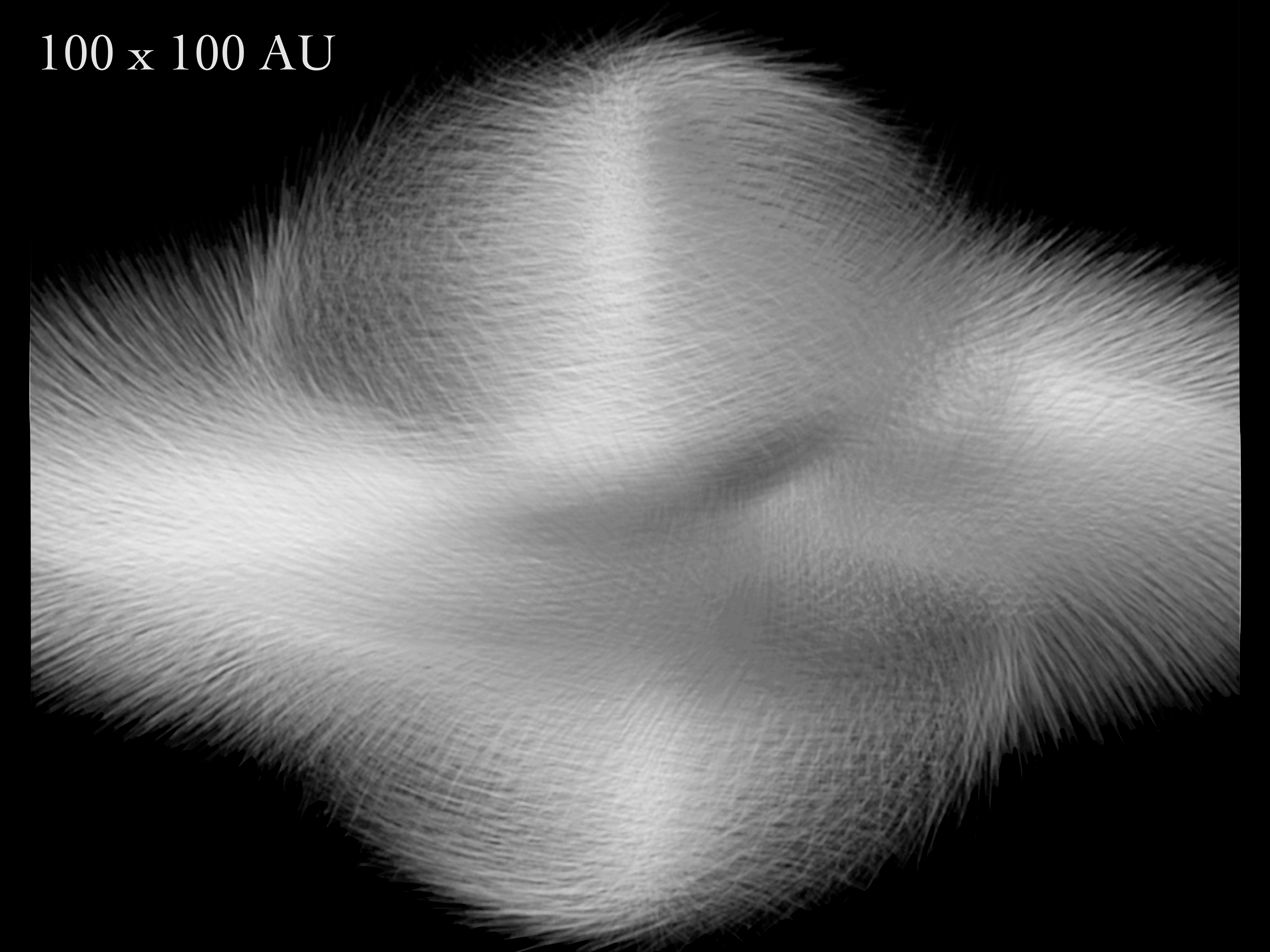


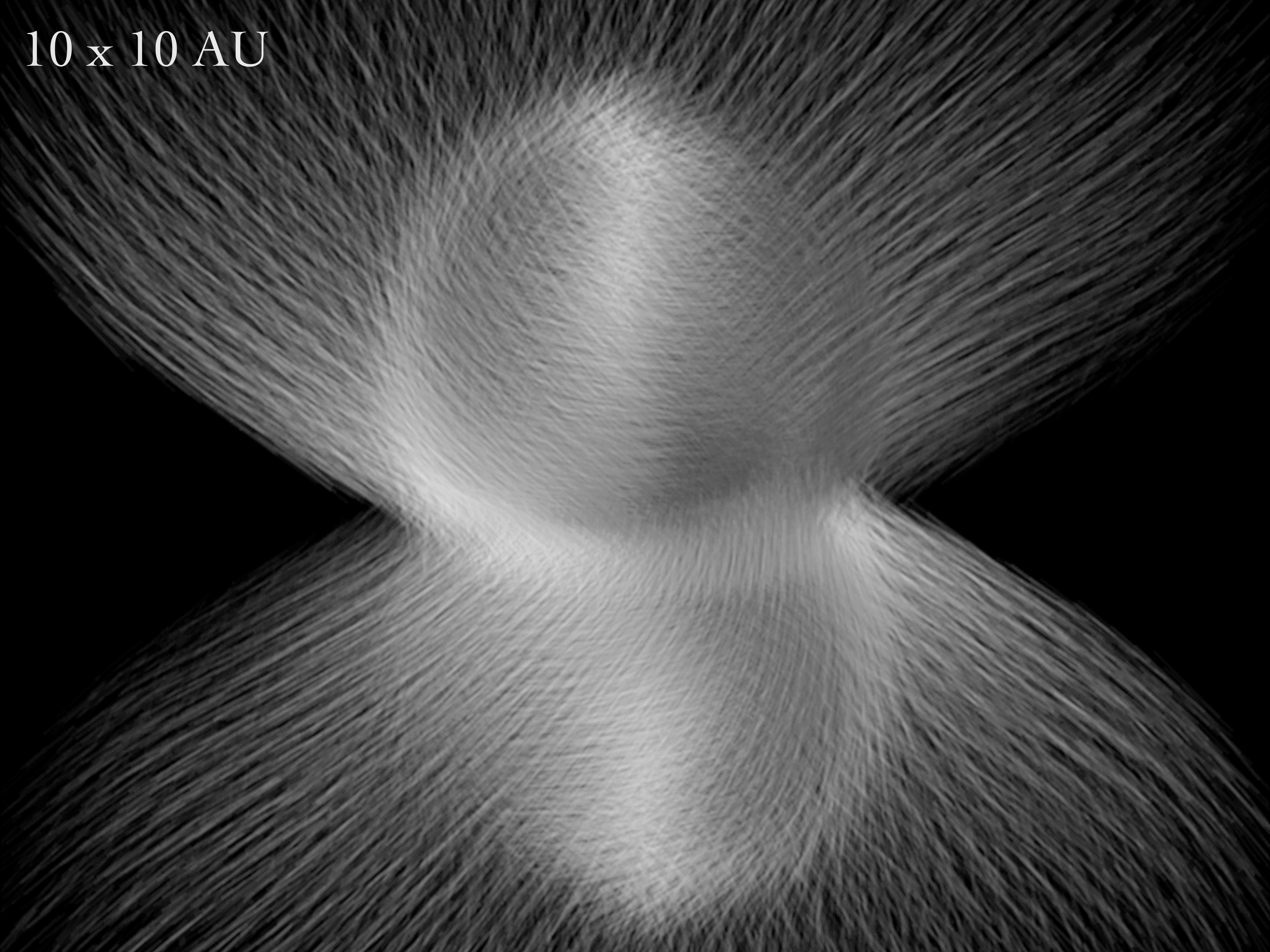
The outflow is slow (characteristic velocity of 2.9 km s−1), shows a jet-like morphology (opening semi-angles ~8° for both lobes), and extends to the edges of the primary beam.

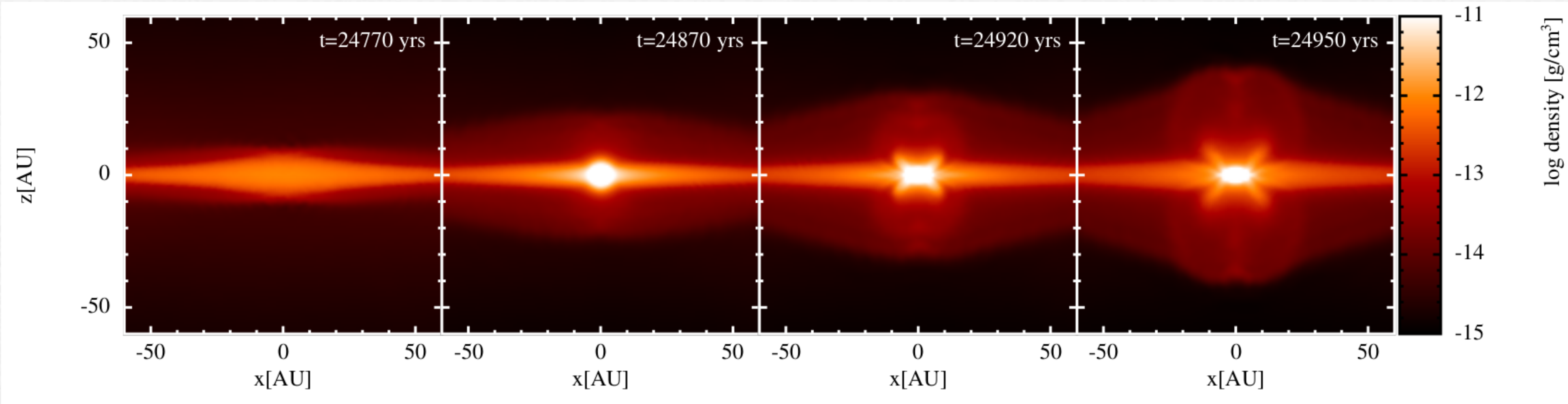
^{12}CO J = 2–1 emission integrated from 0.3 to 7.3 km s^{-1}, while the red contours show redshifted emission integrated from 7.3 to 14.3 km s^{-1}. The solid blue

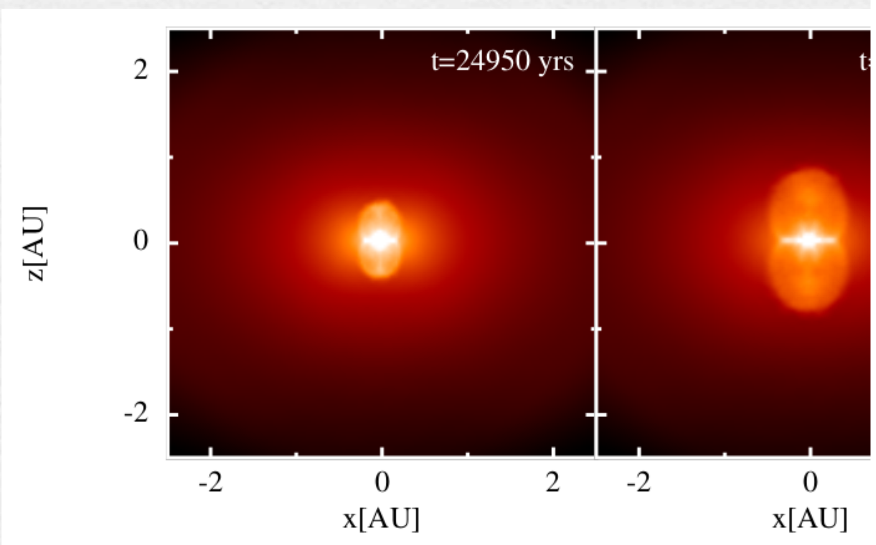
Radiation-MHD to the second core

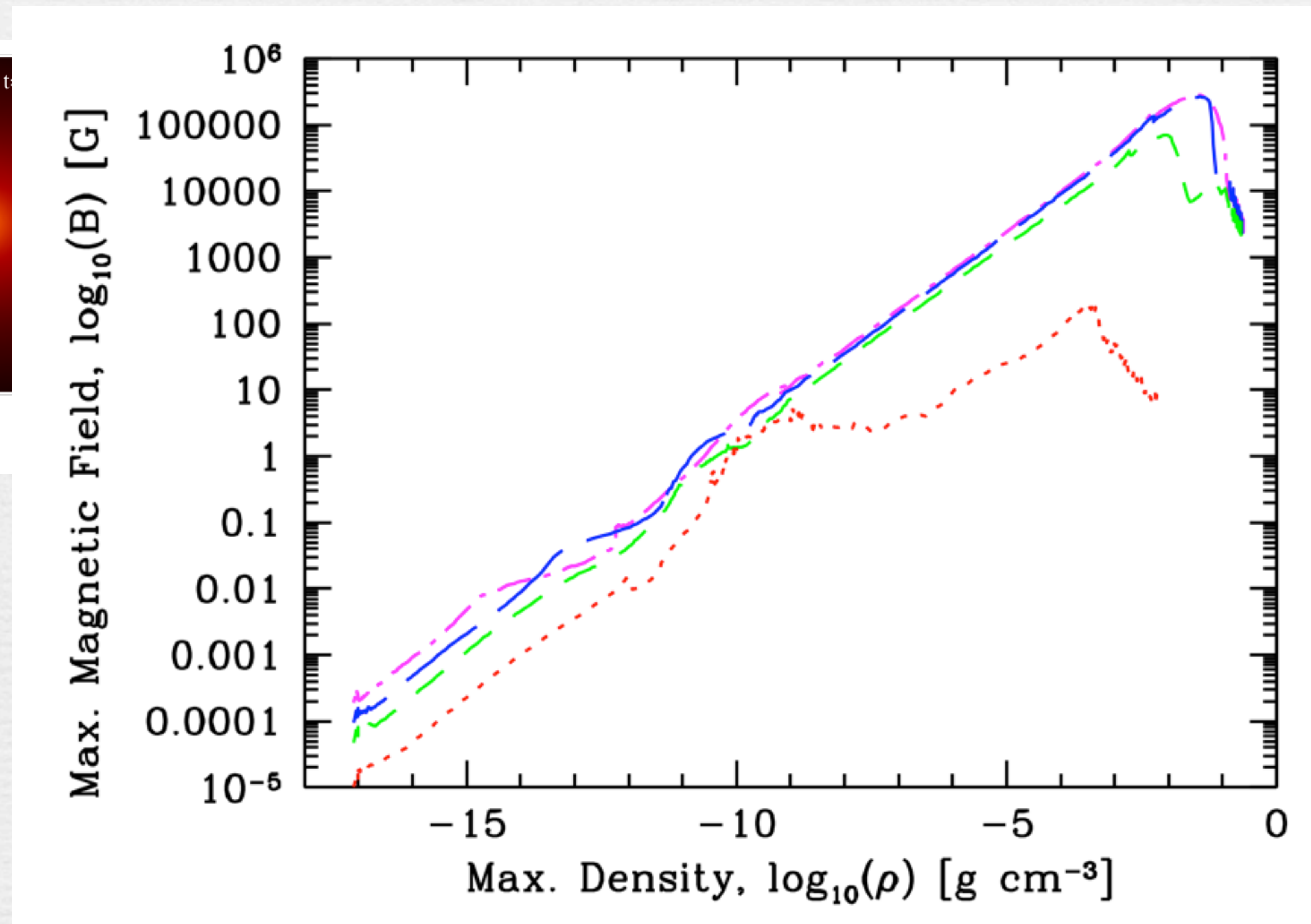



100 x 100 AU

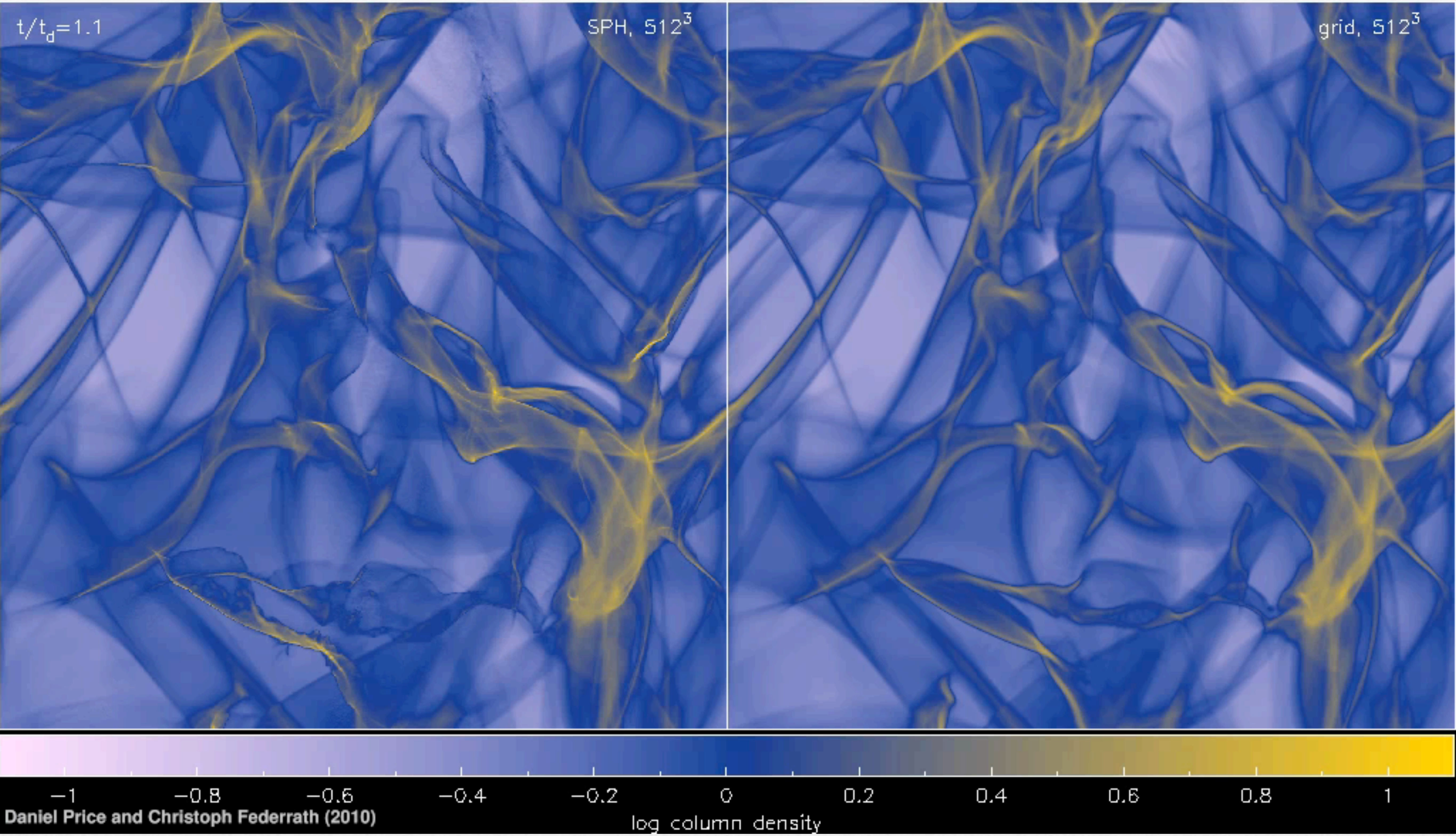

10 x 10 AU


t=24770 yrs
t=24870 yrs
t=24920 yrs
t=24950 yrs
z[AU]
50
0
-50
-50
0
50
x[AU]
log density [g/cm^3]
-11
-12
-13
-14
-15


t=24950 yrs
z[AU]
2
0
-2
-2
0
2
x[AU]


Max. Magnetic Field, $\log_{10}(B)$ [G]
10^6
100000
10000
1000
100
10
1
0.1
0.01
0.001
0.0001
10^{-5}
−15
−10
−5
0
Max. Density, $\log_{10}(\rho)$ [g cm^{-3}]

Comparison of Mach 10, hydro turbulence



SPH=PHANTOM grid=FLASH

Good agreement between methods...

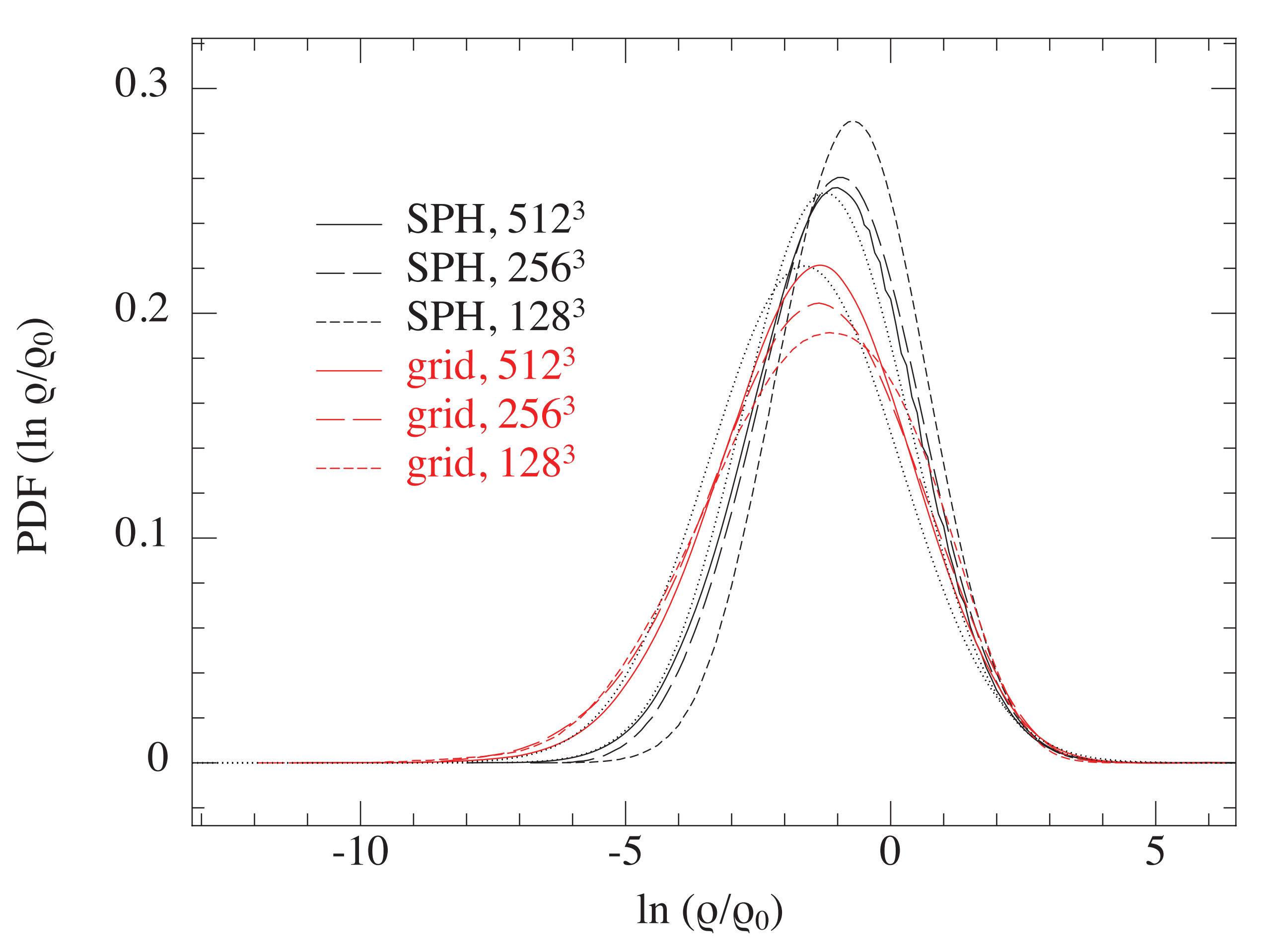
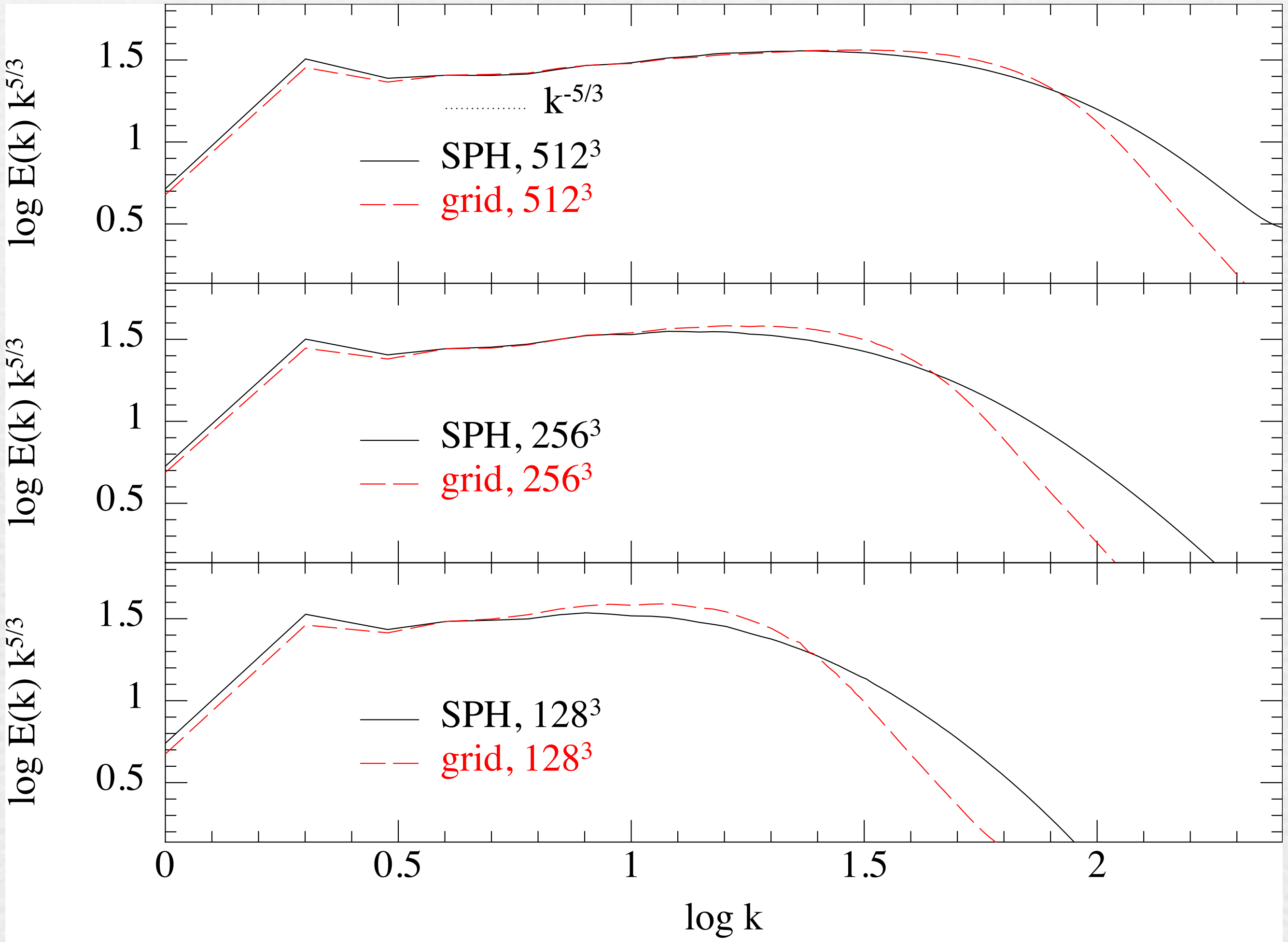


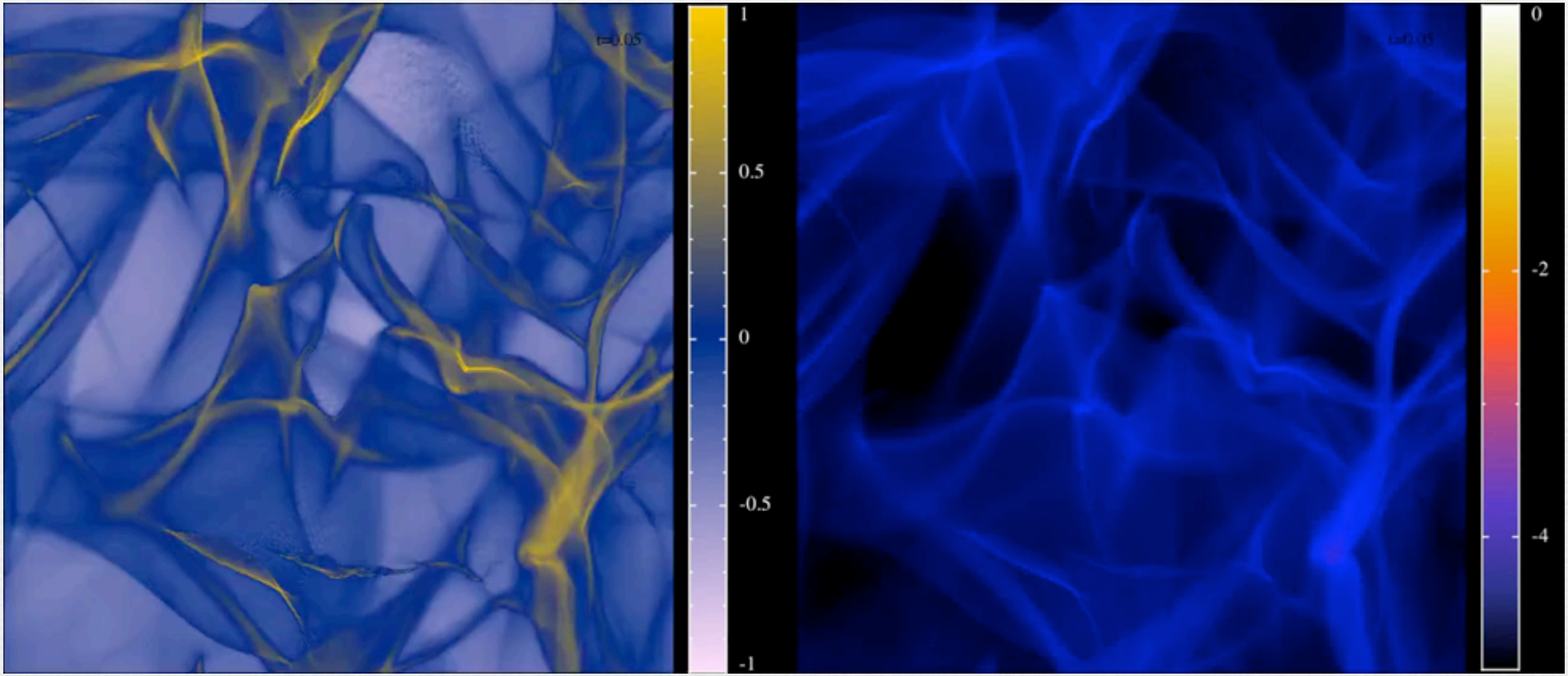
Figure 6. Time-averaged PDF of the logarithm of the density field $s \equiv \ln \rho$ from the PHANTOM [SPH, dark (black in online version) lines other than dotted] and FLASH [grid, lighter (red in online version) lines] calculations, each at resolutions of 128^3, 256^3 and 512^3 particles/grid cells. The PDFs are

$(\rho^{1/3}\mathbf{v})$



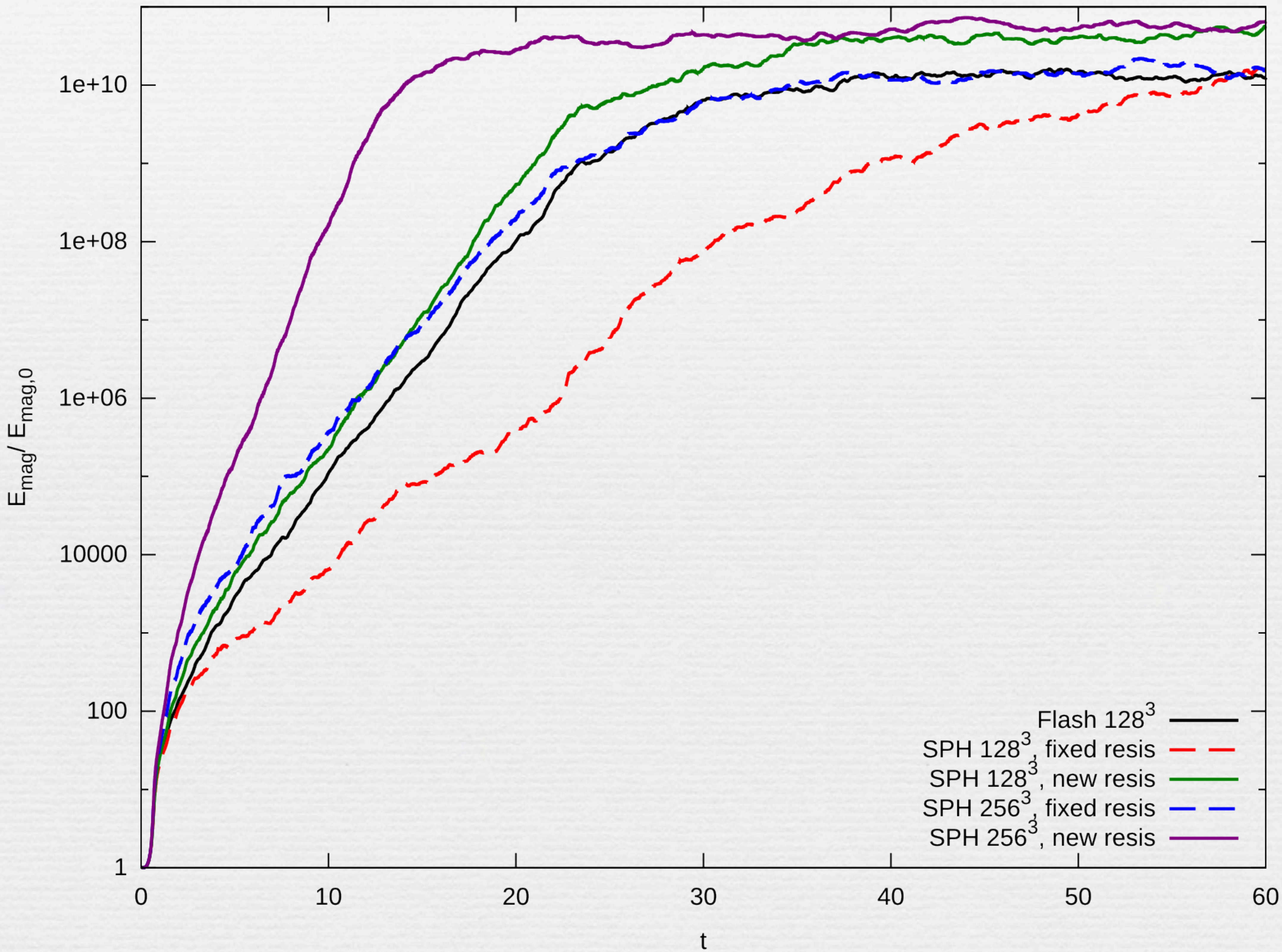
Price & Federrath, 2010, MNRAS 406, 1659

Small-scale magnetic dynamo



Tricco, Price & Federrath (2013, in prep)

Magnetic energy



Tricco, Price & Federrath (2013, in prep)

Two fluids

Dust + Gas: A simple example of a two-fluid mixture

- Two fluids coupled by a drag term

$$
\begin{aligned}
\frac{\partial \rho_{\rm g}}{\partial t} + \nabla . \left(\rho_{\rm g} \mathbf{v}_{\rm g}\right) &= 0, \\
\frac{\partial \rho_{\rm d}}{\partial t} + \nabla . \left(\rho_{\rm d} \mathbf{v}_{\rm d}\right) &= 0, \\
\frac{\partial \mathbf{v}_{\rm g}}{\partial t} + \left(\mathbf{v}_{\rm g} . \nabla\right) \mathbf{v}_{\rm g} &= -\frac{\nabla P_{\rm g}}{\rho_{\rm g}} + K(\mathbf{v}_{\rm d} - \mathbf{v}_{\rm g}) + \mathbf{f}, \\
\frac{\partial \mathbf{v}_{\rm d}}{\partial t} + \left(\mathbf{v}_{\rm d} . \nabla\right) \mathbf{v}_{\rm d} &= -K(\mathbf{v}_{\rm d} - \mathbf{v}_{\rm g}) + \mathbf{f},
\end{aligned}
$$

"Stopping time"

$$t_{\rm stop} \equiv \frac{\rho_{\rm d} \rho_{\rm g}}{K(\rho_{\rm d} + \rho_{\rm g})}$$

If you don't understand simple examples, you'll be really flummoxed when it comes to the complicated stuff

- Phil Collella, yesterday

Dustywave: Waves in a two fluid medium

Laibe & Price, 2011, MNRAS 418, 1491

$$\delta v = Ae^{i(kx-\omega t)}$$

Dispersion relation:

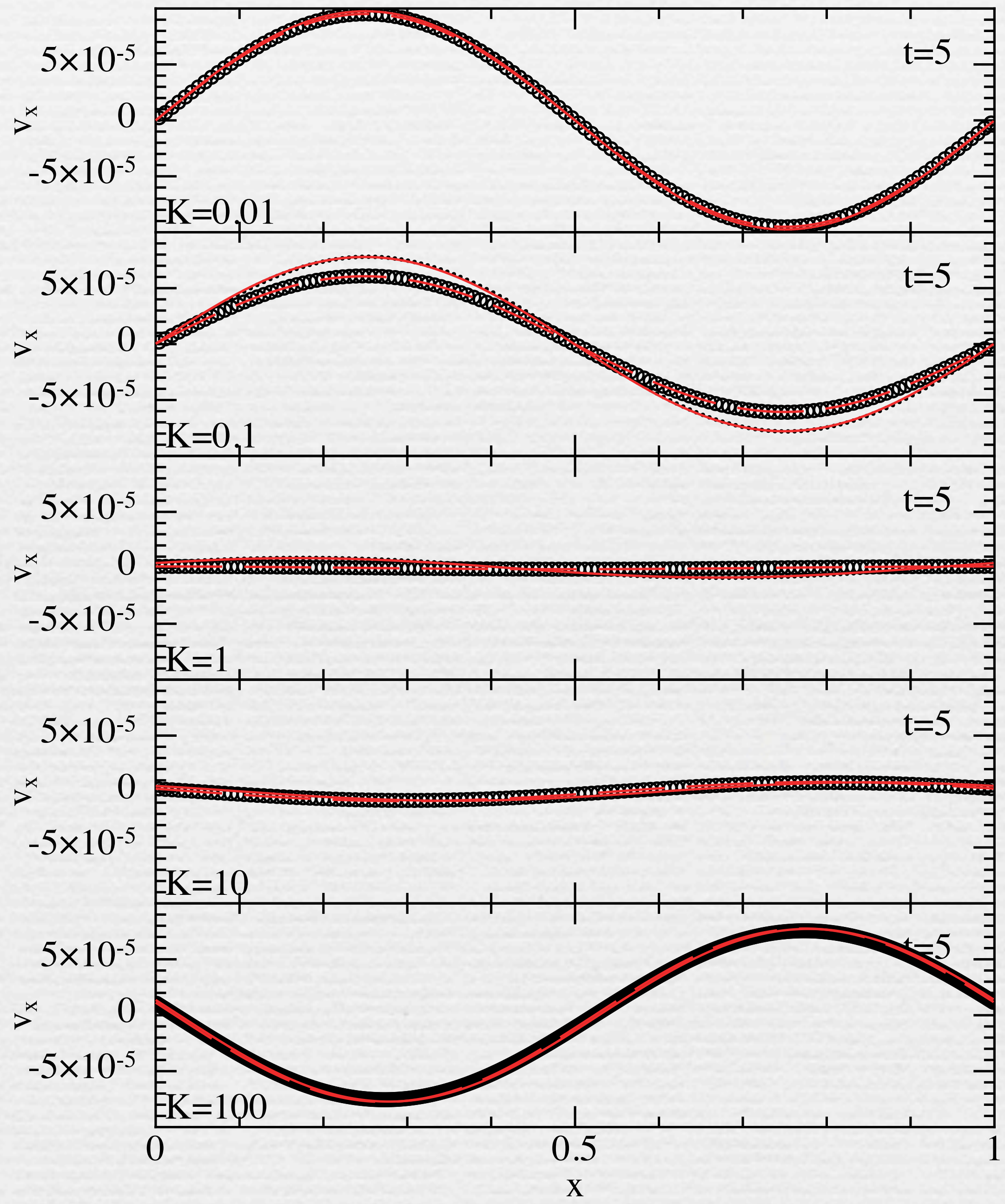
$$\omega^3 + iK\left(\frac{1}{\hat{\rho}_{\rm g}} + \frac{1}{\hat{\rho}_{\rm d}}\right)\omega^2 - k^2 c_{\rm s}^2 \omega - iK\frac{k^2 c_{\rm s}^2}{\hat{\rho}_{\rm d}} = 0$$

Limit of strong drag:

$$\omega = \pm k\tilde{c}_{\rm s} - i\frac{\hat{\rho}_{\rm g}\hat{\rho}_{\rm d}}{K\left(\hat{\rho}_{\rm g} + \hat{\rho}_{\rm d}\right)} k^2 c_{\rm s}^2 \left(\frac{1 - A^2}{2}\right)$$

Effective sound speed:

$$\tilde{c}_{\rm s} \equiv c_{\rm s} A = c_{\rm s}\left(1 + \frac{\hat{\rho}_{\rm d}}{\hat{\rho}_{\rm g}}\right)^{-\frac{1}{2}}$$



Dustywaves: Analytic solution

Laibe & Price, 2011, MNRAS 418, 1491

```
!-------------------------------
! DUST VELOCITIES
!-------------------------------
vd3r = - (rhogeq*cs**2*k**2*w2r**2*w1r**2*rhogsol + rhogeq**2*w3i**4*k*w1r*vgsol - w3i*cs**2*k**3*rhogeq*Kdrag*vdsol*w2r - w3i*cs**2*k**3*rhogeq*Kdrag*vdsol*w1r +
w3i*cs**2*k**3*rhogeq*Kdrag*vgsol*w2r + w3i*cs**2*k**3*rhogeq*Kdrag*vgsol*w1r -rhogeq**2*cs**2*k**3*w2i**2*w1r*vgsol + rhogeq*cs**4*k**4*w2r*w1r*rhogsol -
rhogeq**2*cs**2*k**3*w2r**2*w1r*vgsol - rhogeq*cs**2*k**3*w2r*w1i*Kdrag*vgsol -rhogeq**2*cs**2*k**3*w2r*w1i**2*vgsol + rhogeq*cs**2*k**3*w2r*w1i*Kdrag*vdsol -
rhogeq**2*cs**2*k**3*w2r*w1r**2*vgsol - rhogeq*w3i**4*cs**2*k**2*rhogsol - rhogeq*cs**4*k**4*w2i*w1i*rhogsol + rhogeq*cs**2*k**2*w1i**2*w2r**2*rhogsol + rhogeq**2*w3i**4*w2r*k*vgsol -
rhogeq*w3i**3*k*Kdrag*vdsol*w1r +  2*rhogeq*w3i**3*k*Kdrag*vgsol*w2r + rhogeq*w2i**2*w1i**2*cs**2*k**2*rhogsol +  rhogeq*w2i**2*w1r**2*cs**2*k**2*rhogsol - rhogeq*w3i**4*w2r*w1r*rhogsol +
rhogeq*w3i**4*w1i*rhogsol*w2i +  w3i*cs**4*k**4*rhogeq*rhogsol*w2i + w3i*cs**4*k**4*rhogeq*rhogsol*w1i + w3i**2*Kdrag**2*k*vgsol*w1r -w3i**3*Kdrag*rhogsol*k**2*cs**2 -
w3i**3*Kdrag*rhogsol*w2r*w1r + w3i**3*Kdrag*rhogsol*w2i*w1i - w3i*cs**2*k**2*rhogeq*rhogsol*w2r**2*w1i - w3i*cs**2*k**2*rhogeq*rhogsol*w1i*w2i**2 +  rhogeq*w3i**3*rhogsol*k**2*cs**2*w2i +
rhogeq*w3i**3*rhogsol*k**2*cs**2*w1i - rhogeq*w3i**3*rhogsol*w2r**2*w1i  - rhogeq*w3i**3*rhogsol*w1i*w2i**2 - rhogeq*w3i**3*rhogsol*w2i*w1r**2 -  w3i*cs**2*k**2*rhogeq*rhogsol*w2i*w1r**2 -
w3i*cs**2*k**2*rhogeq*rhogsol*w2i*w1i**2 - rhogeq*w3i**3*rhogsol*w2i*w1i**2 - rhogeq*w3i**3*k*Kdrag*vdsol*w2r + 2*rhogeq*w3i**3*k*Kdrag*vgsol*w1r + w3r*rhogeq**2*cs**2*k**3*w2i**2*vgsol +
w3r*rhogeq*w2r*w3i**2*cs**2*k**2*rhogsol - w3r*rhogeq**2*w3i**2*k*w1r**2*vgsol + w3r*rhogeq*w3i**2*cs**2*k**2*rhogsol*w1r +  2*w3r*rhogeq*cs**2*k**3*w3i*Kdrag*vdsol -
w3r*rhogeq*cs**4*k**4*w2r*rhogsol +  w3r*rhogeq**2*cs**2*k**3*w2r**2*vgsol + w3r**4*rhogeq**2*w2r*vgsol*k - w3r**4*rhogeq*w2r*w1r*rhogsol +  w3r**4*rhogeq**2*k*w1r*vgsol -
w3r**4*rhogeq*rhogsol*k**2*cs**2 + w3r**4*rhogeq*w2i*rhogsol*w1i +2*w3r*rhogeq*w2i*w3i*w1i*k*Kdrag*vgsol + 2*w3r*rhogeq**2*w2i*k*vgsol*w3i*w1r**2 +
2*w3r*rhogeq*w2r*w3i*w1r*k*Kdrag*vdsol - 2*w3r*rhogeq*w2i*w3i**2*k*Kdrag*vgsol - w3r*w3i**2*Kdrag**2*k*vgsol + w3r*w3i**2*Kdrag**2*k*vdsol - w3r*rhogeq**2*w1i**2*k*vgsol*w3i**2 -
2*w3r*rhogeq**2*w3i**2*k*w2i*w1i*vgsol + 2*w3r*rhogeq**2*w2i*w1i**2*w3i*k*vgsol + w3r*w3i**2*Kdrag*rhogsol*w1i*w2r + w3r*w3i**2*Kdrag*rhogsol*w2i*w1r -
w3r*Kdrag*rhogsol*k**2*cs**2*w1i*w2r - w3r*Kdrag*rhogsol*k**2*cs**2*w2i*w1r +  w3r*Kdrag*k*rhogeq*vgsol*w2i*w1r**2 + w3r*Kdrag*k*rhogeq*vgsol*w2i*w1i**2 + w3r*Kdrag**2*k*vdsol*w2r*w1r
-w3r*Kdrag**2*k*vdsol*w2i*w1i - w3r*Kdrag**2*k*vgsol*w2r*w1r + w3r*Kdrag**2*k*vgsol*w2i*w1i + w3r*Kdrag*k*rhogeq*vgsol*w2r**2*w1i + w3r*Kdrag*k*rhogeq*vgsol*w1i*w2i**2 -
w3r*rhogeq**2*w2r**2*w3i**2*k*vgsol  - 2*w3r*rhogeq*w2r*w3i*w1r*k*Kdrag*vgsol + w3r*rhogeq*w2i*w3i**2*k*Kdrag*vdsol - 2*w3r*rhogeq**2*w2r*w3i**2*k*w1r*vgsol +
w3r*rhogeq*w2r**2*w3i**2*w1r*rhogsol -  2*w3r*rhogeq*w1i*w3i**2*k*Kdrag*vgsol + w3r*rhogeq*w1i*w3i**2*k*Kdrag*vdsol + 2*w3r*rhogeq**2*w3i*w2r**2*w1i*k*vgsol +
w3r*rhogeq*w1i**2*w3i**2*w2r*rhogsol - w3r*rhogeq**2*w3i**2*w2i**2*k*vgsol - w3r*rhogeq*cs**2*k**2*w1i**2*w2r*rhogsol -   w3r*rhogeq*cs**2*k**2*w2r*w1r**2*rhogsol -
w3r*rhogeq*cs**2*k**2*w2r**2*w1r*rhogsol + w3r*rhogeq*w2i**2*w3i**2*w1r*rhogsol + w3r*rhogeq*w2r*w3i**2*w1r**2*rhogsol - 2*w3r*rhogeq**2*cs**2*k**3*w3i*w2i*vgsol -
w3r*rhogeq*w2i**2*w1r*cs**2*k**2*rhogsol +  2*w3r*rhogeq**2*w2i**2*w3i*w1i*k*vgsol + w3r*rhogeq*cs**2*k**3*w1i*Kdrag*vgsol - w3r*rhogeq*cs**2*k**3*w2i*Kdrag*vdsol +
w3r*rhogeq*cs**2*k**3*w2i*Kdrag*vgsol - w3r*rhogeq*cs**2*k**3*w1i*Kdrag*vdsol + w3r*rhogeq**2*cs**2*k**3*w1i**2*vgsol -  2*w3r*rhogeq**2*cs**2*k**3*w3i*w1i*vgsol +
w3r*rhogeq**2*cs**2*k**3*w1r**2*vgsol - w3r*rhogeq*cs**4*k**4*w1r*rhogsol - 2*w3r*rhogeq*cs**2*k**3*w3i*Kdrag*vgsol + 2*w3r*rhogeq**2*cs**2*k**3*w2r*w1r*vgsol -
2*w3r*rhogeq*w2i*w3i*w1i*k*Kdrag*vdsol + 2*w3r*rhogeq**2*cs**2*k**3*w2i*w1i*vgsol + w3r**2*w3i*rhogeq*rhogsol*k**2*cs**2*w1i - w3r**2*w3i*rhogeq*rhogsol*w2r**2*w1i -
w3r**2*w3i*rhogeq*rhogsol*w1i*w2i**2 - w3r**2*w3i*rhogeq*rhogsol*w2i*w1r**2 - w3r**2*w3i*rhogeq*rhogsol*w2i*w1i**2 - w3r**2*w3i*rhogeq*k*Kdrag*vdsol*w2r -
w3r**2*Kdrag*rhogsol*w3i*w2r*w1r + w3r**2*Kdrag*rhogsol*w2i*w3i*w1i - w3r**2*w3i*rhogeq*k*Kdrag*vdsol*w1r + w3r**2*w3i*rhogeq*rhogsol*k**2*cs**2*w2i -
w3r**2*Kdrag*rhogsol*k**2*cs**2*w3i + w3r**2*Kdrag*rhogsol*k**2*cs**2*w2i + w3r**2*Kdrag*rhogsol*k**2*cs**2*w1i + 2*w3r**2*Kdrag*k*rhogeq*vgsol*w2r*w3i +
2*w3r**2*Kdrag*k*rhogeq*vgsol*w3i*w1r + 2*w3r**2*rhogeq**2*w2r*k*vgsol*w3i**2 - w3r**2*rhogeq**2*cs**2*k**3*w2r*vgsol + w3r**2*rhogeq*w2r*w1i*k*Kdrag*vgsol +
w3r**2*rhogeq**2*w2r*w1i**2*k*vgsol - w3r**2*rhogeq*w2r*w1i*k*Kdrag*vdsol + w3r**2*rhogeq**2*w2r*w1r**2*k*vgsol - w3r**2*rhogeq**2*cs**2*k**3*w1r*vgsol - w3r**2*Kdrag**2*k*vdsol*w2r -
w3r**2*rhogeq*w2i**2*w1i**2*rhogsol - w3r**2*rhogeq*w1r**2*w2i**2*rhogsol - w3r**2*rhogeq*w2r**2*w1r**2*rhogsol + w3r**2*rhogeq*cs**4*k**4*rhogsol - w3r**2*rhogeq*w1i**2*w2r**2*rhogsol -
w3r**2*Kdrag*rhogsol*w1i*w2i**2 - w3r**2*Kdrag*rhogsol*w2i*w1r**2 - w3r**2*Kdrag*rhogsol*w2i*w1i**2 - w3r**2*Kdrag*rhogsol*w2r**2*w1i - w3r**2*Kdrag**2*k*vdsol*w1r +
w3r**2*Kdrag**2*k*vgsol*w2r + w3r**2*Kdrag**2*k*vgsol*w1r + w3r**3*Kdrag*rhogsol*w2i*w1r + w3r**3*rhogeq*w2i*k*Kdrag*vdsol - w3r**3*rhogeq**2*w2i**2*k*vgsol +
w3r**2*rhogeq**2*w2i**2*w1r*k*vgsol - 2*w3r**2*rhogeq*w2r*w3i**2*w1r*rhogsol + 2*w3r**2*rhogeq**2*w3i**2*k*w1r*vgsol + 2*w3r**2*rhogeq*w1i*w3i**2*rhogsol*w2i -
w3r**2*rhogeq*w2i*w1r*k*Kdrag*vdsol + w3r**2*rhogeq*w2i*w1r*k*Kdrag*vgsol - w3r**3*rhogeq**2*w1i**2*k*vgsol - 2*w3r**3*rhogeq**2*w2r*k*w1r*vgsol + w3r**3*rhogeq*w2r**2*w1r*rhogsol +
w3r**3*rhogeq*w2r*cs**2*k**2*rhogsol - 2*w3r**3*rhogeq*w2i*k*Kdrag*vgsol - w3r**3*rhogeq**2*w1r**2*k*vgsol + w3r**3*rhogeq*w1r*cs**2*k**2*rhogsol - w3r**3*rhogeq**2*w2r**2*k*vgsol +
w3r**3*rhogeq*w1i*k*Kdrag*vdsol - 2*w3r**3*rhogeq**2*k*w2i*w1i*vgsol + w3r**3*rhogeq*w2r*w1r**2*rhogsol + w3r**3*rhogeq*w1i**2*w2r*rhogsol + w3r**3*rhogeq*w2i**2*w1r*rhogsol -
2*w3r**3*rhogeq*w1i*k*Kdrag*vgsol + w3r**3*Kdrag*rhogsol*w1i*w2r + w3r**2*rhogeq**2*w2r**2*k*w1r*vgsol - 2*w3r**2*rhogeq*w3i**2*cs**2*k**2*rhogsol + w3r**3*Kdrag**2*k*vdsol +
w3i**2*rhogeq**2*cs**2*k**3*w2r*vgsol - w3r**3*Kdrag**2*k*vgsol + w3i**2*rhogeq*w2i*w1r*k*Kdrag*vdsol - w3i**2*Kdrag**2*k*vdsol*w2r + w3i**2*Kdrag**2*k*vgsol*w2r -
w3i**2*Kdrag*rhogsol*w2r**2*w1i - w3i**2*Kdrag*rhogsol*w1i*w2i**2 - w3i**2*Kdrag*rhogsol*w2i*w1r**2 - w3i**2*Kdrag*rhogsol*w2i*w1i**2 + w3i**2*Kdrag*rhogsol*k**2*cs**2*w2i +
w3i**2*Kdrag*rhogsol*k**2*cs**2*w1i - Kdrag*k*rhogeq*vgsol*w3i*w1r*w2r**2 - Kdrag*k*rhogeq*vgsol*w3i*w2r*w1r**2 - Kdrag*k*rhogeq*vgsol*w3i*w2r*w1i**2 -
Kdrag*k*rhogeq*vgsol*w3i*w1r*w2i**2 + Kdrag*rhogsol*k**2*cs**2*w3i*w2r*w1r - Kdrag*rhogsol*k**2*cs**2*w2i*w3i*w1i + Kdrag*rhogsol*w2r**2*w1i**2*w3i + Kdrag*rhogsol*w3i*w1r**2*w2r**2 +
Kdrag*rhogsol*w1i**2*w2i**2*w3i + Kdrag*rhogsol*w2i**2*w1r**2*w3i + Kdrag**2*k*vdsol*w2r*w1i*w3i + Kdrag**2*k*vdsol*w2i*w3i*w1r - Kdrag**2*k*vgsol*w2r*w1i*w3i -
Kdrag**2*k*vgsol*w2i*w3i*w1r - w3i**2*Kdrag**2*k*vdsol*w1r + rhogeq*cs**2*k**3*w2i*w1r*Kdrag*vdsol - rhogeq*cs**2*k**3*w2i*w1r*Kdrag*vgsol - w3i**2*rhogeq**2*w2i**2*w1r*k*vgsol -
w3i**2*rhogeq**2*w2r**2*k*w1r*vgsol - w3i**2*rhogeq*w2i*w1r*k*Kdrag*vgsol - w3i**2*rhogeq*w2r*w1i*k*Kdrag*vgsol - w3i**2*rhogeq**2*w2r*w1i**2*k*vgsol +
w3i**2*rhogeq*w2r*w1i*k*Kdrag*vdsol - w3i**2*rhogeq**2*w2r*w1r**2*k*vgsol + w3i**2*rhogeq**2*cs**2*k**3*w1r*vgsol + w3i**2*rhogeq*w2i**2*w1i**2*rhogsol +
w3i**2*rhogeq*w1r**2*w2i**2*rhogsol + w3i**2*rhogeq*w2r**2*w1r**2*rhogsol - w3i**2*rhogeq*cs**4*k**4*rhogsol + w3i**2*rhogeq*w1i**2*w2r**2*rhogsol)/(w1i**2 - 2*w3i*w1i + w3r**2 + w1r**2 +
w3i**2 - 2*w3r*w1r)/(w2r**2 - 2*w3r*w2r + w2i**2 + w3i**2 - 2*w2i*w3i + w3r**2)/k/rhogeq/Kdrag
```

Resolution study

Laibe & Price, 2012, MNRAS 420, 2345

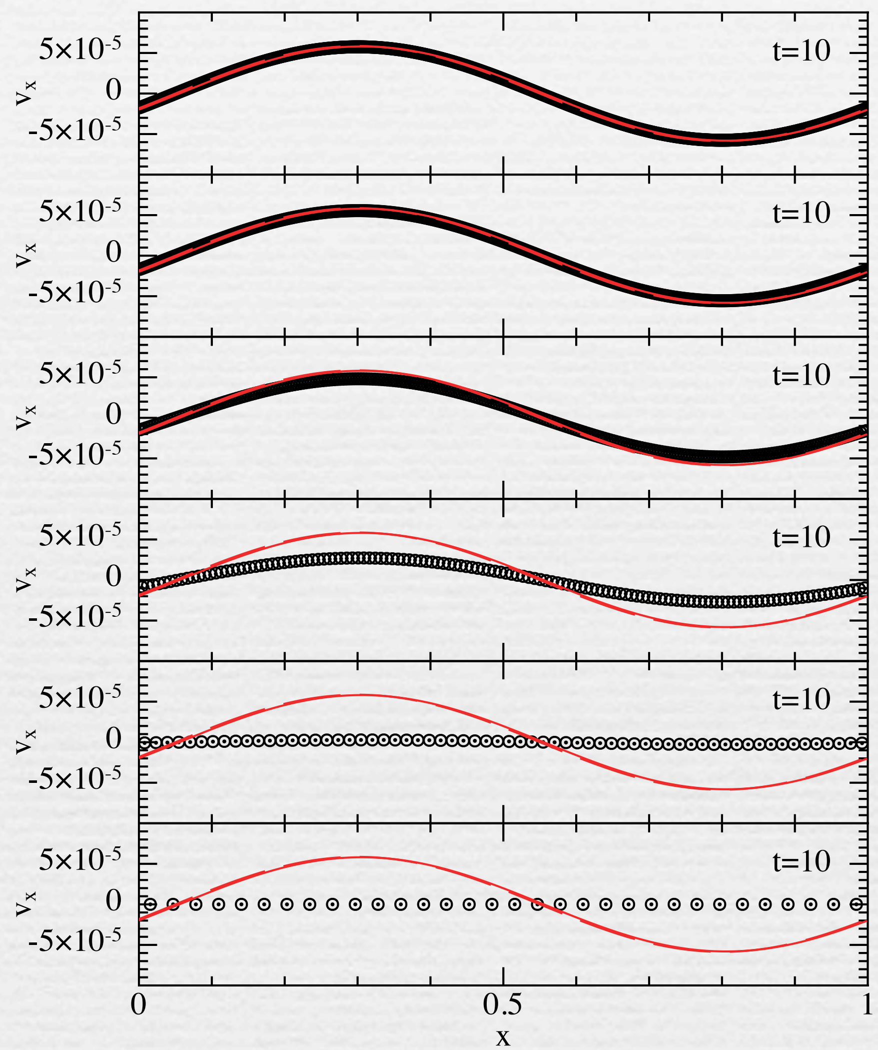
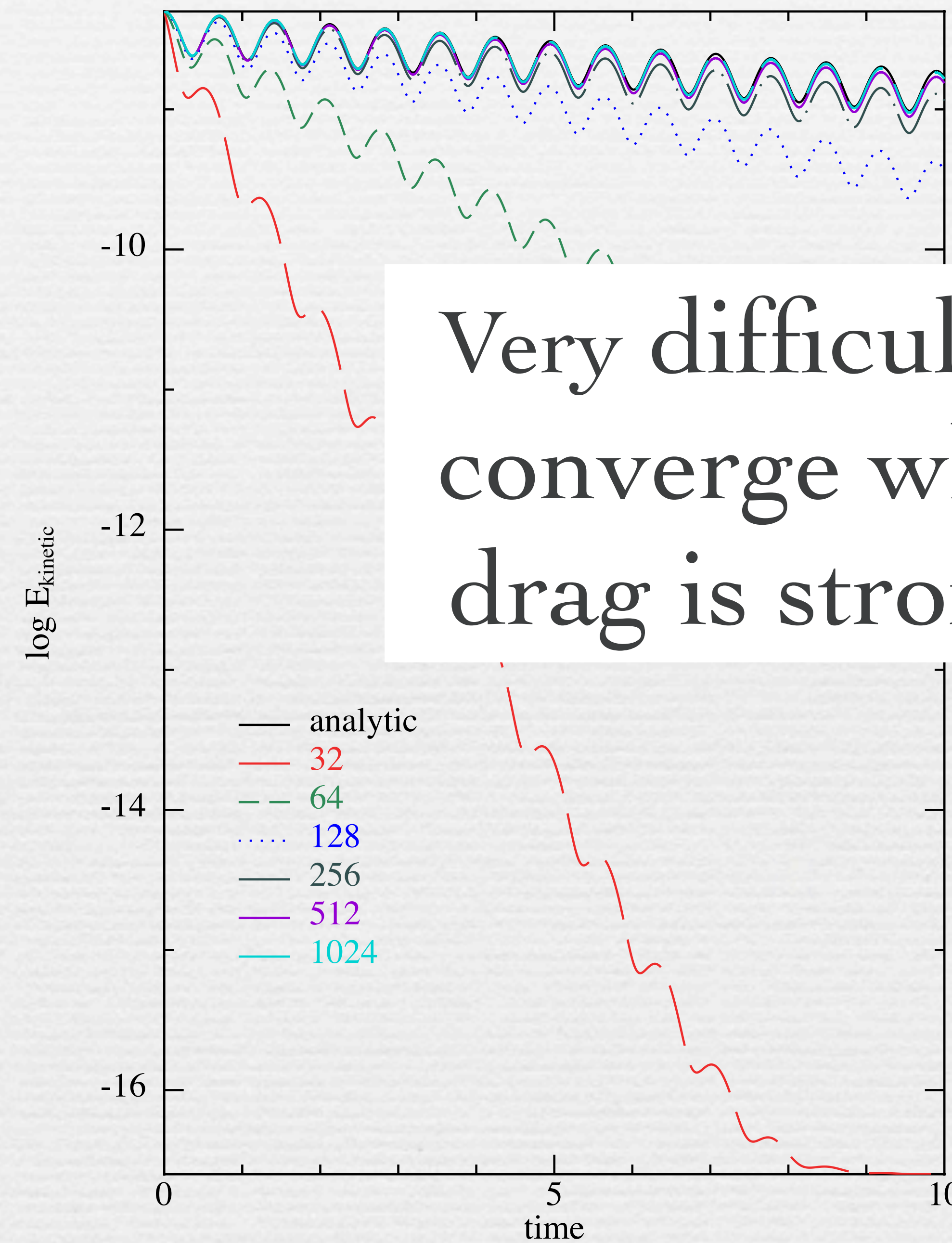


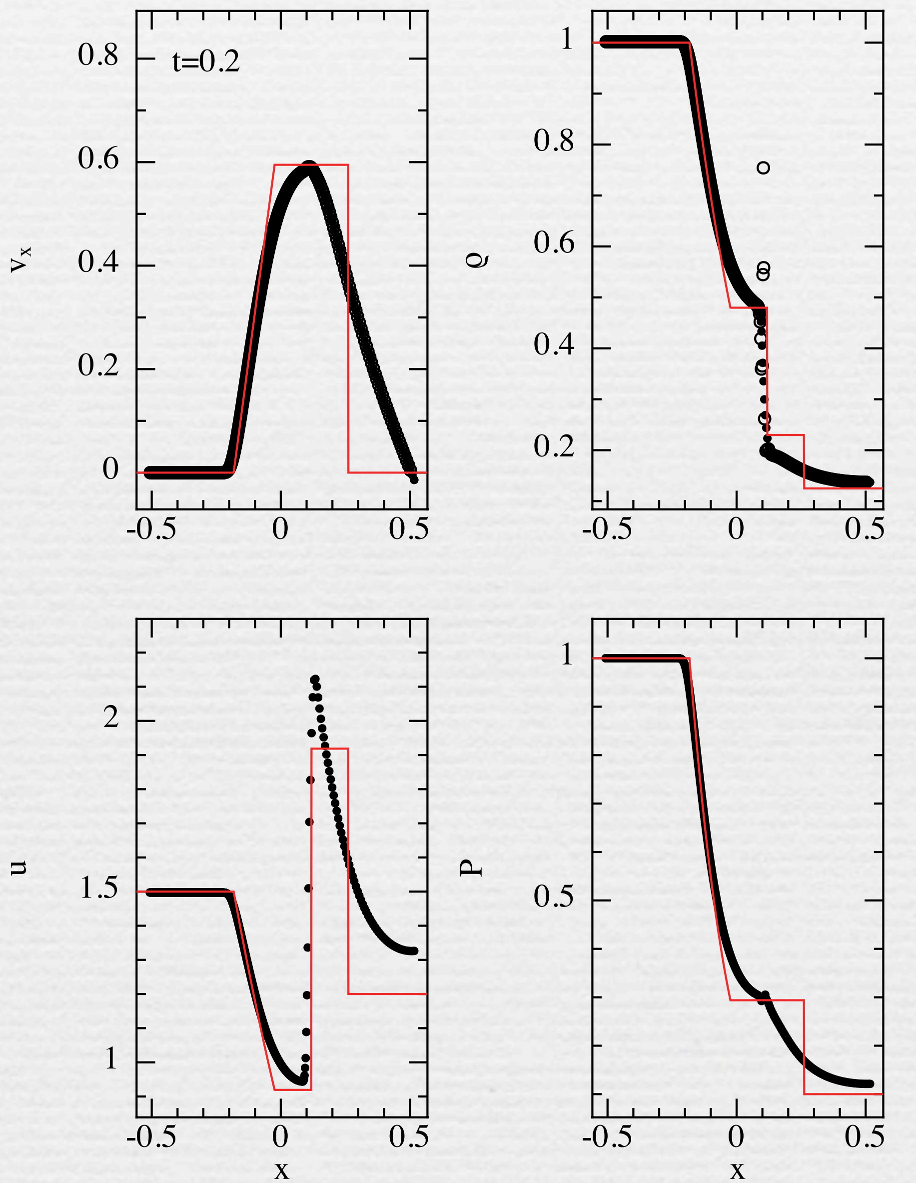
Figure 8. Resolution study for the DUSTYWAVE test in 1D using a high drag coefficient (K = 100) and a dust-to-gas ratio of unity using 32, 64, 128, 256, 512 and 1024 particles from bottom to top. At large drag high resolution is required to resolve the small differential motions between the fluids and thus prevent over-damping of the numerical solution, corresponding to the criterion $h \lesssim c_s t_s$, here implying $\gtrsim$ 240 particles. See also Fig. 9.



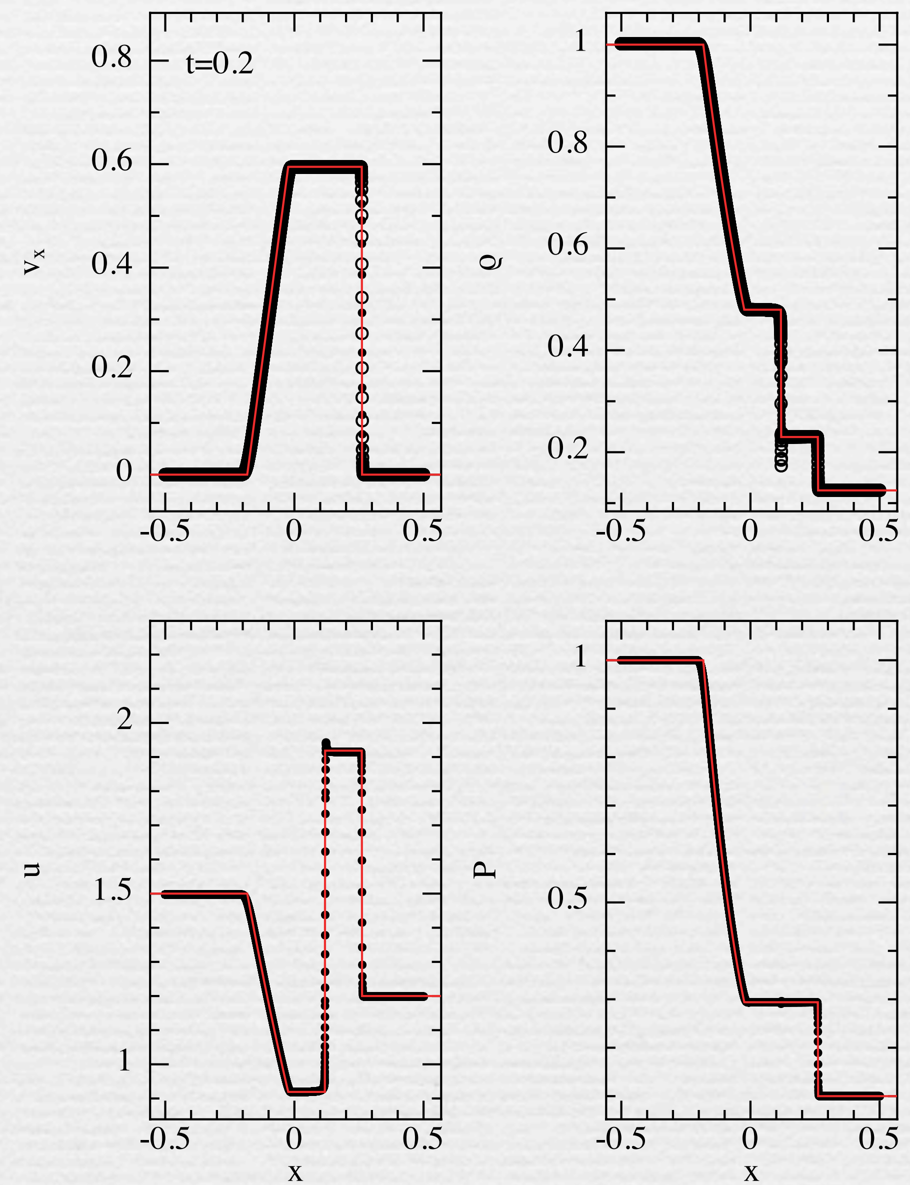
Very difficult to converge when drag is strong!

Dustyshock

Laibe & Price, 2012, MNRAS 420, 2345



sensible resolution



ludicrous resolution

RESOLUTION CRITERION

Laibe & Price, 2012, MNRAS 420, 2345

Temporal: $\Delta t < t_{\rm stop}$ (can be fixed with implicit timestepping methods)

Spatial: $\Delta x \lesssim t_{\rm stop} c_{\rm s}$ (cannot be fixed)

$t_{\rm stop} \to 0$ $(K \to \infty)$ implies $\Delta t \to 0$, $\Delta x \to 0$

- Require infinite timesteps AND infinite resolution in the obvious limit of perfect coupling!

DUSTY GAS WITH ONE FLUID

Laibe & Price (2013, submitted to MNRAS)

- Reformulate equations on the barycentre of both fluids

$$\mathbf{v} \equiv \frac{\rho_{\rm g}\mathbf{v}_{\rm g} + \rho_{\rm d}\mathbf{v}_{\rm d}}{\rho_{\rm g} + \rho_{\rm d}}$$

- Change of variables, from $\mathbf{v}_{\rm g}, \mathbf{v}_{\rm d}, \rho_{\rm g}, \rho_{\rm d}$

  to $\mathbf{v}, \Delta\mathbf{v}, \rho, \rho_{\rm d}/\rho_{\rm g}$

TWO BECOME ONE

A phoenix from the ashes

- One mixture with decay of differential velocity

$$
\begin{aligned}
\frac{\mathrm{d}\rho}{\mathrm{d}t} &= -\rho(\nabla.\mathbf{v}), \\
\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} &= \mathbf{f} - \frac{\nabla P_{\mathrm{g}}}{\rho} - \frac{1}{\rho}\nabla\left(\frac{\rho_{\mathrm{g}}\rho_{\mathrm{d}}}{\rho}\Delta\mathbf{v}^2\right), \\
\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\rho_{\mathrm{d}}}{\rho_{\mathrm{g}}}\right) &= -\frac{\rho}{\rho_{\mathrm{g}}^2}\nabla\cdot\left(\frac{\rho_{\mathrm{g}}\rho_{\mathrm{d}}}{\rho}\Delta\mathbf{v}\right), \\
\frac{\mathrm{d}\Delta\mathbf{v}}{\mathrm{d}t} &= -\frac{\Delta\mathbf{v}}{t_{\mathrm{s}}} + \frac{\nabla P_{\mathrm{g}}}{\rho_{\mathrm{g}}} - (\Delta\mathbf{v}\cdot\nabla)\mathbf{v} + \frac{1}{2}\nabla\left(\frac{\rho_{\mathrm{d}} - \rho_{\mathrm{g}}}{\rho_{\mathrm{d}} + \rho_{\mathrm{g}}}\Delta\mathbf{v}^2\right).
\end{aligned}
$$

Laibe & Price (2013, submitted to MNRAS)

Strong drag/small grains

Laibe & Price (2013, submitted)

- Equations simplify further in this limit

$$\Delta \mathbf{v} = \frac{\nabla P_{\mathrm{g}}}{\rho_{\mathrm{g}}} t_{\mathrm{s}}$$

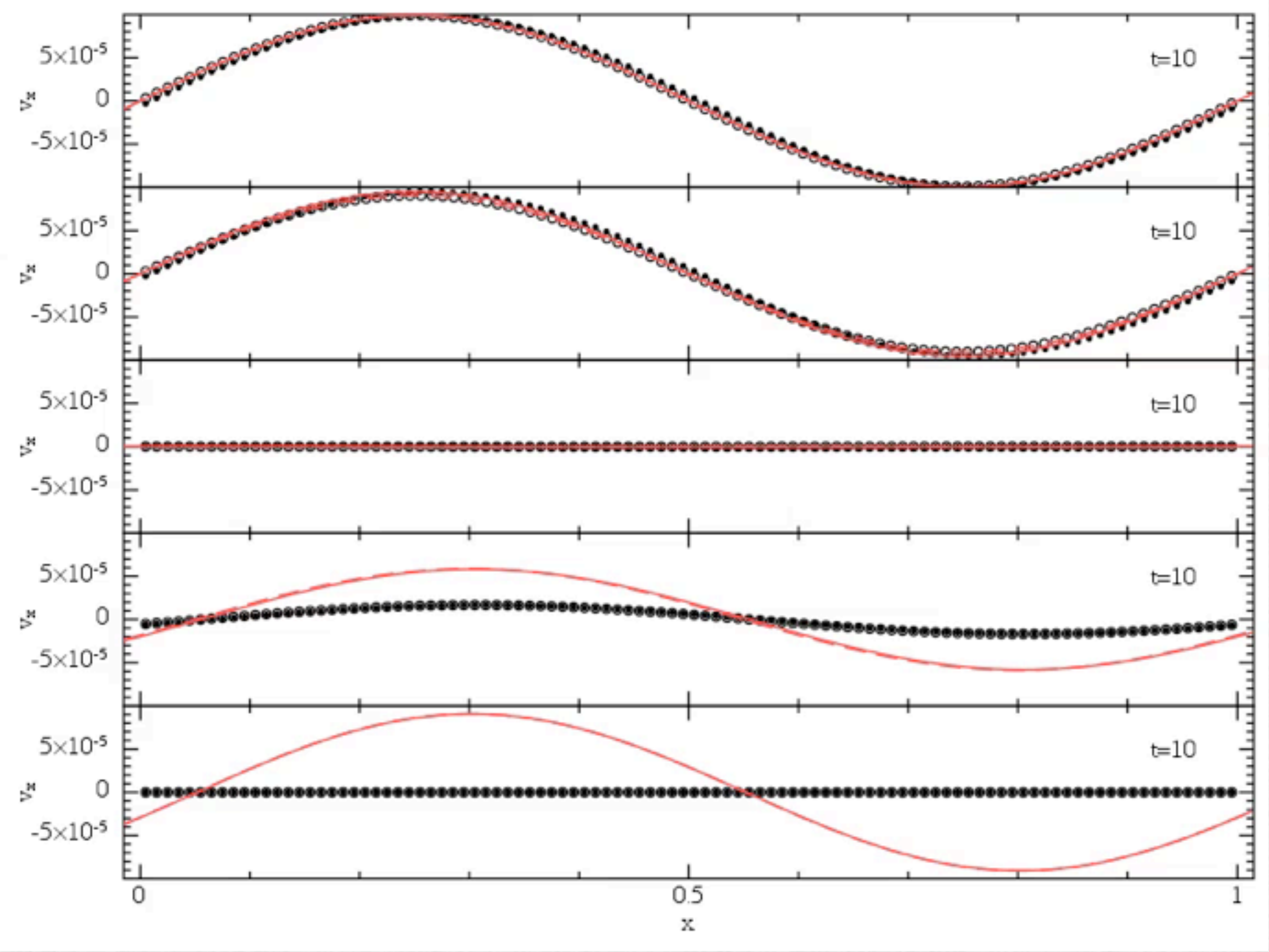
$$\begin{aligned}
\frac{\mathrm{d}\rho}{\mathrm{d}t} &= -\rho(\nabla.\mathbf{v}), \\
\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} &= \mathbf{f} - \frac{\nabla P_{\mathrm{g}}}{\rho}, \\
\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\rho_{\mathrm{d}}}{\rho_{\mathrm{g}}}\right) &= -\frac{\rho}{\rho_{\mathrm{g}}^{2}} \nabla . \left(\frac{\rho_{\mathrm{g}}\rho_{\mathrm{d}}}{\rho}\left[\frac{\nabla P_{\mathrm{g}}}{\rho_{\mathrm{g}}} t_{\mathrm{s}}\right]\right).
\end{aligned}$$

Note: $P_{\mathrm{g}} = \tilde{c}_{\mathrm{s}}\rho$

Valid when $t_{\mathrm{stop}} < \Delta t$

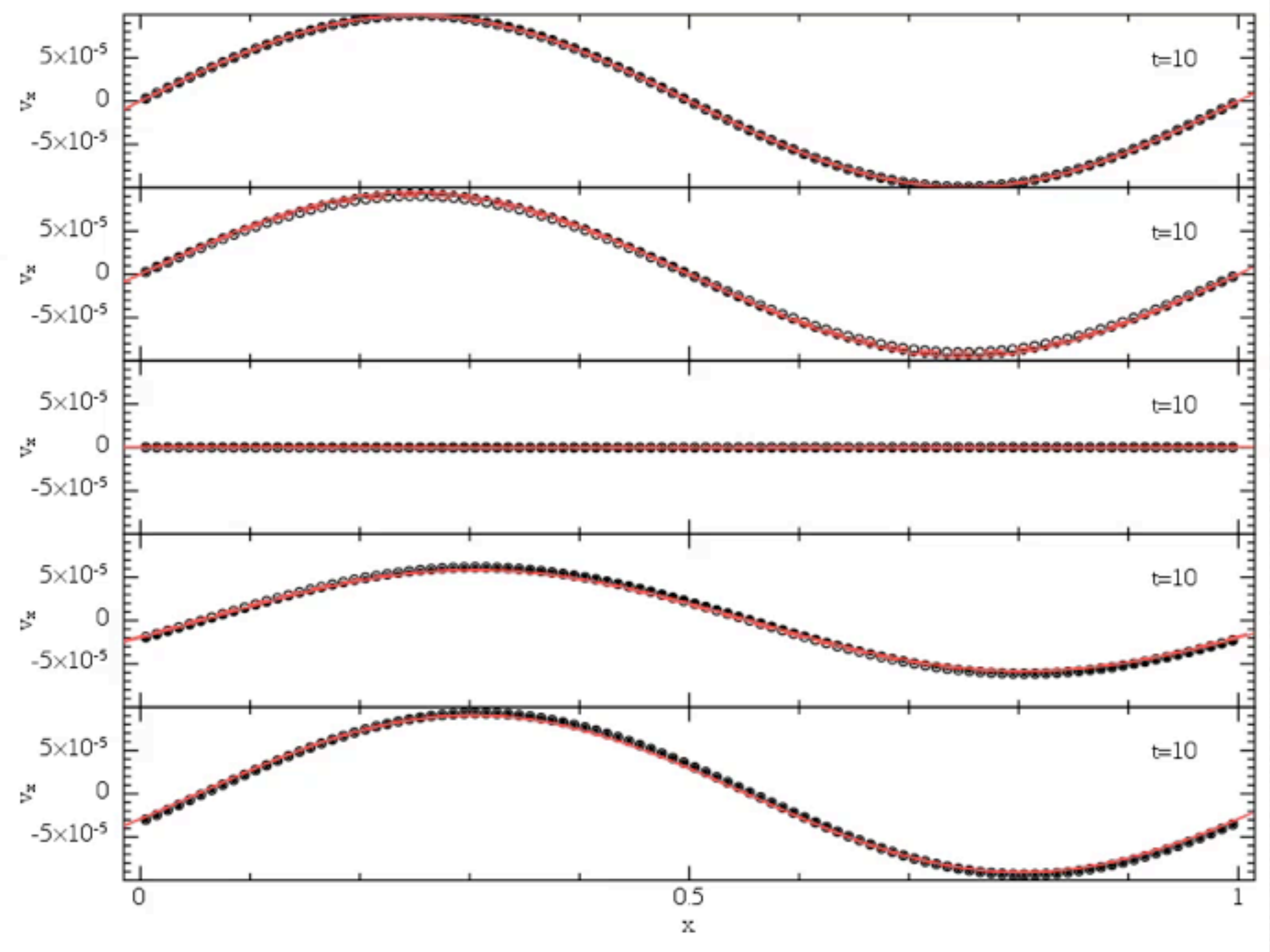
DUSTY WAVES: TWO FLUIDS

Laibe & Price (2012a)

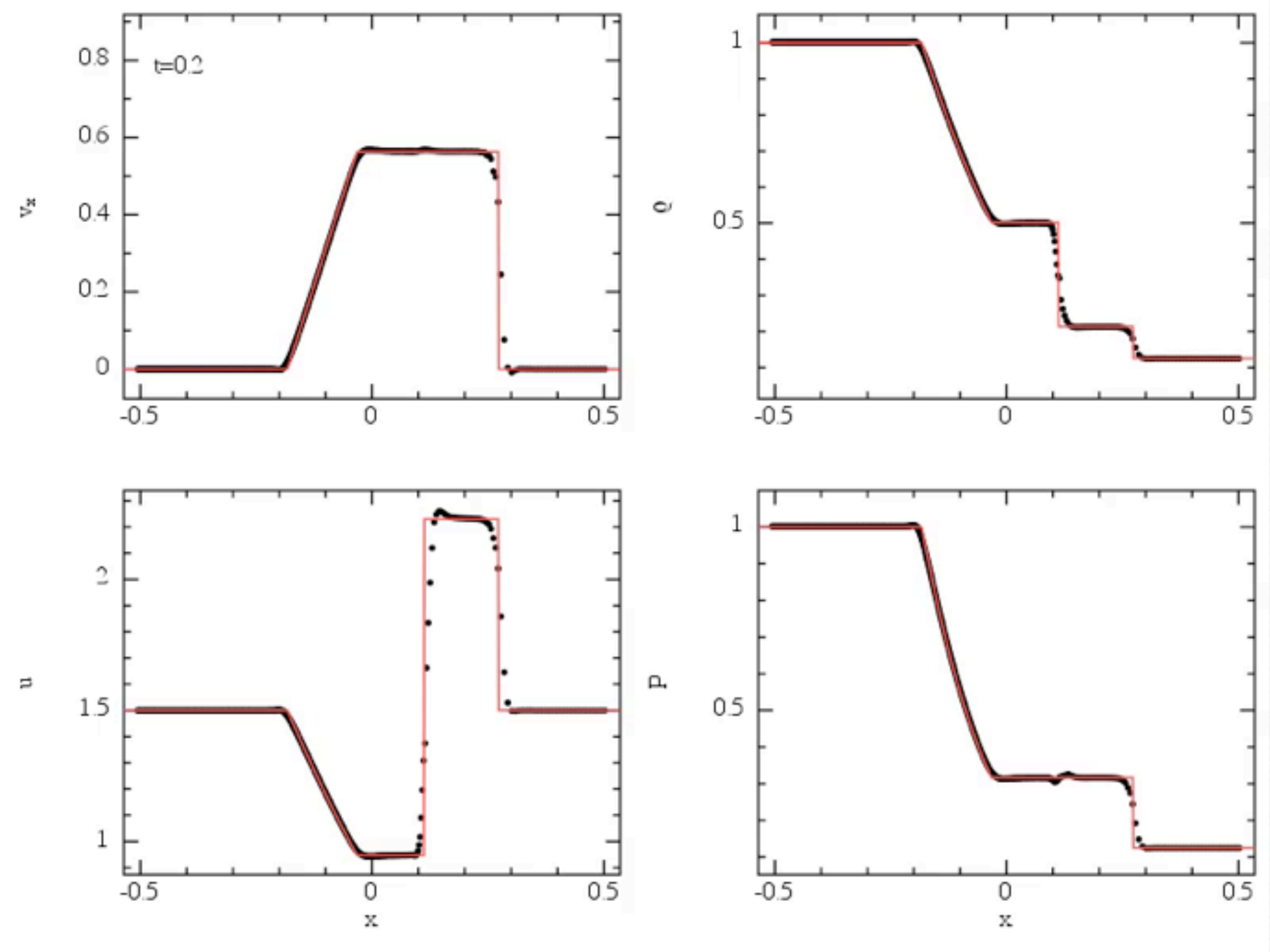


DUSTY WAVES: ONE FLUID

Laibe & Price (2013, in prep)



Dustyshock with one fluid



Summary

- New conservative formulation of hyperbolic divergence cleaning
- Enables robust maintenance of divergence-free condition in Smoothed Particle Magnetohydrodynamics
- Applications to magnetic jets, dynamos
- General spatial resolution issue in two-fluid mixtures in limit of strong drag