Metamorphoses of Graph Designs

Elizabeth Billington

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Outline

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- Metamorphosis of a graph design: what it is

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- Metamorphosis of graph designs: results to date
- A typical construction (an easy case)
- "Complete sets" of metamorphoses from paired graph designs
- Some open questions

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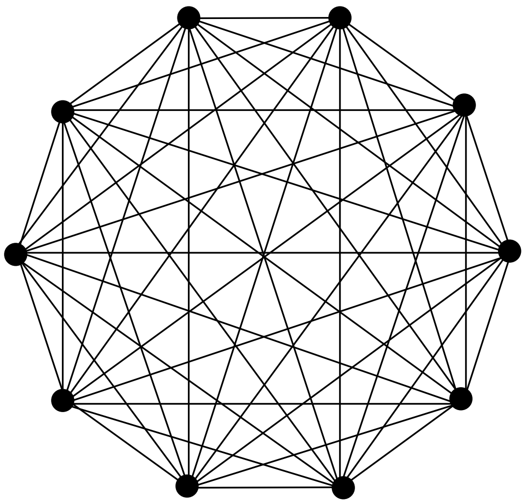

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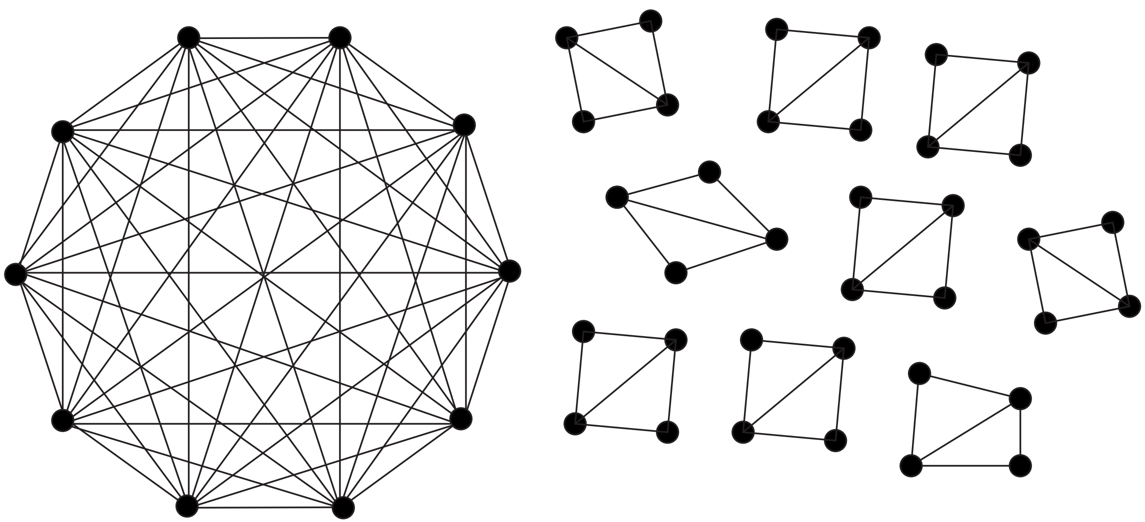

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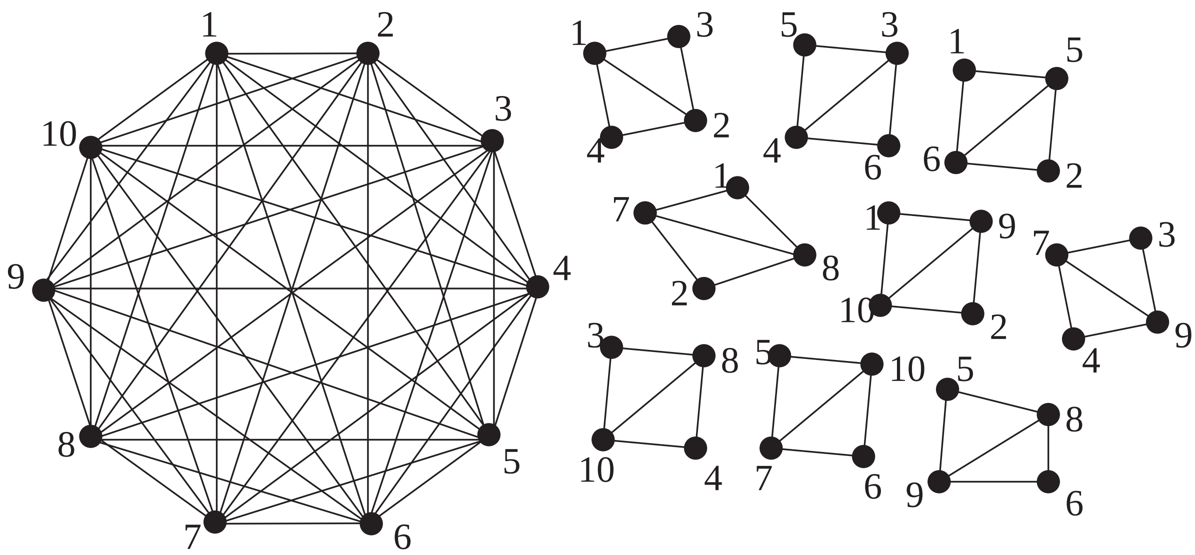

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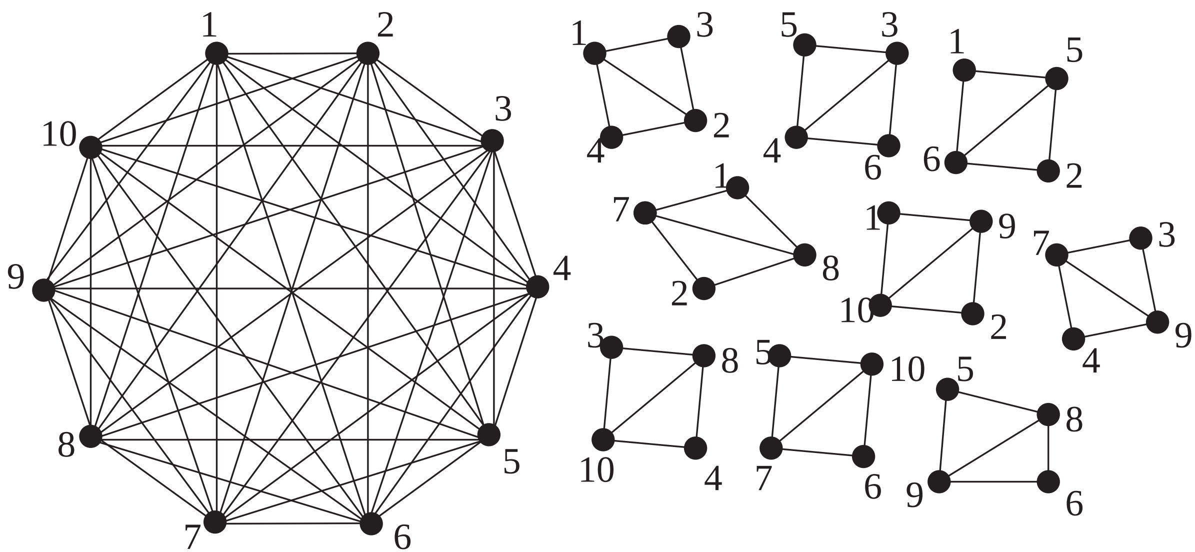

This is a G-design of order 10, where $G = K_4 - e$.

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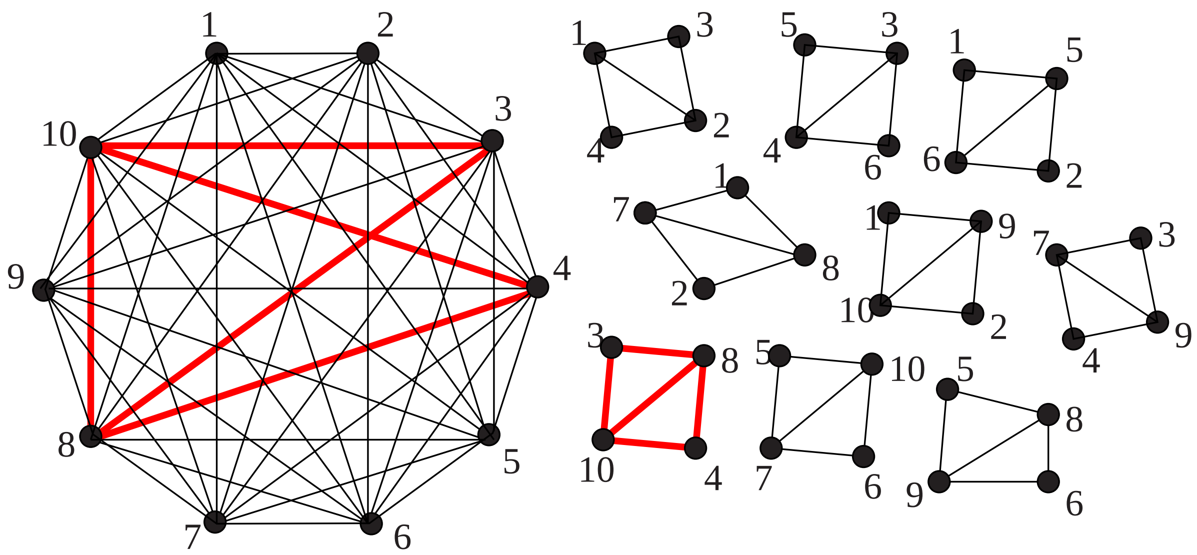

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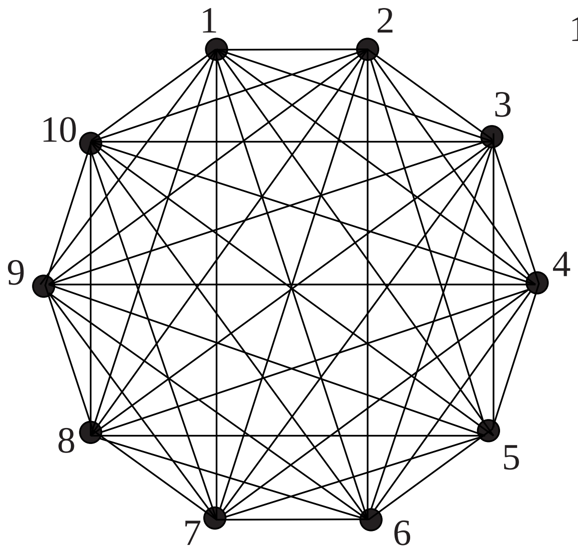

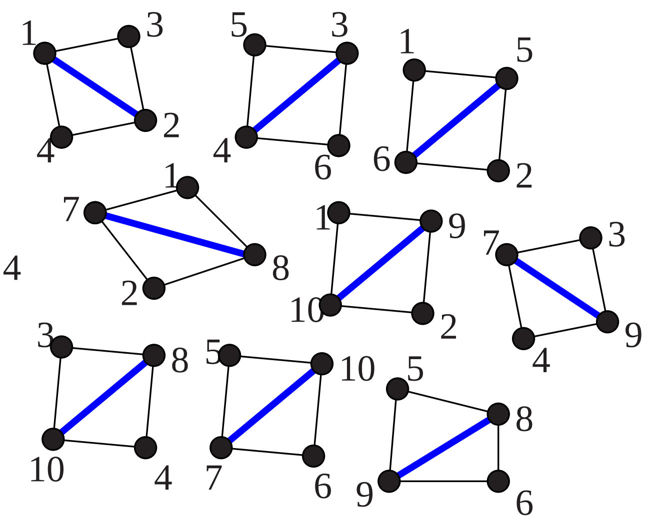

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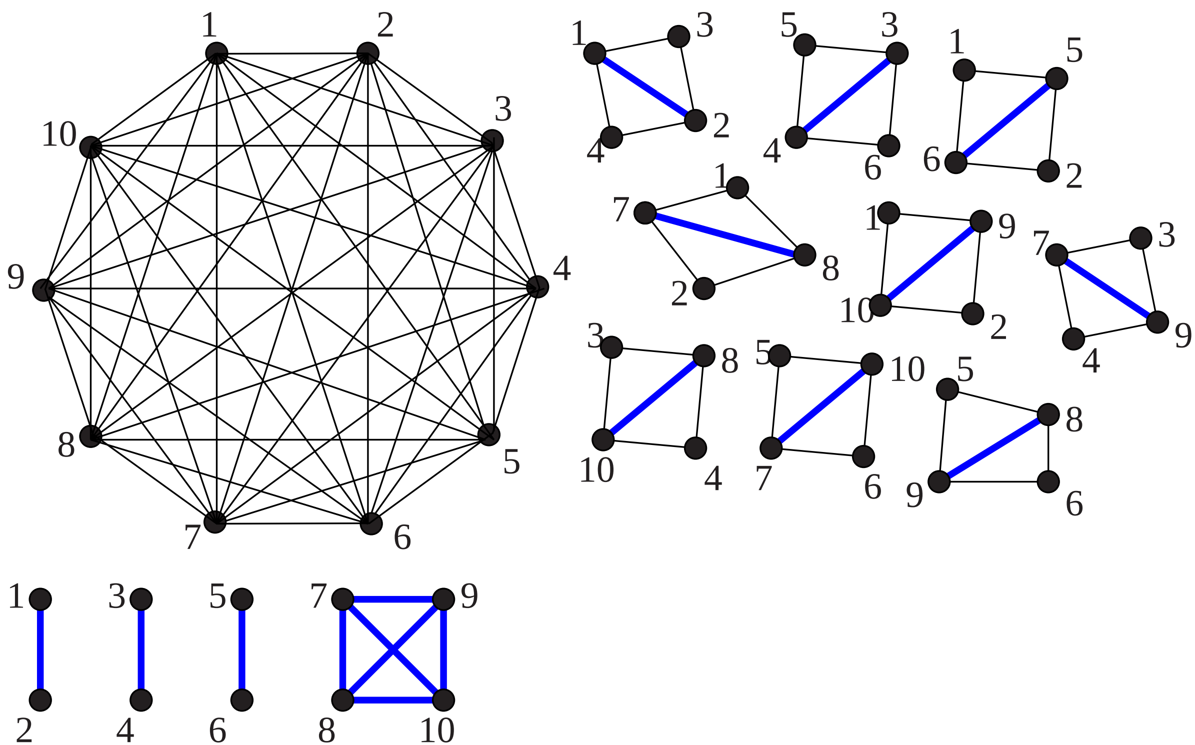

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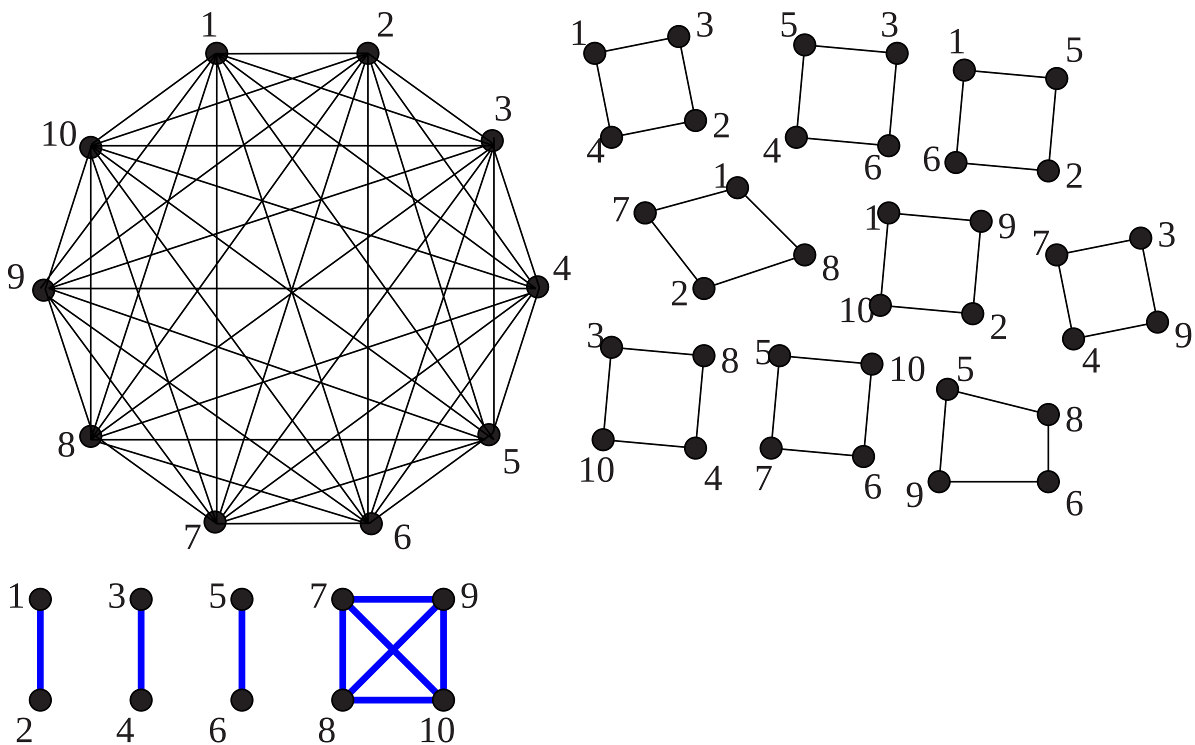

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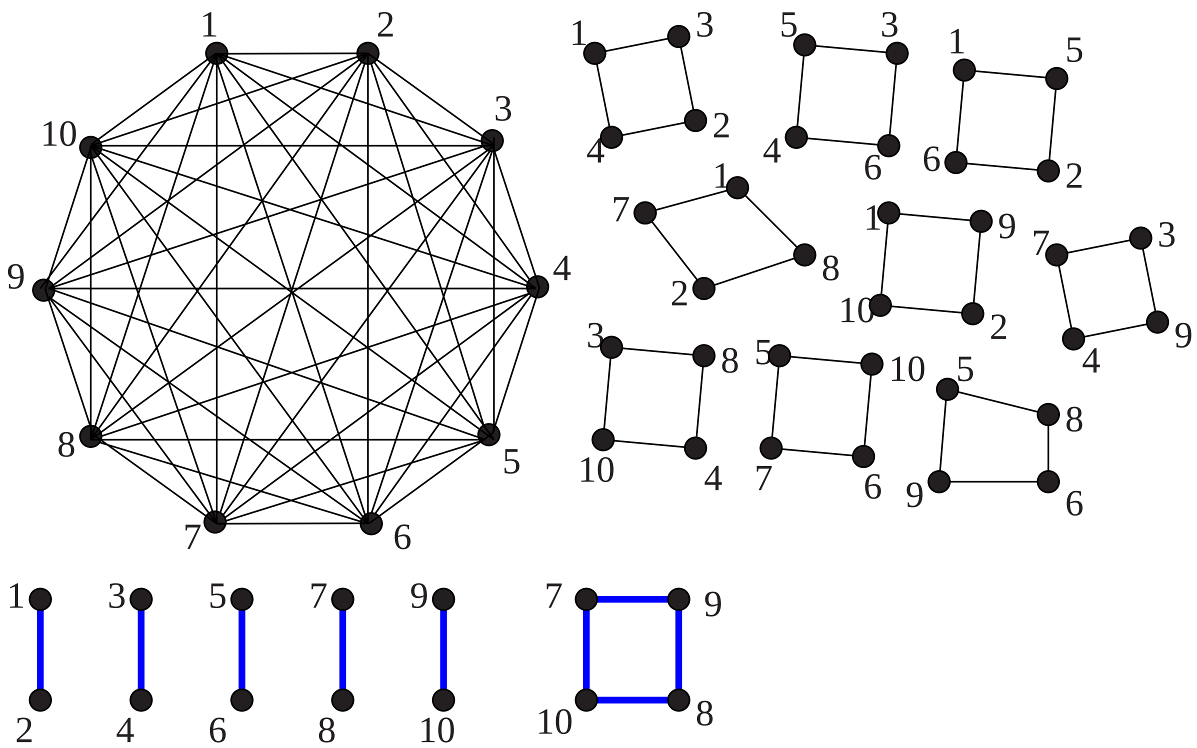

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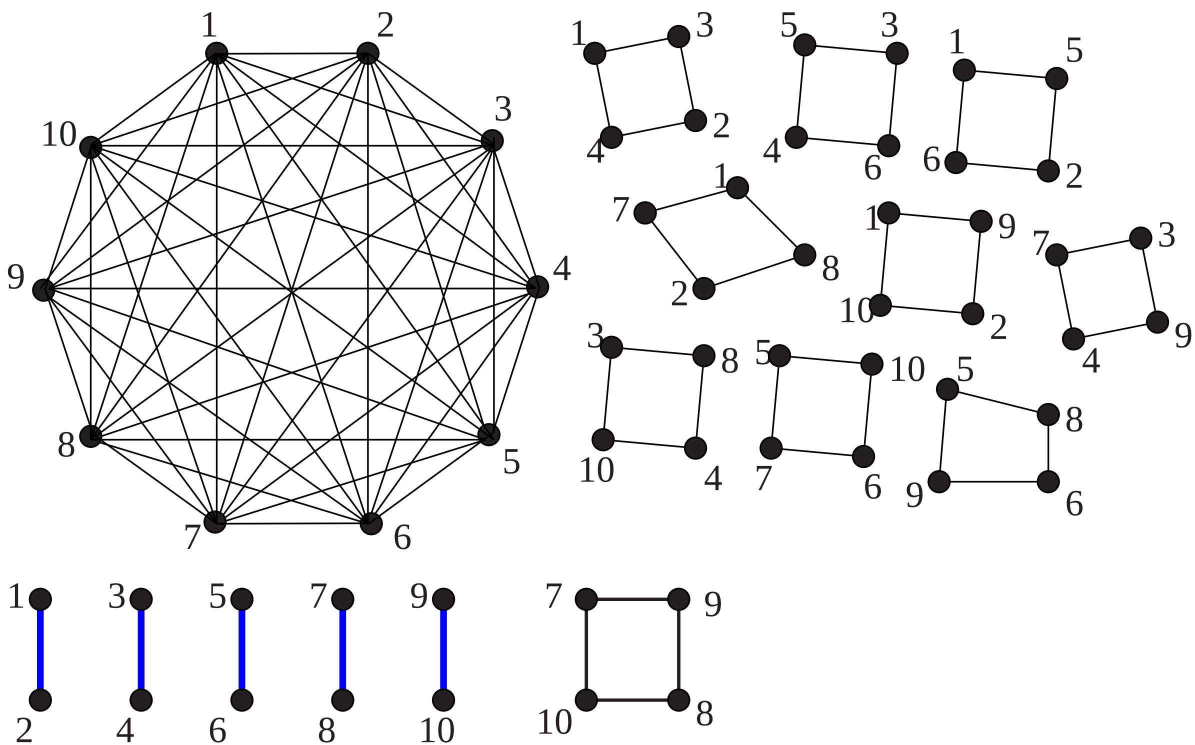

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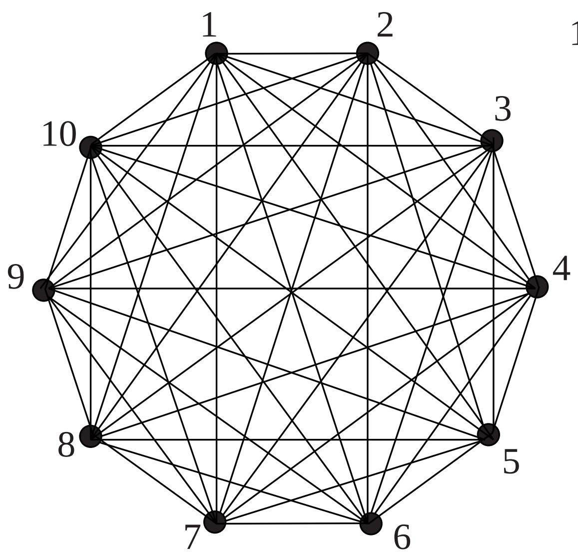

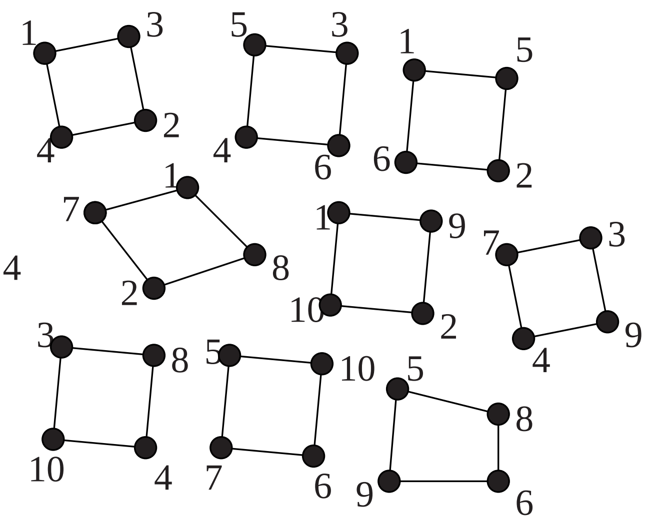

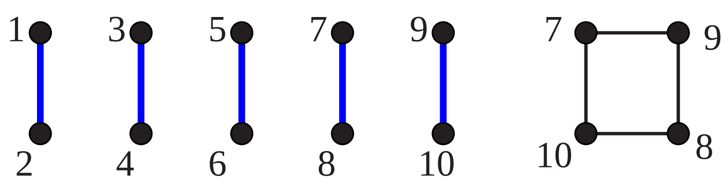

This is a metamorphosis from a $K_4 - e$ design of order 10 into a 4-cycle packing of order 10 with leave a 1-factor.

Graph decomposition and metamorphosis

Such a metamorphosis from some $K_4 - e$ design of order n into a 4-cycle packing (of order n) exists for all orders 0 or 1 (mod 5), but NOT order 11.

(not 5) (Lindner & Tripodi, 2005)

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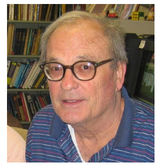

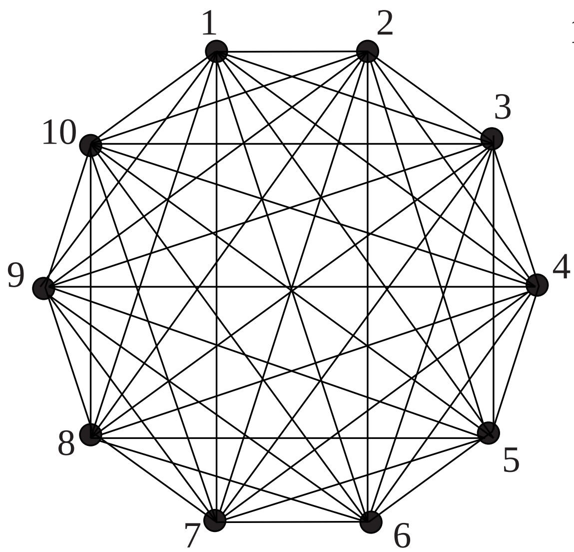

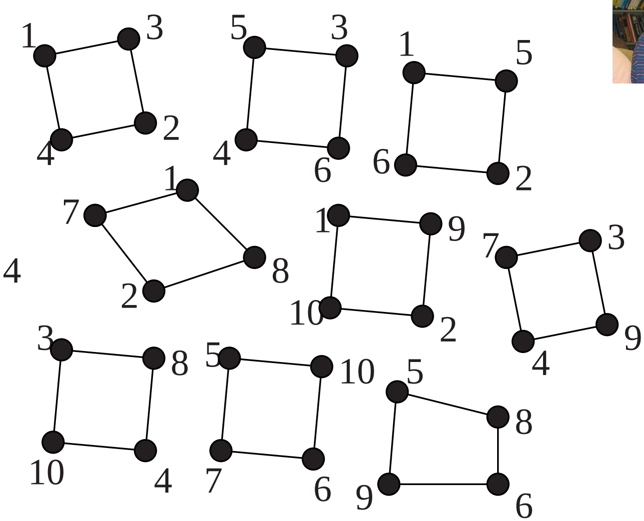

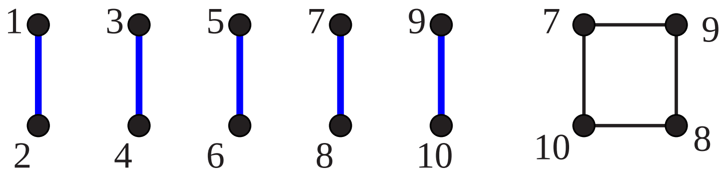

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- $\lambda(n-1)$ must be divisible by gcd of degrees of the vertices in G

G-design to H-design metamorphosis

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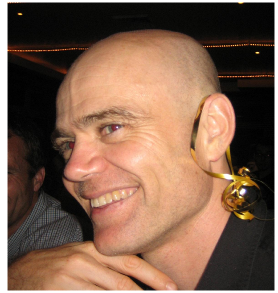
Result is a metamorphosis from a G-design into an H-design of the same order.

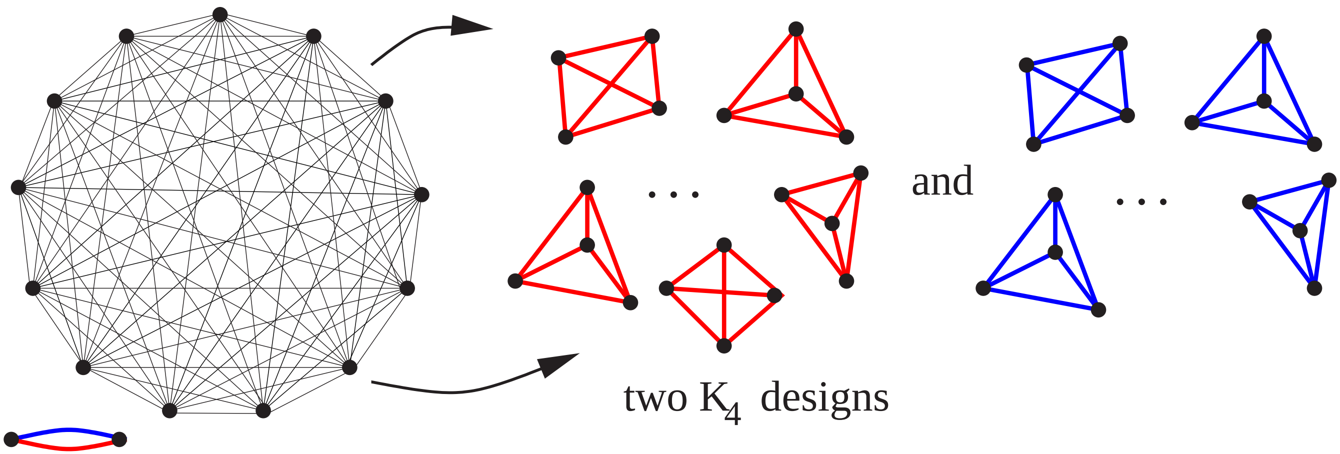
(Or try to get a *maximum packing* of an H-design if the order n isn't right for H!)

Some metamorphosis pre-history

1996: Darryn Bryant
There exist pairs of K_4-designs of order n so that removal of a 3-star (*a point and its adjacent edges*) from each block in both designs (keeping remaining triangles) results in a K_3-design (or Steiner Triple System) if and only if $n \equiv 1 \pmod{12}$.



"Partitionable nested Steiner triple systems".



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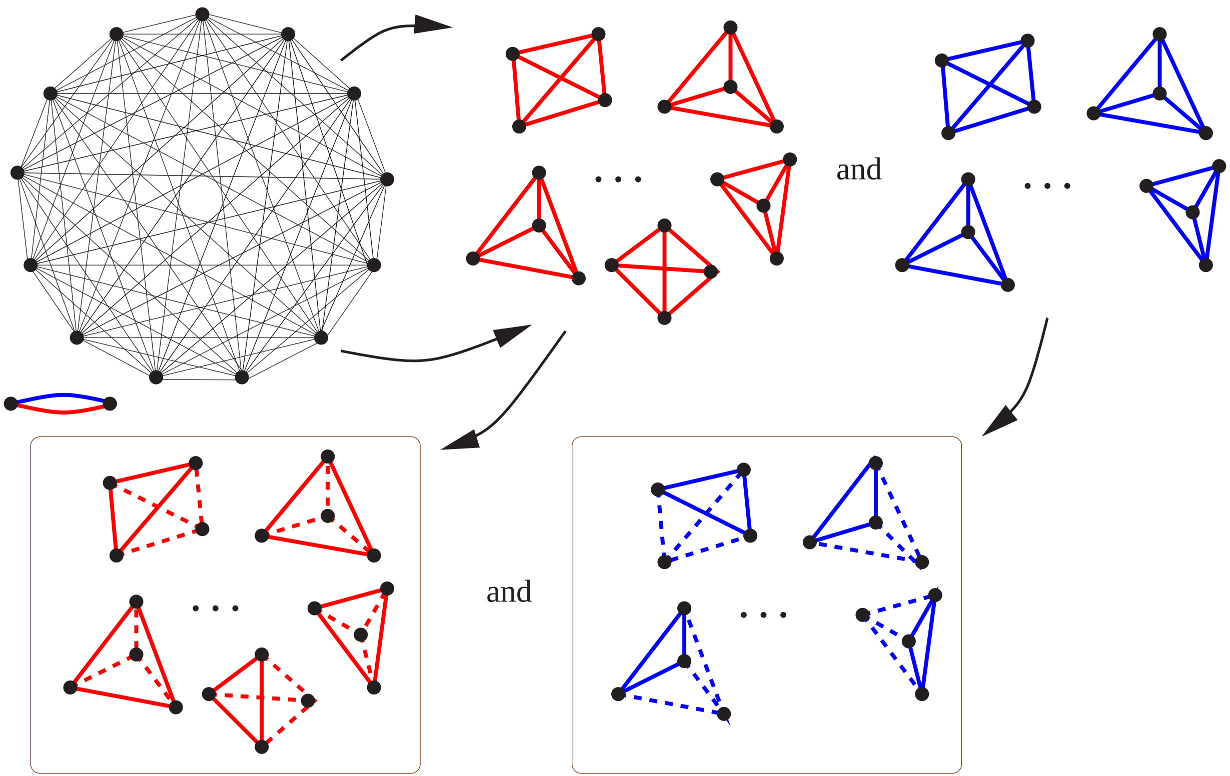

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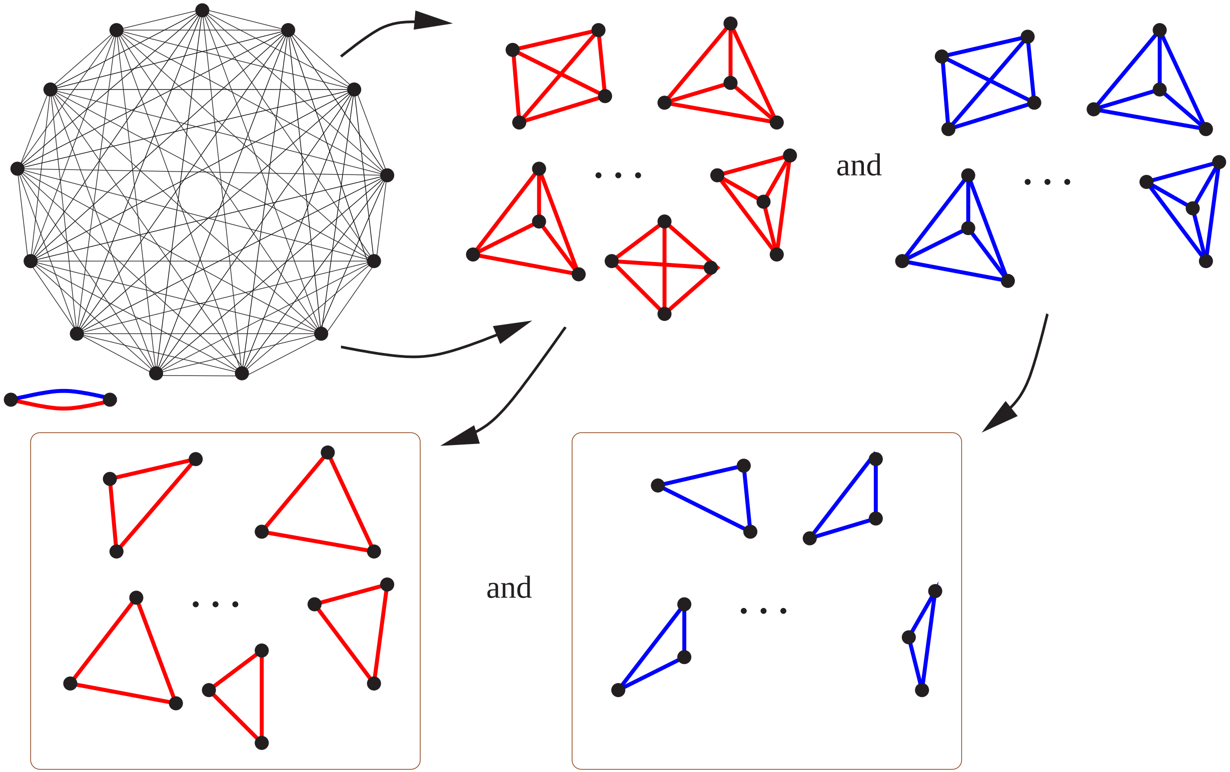

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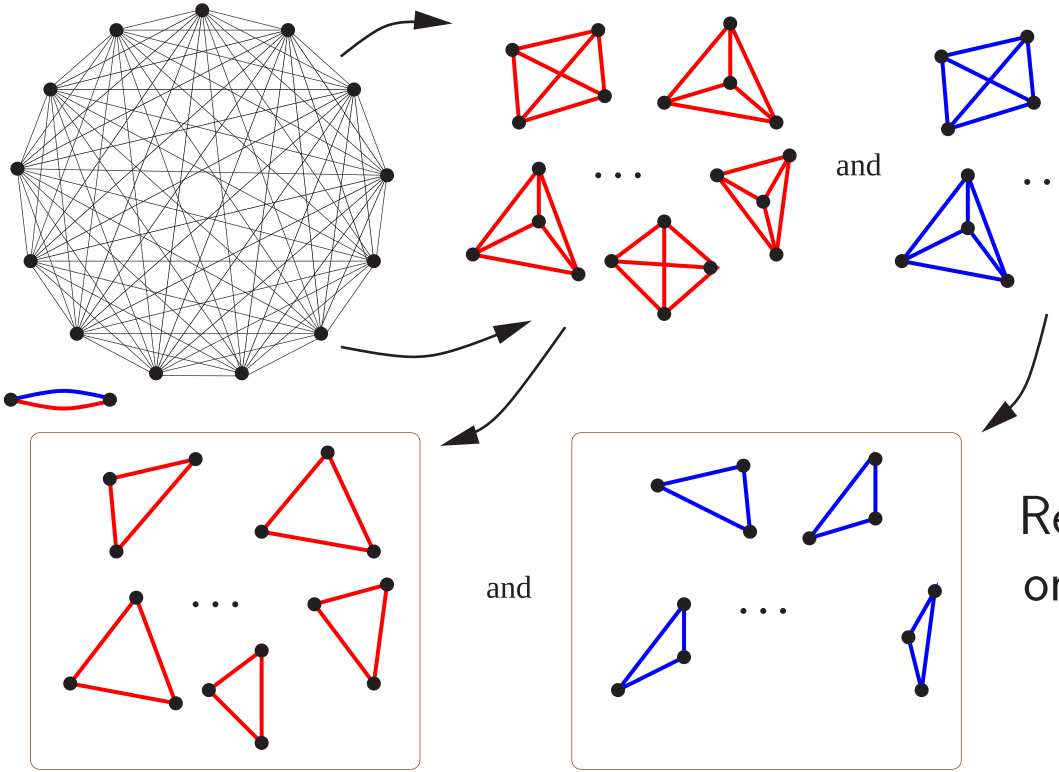

Red and blue triangles form one Steiner triple system

Metamorphoses results: K_4 design into subgraphs

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LINDNER STREET ROSA

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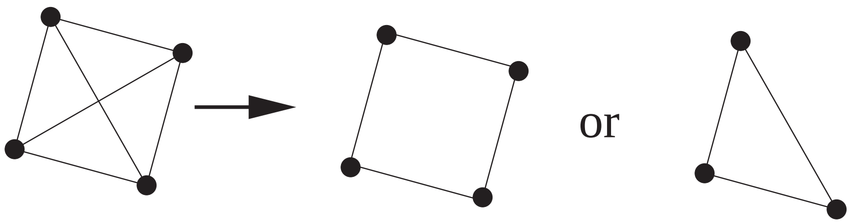
3-cycle Lindner & Rosa (2002)

(Bryant's 1996 "partitionable nesting" did $\lambda = 1$)



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Dancer Smith Yazici
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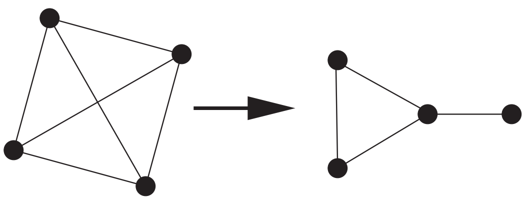

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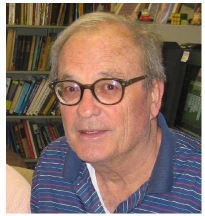
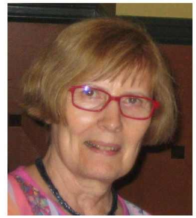


Lindner



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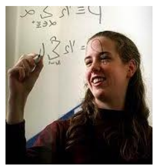


Dancer

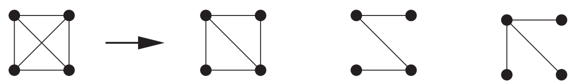
Küçükçifçi

Smith

Yazici

$K_4 - e$ Lindner & Rosa (2002); Lindner & Küçükçifçi (λ-fold, 2003)

P_4; P_3; $K_{1,3}$; two disjoint edges; various.



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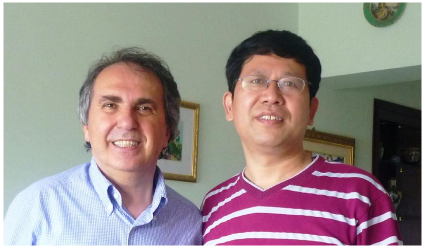
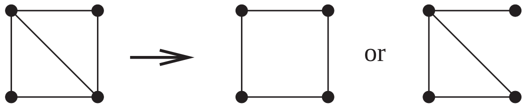


Lo Faro Chang



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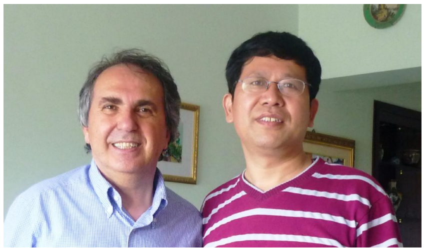

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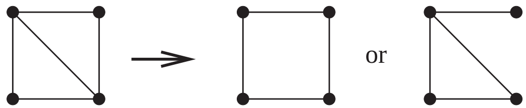

3-cycle + pendant edge ("kite") λ-fold designs into
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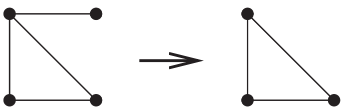

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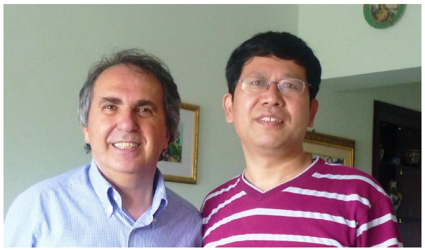
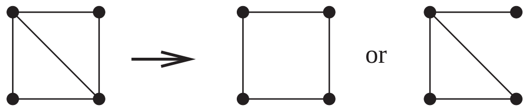
TRIPODI



LO FARO   CHANG



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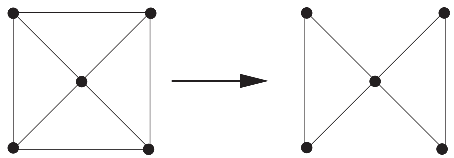
4-wheel into bowtie (λ-fold)   EJB (2000)



Metamorphoses results

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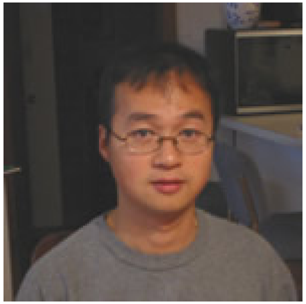

LING QUATTROCCHI

(a) take a maximum packing, already considered;

Metamorphoses results

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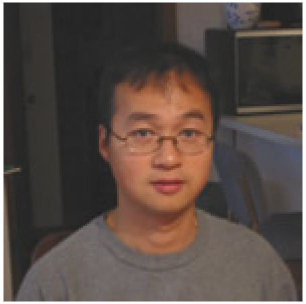

LING QUATTROCCHI

(a) take a maximum packing, already considered;

(b) (reminiscent of embeddings)
with H a subgraph of G, a G-design of order n
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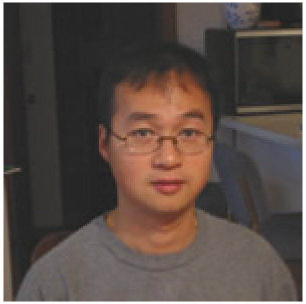 

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Ling and Quattrocchi use attack (b) for λ-fold K_4-designs into λ-fold K_3-designs. They add $v - n = 0, 1$ or 3 new vertices.

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Other G-designs and metamorphoses:

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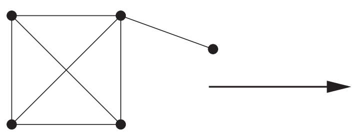

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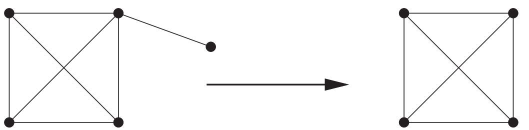

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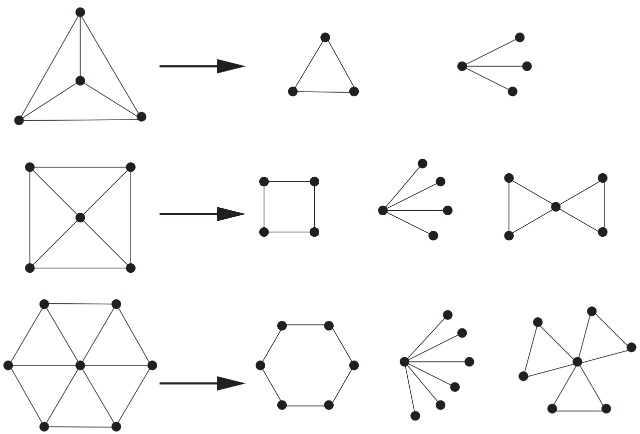
Hypergraph metamorphosis, (3-uniform), $K_4^{(3)}$ into $K_4^{(3)} - e$

Chang, Feng, Lo Faro & Tripodi (2010)

Metamorphoses results: simultaneous metamorphoses

Adams, EJB, Mahmoodian (2003)

Simultaneous metamorphoses of small k-wheel designs for $k = 3, 4, 6$.



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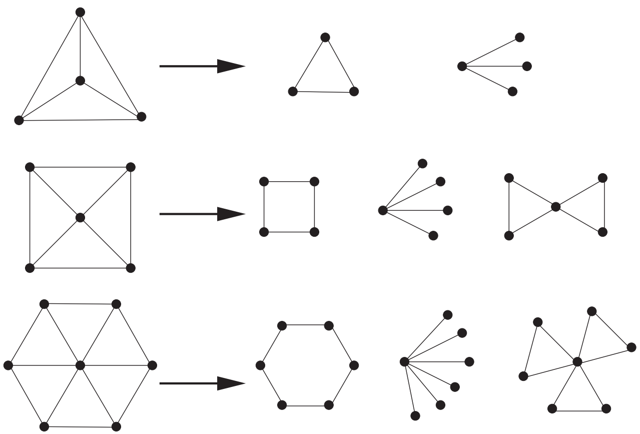

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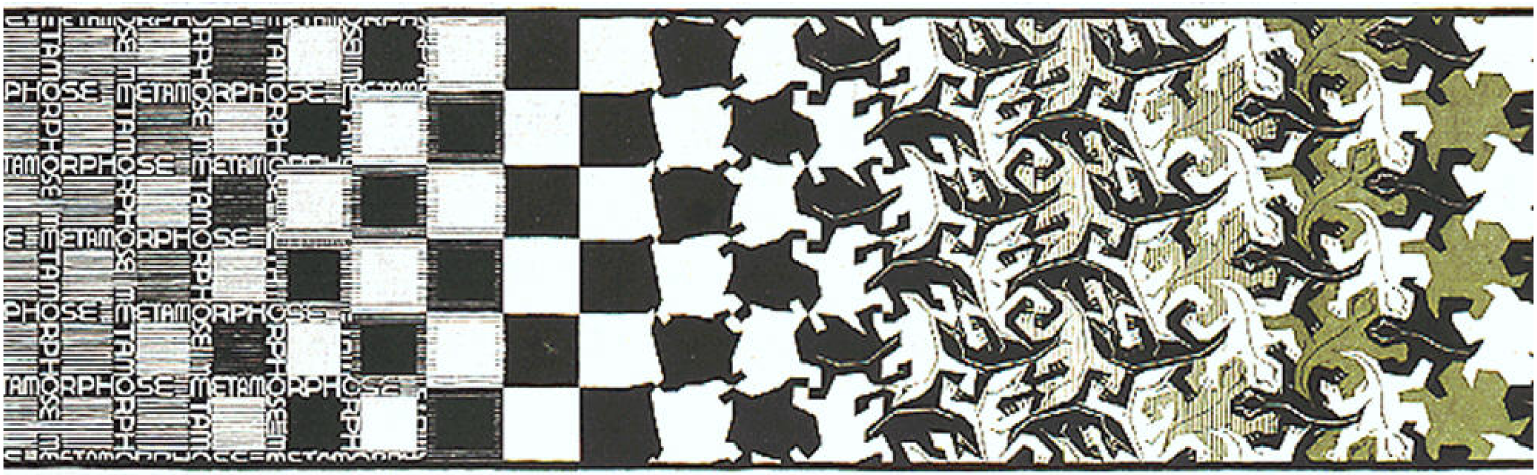
Ragusa (2010)

Simultaneous metamorphoses of λ-fold $K_3 + e$ designs (kite designs) into all possible subgraphs.

Ragusa

Metamorphoses results: a typical construction



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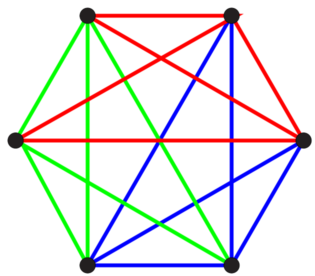

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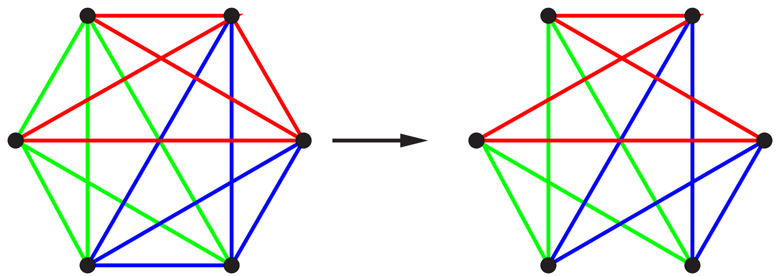

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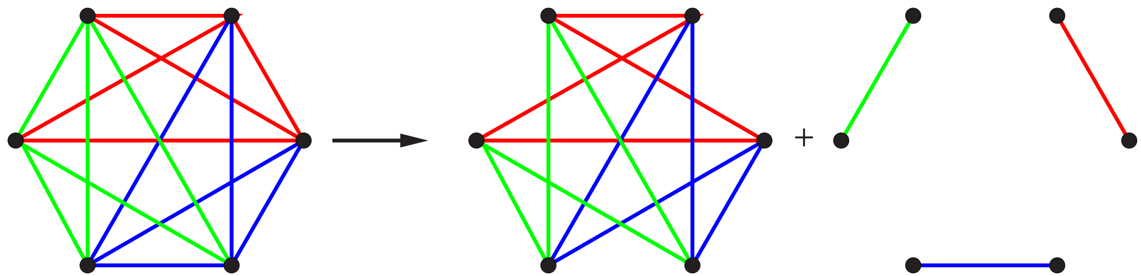

Metamorphoses results: a typical construction

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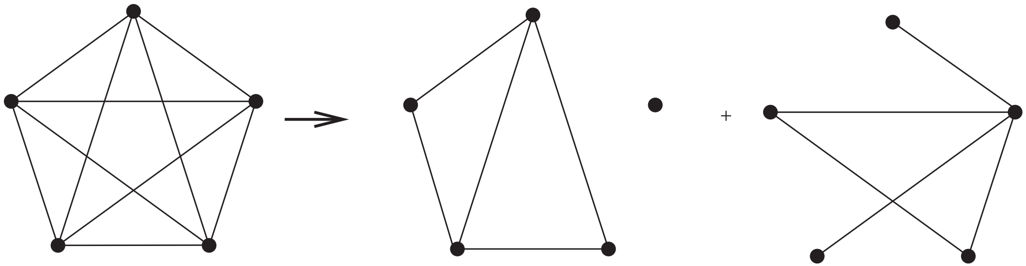

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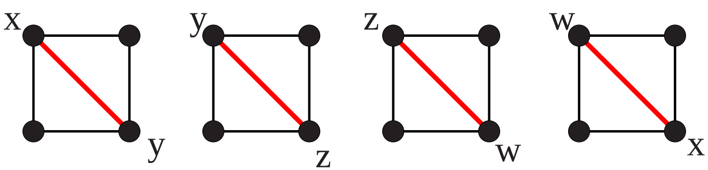
Say we have a 4-cycle (x, y, z, w) from removed "diagonal" edges:



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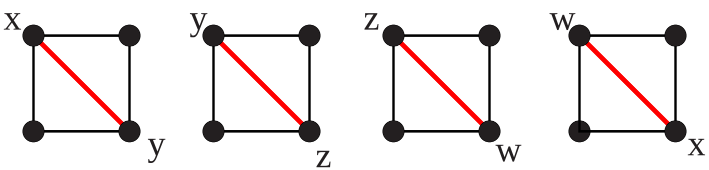

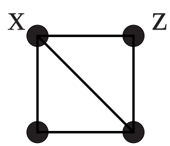

Edge $x\,z$ cannot be in these four $K_4 - e$ blocks, (since $\lambda = 1$) so must have another block with edge $x\,z$, so one of x, z will have total degree $3 + 3 + 3 = 9$, leaving degree 1, impossible!

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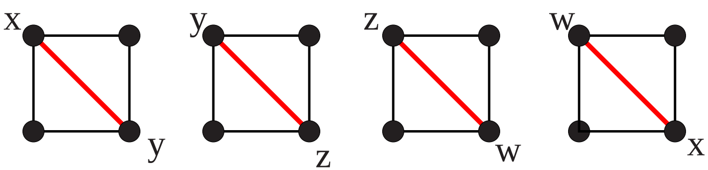

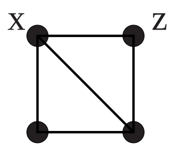

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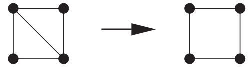
So there is *no* metamorphosis from a $K_4 - e$ design of order 11 into just 11+1 4-cycles, let alone 11+2=13 4-cycles and a triangle leave!

Metamorphoses results: a typical construction



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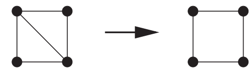

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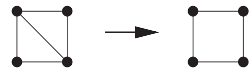

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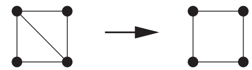

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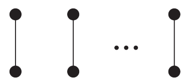

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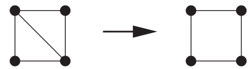

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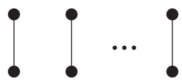

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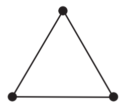

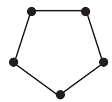

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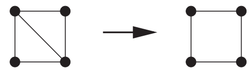

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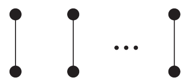

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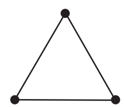

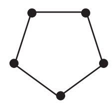

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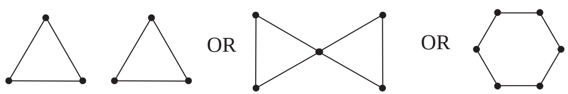

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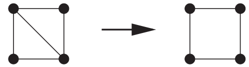

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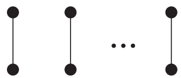

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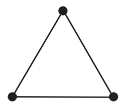

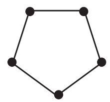

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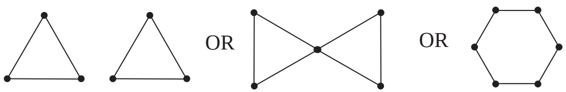

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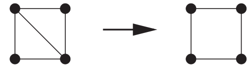

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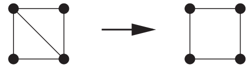

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Illustration of easy case, order 0 (mod 10), when the 4-cycle packing has 1-factor leave.

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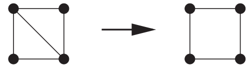

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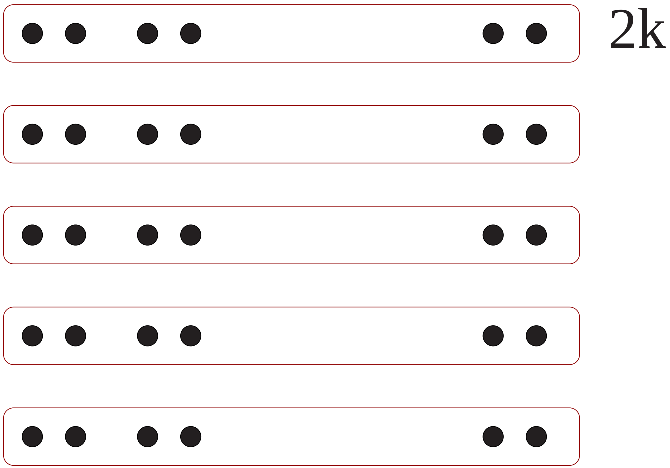

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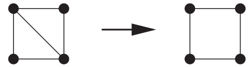

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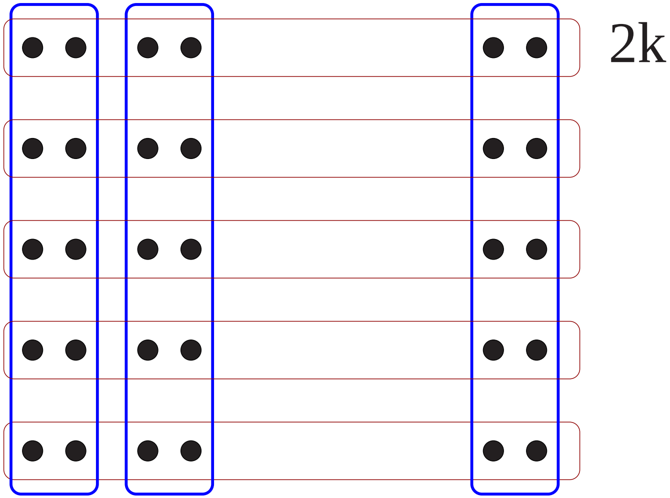

Place a $K_4 - e$ design of order 10 on each blue set of vertices;
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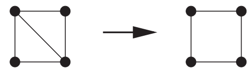

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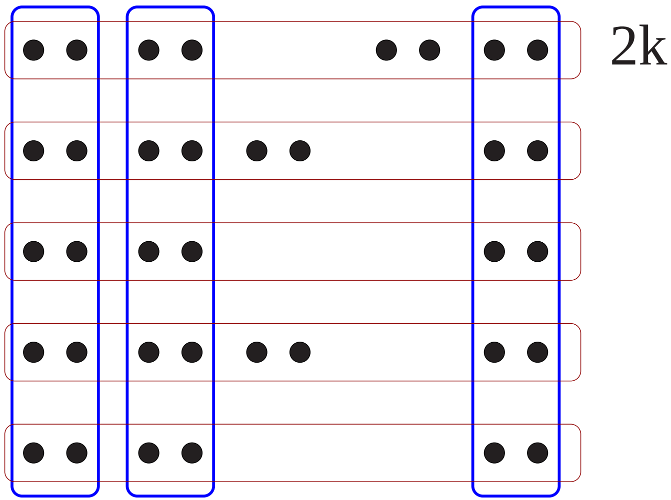

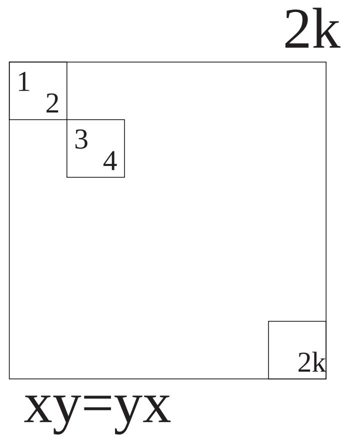

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Then take a commutative quasigroup (order $2k$) with $2{\times}2$ holes on diagonal (ok for $k \geqslant 3$).

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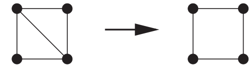

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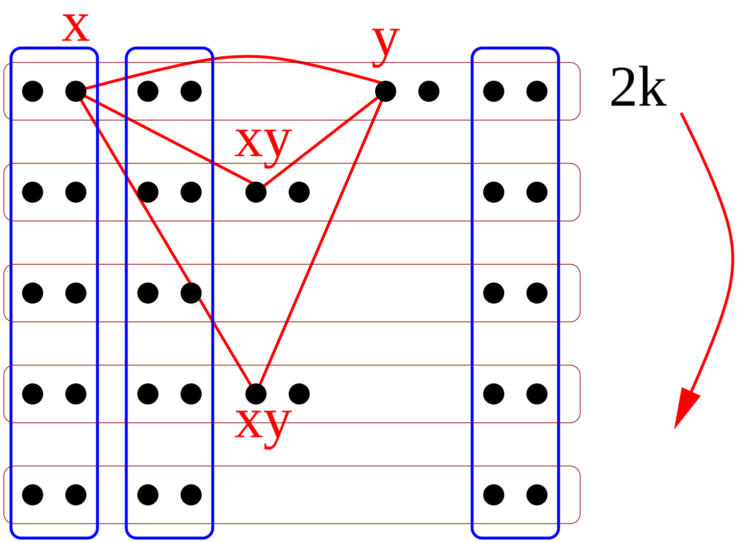
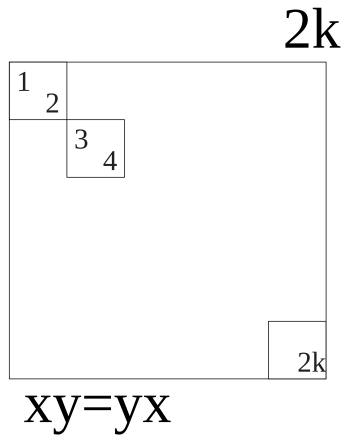
Have $10k$ points





Place a $K_4 - e$ design of order 10 on each blue set of vertices;
have metamorphosis into a 4-cycle packing with 1-factor leave.
Then take a commutative quasigroup (order $2k$) with $2{\times}2$ holes on diagonal (ok for $k \geqslant 3$).
For all x, y in different holes, take red $K_4 - e$ blocks.

Metamorphoses results: a typical construction

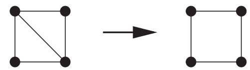
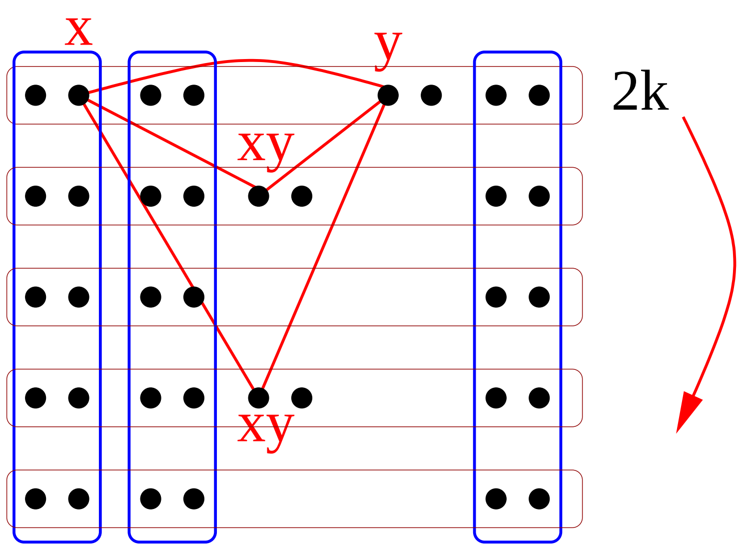
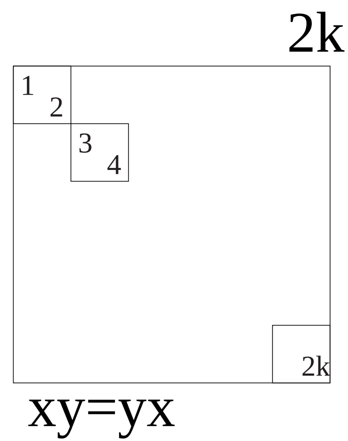


Illustration of easy case, order 0 (mod 10), when the 4-cycle packing has 1-factor leave.

The metamorphosis:





Remove the edges $x\,y$ from all the $K_4 - e$ blocks.

Metamorphoses results: a typical construction

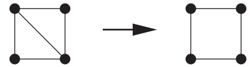


Illustration of easy case, order 0 (mod 10), when the 4-cycle packing has 1-factor leave.

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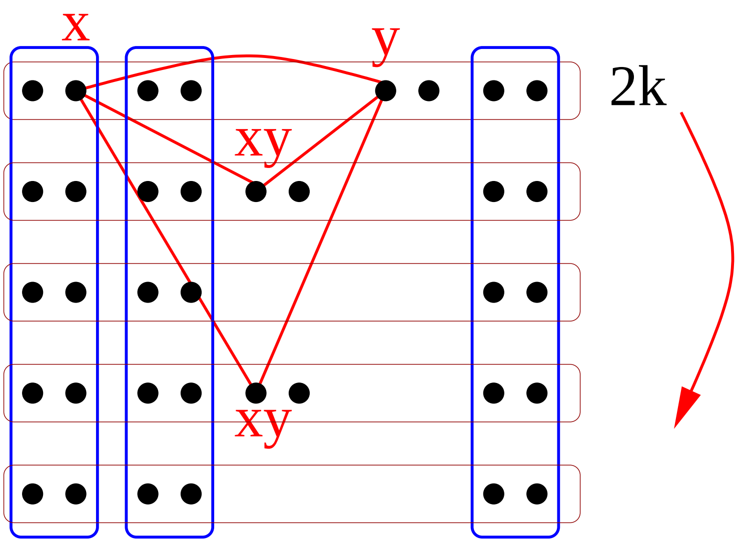
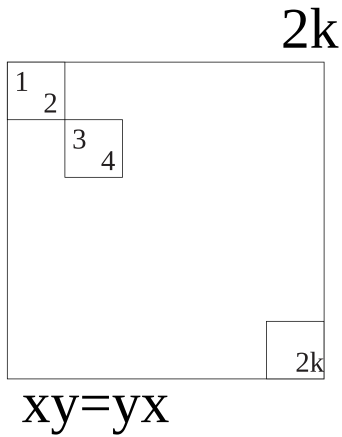
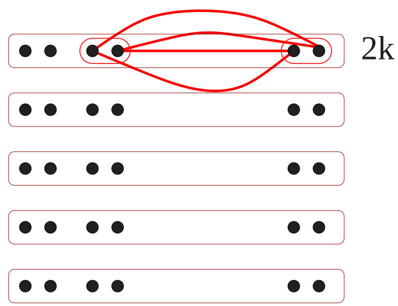


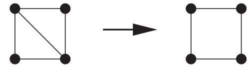


Remove the edges $x\,y$ from all the $K_4 - e$ blocks.



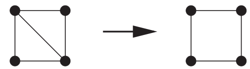
Since x and y are all possible edges, all levels, with x, y in different holes, these removed edges rearrange into 4-cycles:

Metamorphoses results: a typical construction



RESULT: Lindner & Tripodi
There is a metamorphosis from a $K_4 - e$ design into a 4-cycle maximum packing for all orders 0, 1 (mod 5) except for 5 and 11.

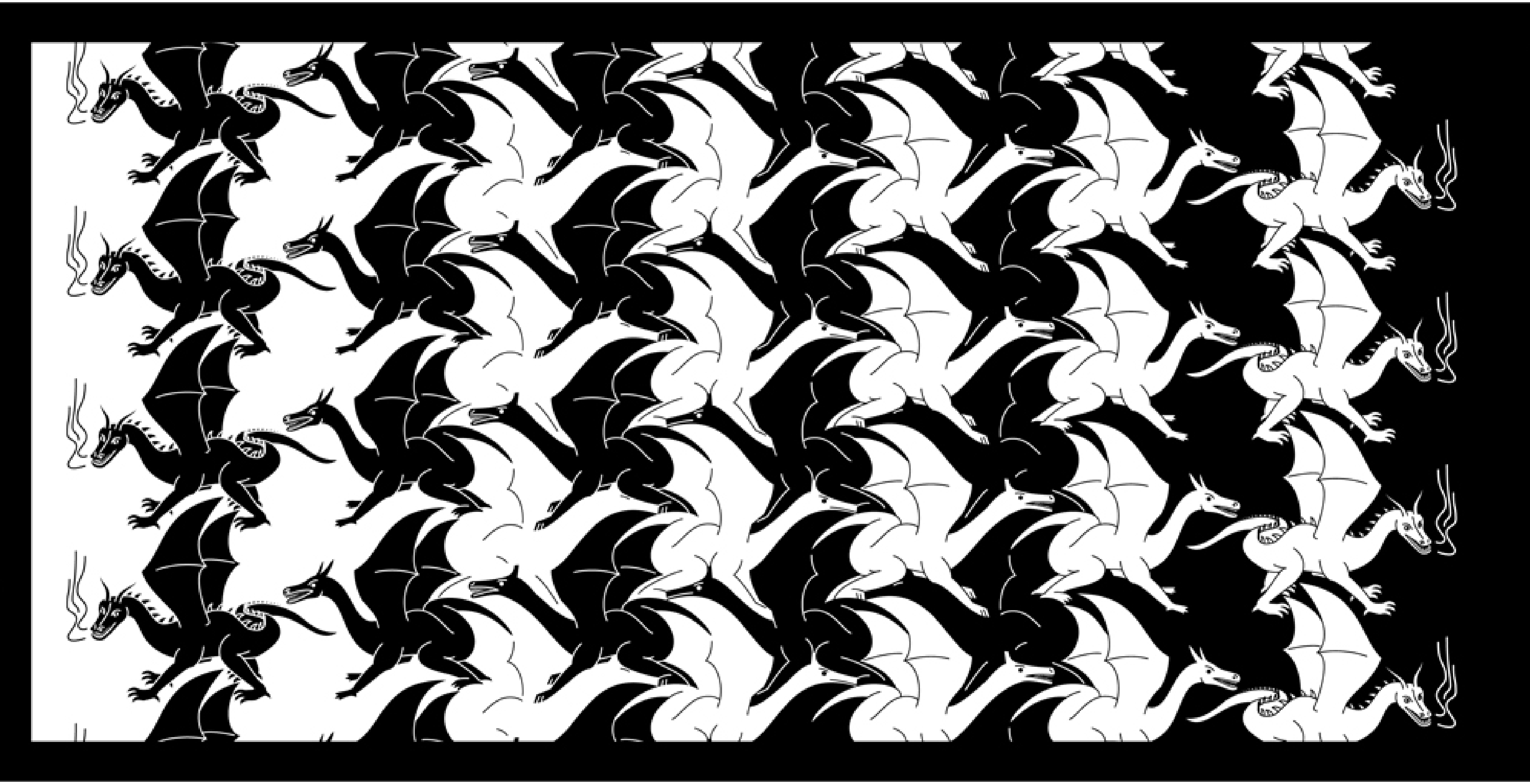
Metamorphoses results: a typical construction



RESULT: Lindner & Tripodi
There is a metamorphosis from a $K_4 - e$ design into a 4-cycle maximum packing for all orders 0, 1 (mod 5) except for 5 and 11.

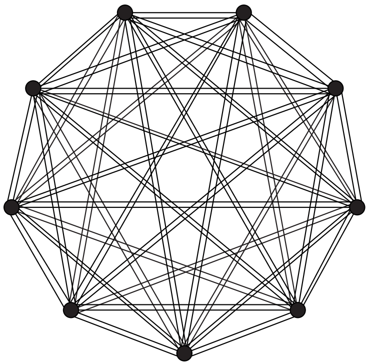
The λ-fold cases: Tripodi, 2003.

Complete sets of metamorphoses



Complete sets of metamorphoses

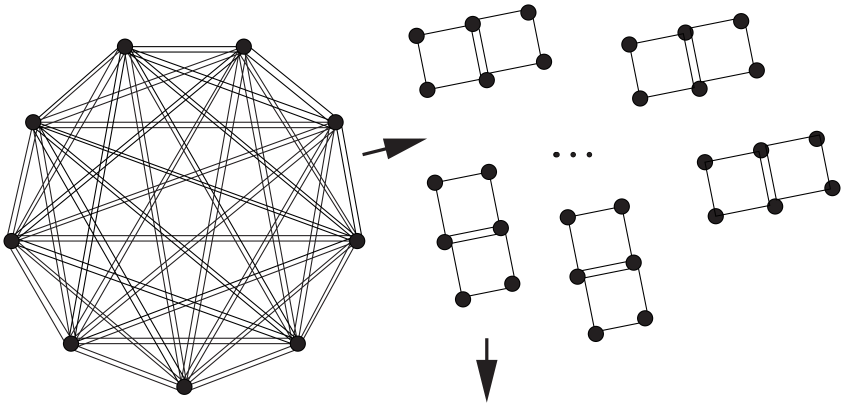
Twofold 4-cycle system into twofold 6-cycle system:



$2K_n$

Complete sets of metamorphoses

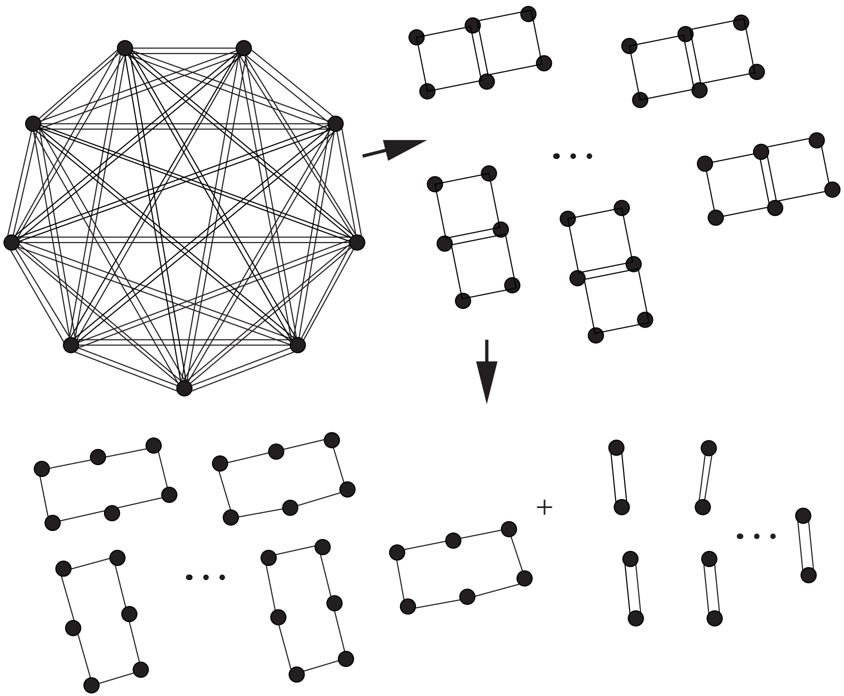
Twofold 4-cycle system into twofold 6-cycle system:



Paired 4-cycle system

Complete sets of metamorphoses

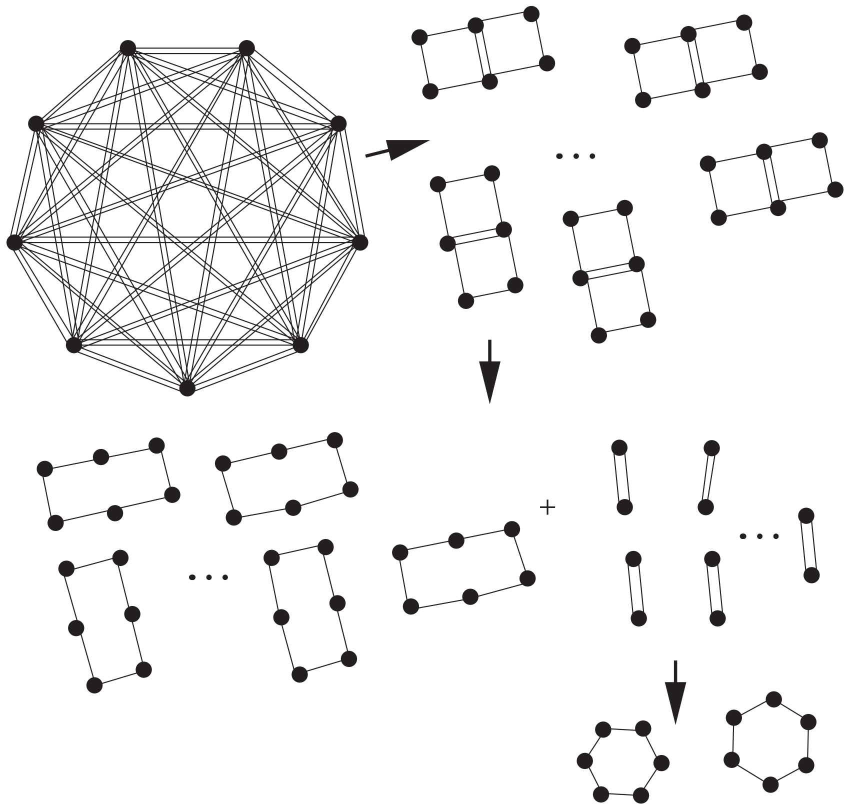
Twofold 4-cycle system into twofold 6-cycle system:



Remove doubled edges from pairs

Complete sets of metamorphoses

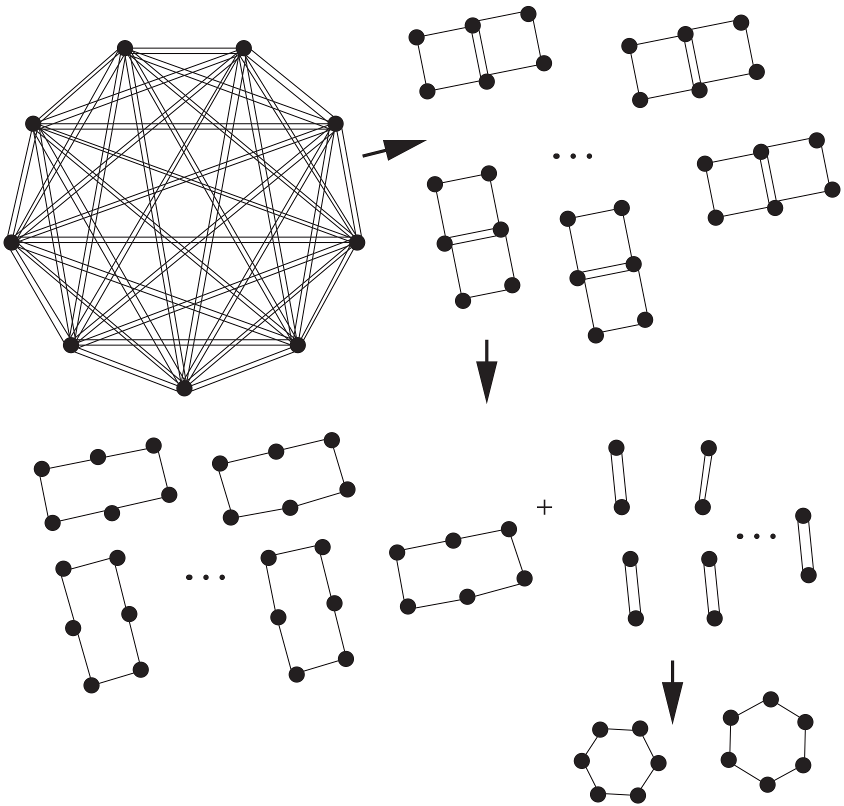
Twofold 4-cycle system into twofold 6-cycle system:



Rearrange double edges into further 6-cycles

Complete sets of metamorphoses

Twofold 4-cycle system into twofold 6-cycle system:



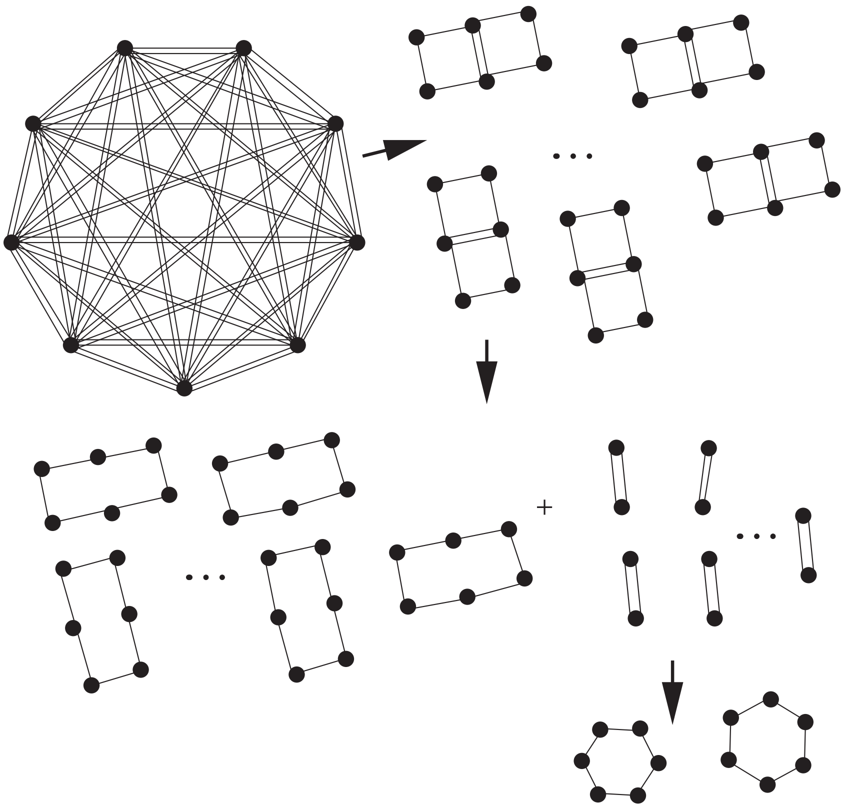
Metamorphosis, 2-fold 4-cycle system to 2-fold 6-cycle system.

Need order $n \equiv 0,1,4$ or 9 (mod 24).

Rearrange double edges into further 6-cycles

Complete sets of metamorphoses

Twofold 4-cycle system into twofold 6-cycle system:



Metamorphosis, 2-fold 4-cycle system to 2-fold 6-cycle system.

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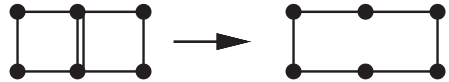
Şule Yazıcı 2005.

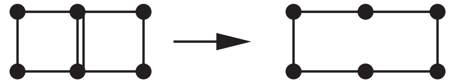
Rearrange double edges into further 6-cycles

Complete sets of metamorphoses



AIM: Take one fixed 2-fold 4-cycle system of order n. then:

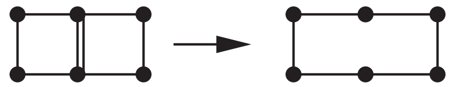
Complete sets of metamorphoses



AIM: Take one fixed 2-fold 4-cycle system of order n. then:

Take four different pairings of the 4-cycles, for four different metamorphoses into 6-cycles, so that:

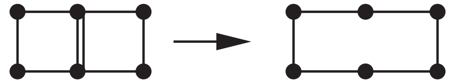
Complete sets of metamorphoses



AIM: Take one fixed 2-fold 4-cycle system of order n. then:

Take four different pairings of the 4-cycles, for four different metamorphoses into 6-cycles, so that: all the double edges exactly cover $2K_n$.

Complete sets of metamorphoses



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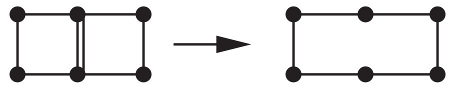
EJB, Cavenagh & Khodkar (2011+)

Complete sets of metamorphoses



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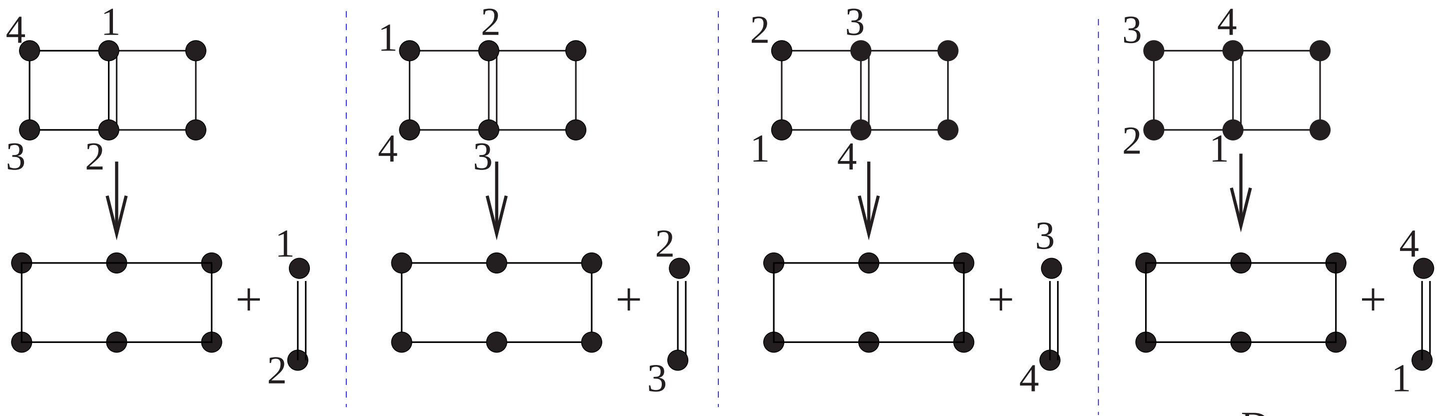
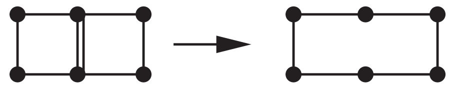


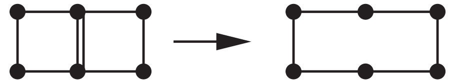
Metamorphosis A B C D

Complete sets of metamorphoses



Hardest part: small cases. Cannot do order 9.

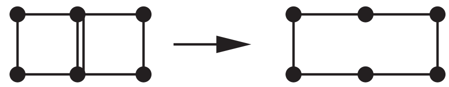
Complete sets of metamorphoses



Hardest part: small cases. Cannot do order 9.

There is a metamorphosis, twofold, order 9

Complete sets of metamorphoses

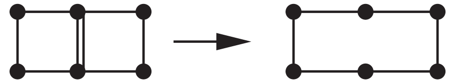


Hardest part: small cases. Cannot do order 9.

There is a metamorphosis, twofold, order 9

But cannot get *four* such metamorphoses, on the *same* fixed twofold 4-cycle system of $2K_9$.
Smallest cases, with $n \equiv 0, 1, 9, 16$ (mod 24), are 16, 24, 25 and 33.

Complete sets of metamorphoses



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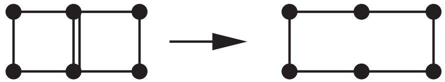
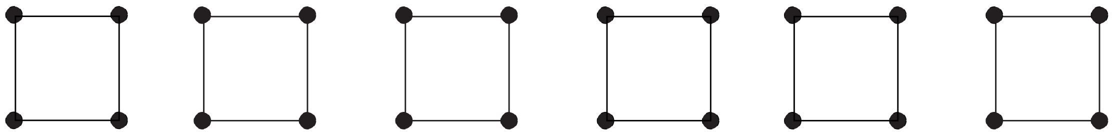
But cannot get *four* such metamorphoses, on the *same* fixed twofold 4-cycle system of $2K_9$.
Smallest cases, with $n \equiv 0, 1, 9, 16$ (mod 24), are 16, 24, 25 and 33.

Order 16: computer search.
Order 25: nice cyclic solution.
Also have orders 24 and 33, *ad hoc* methods.

Complete sets of metamorphoses

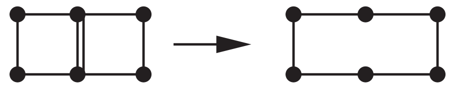


Order 25: $V(K_{25}) = \mathbb{Z}_{25}$. Six starters for 4-cycle system of $2K_{25}$:



Use differences 1,2, . . . 12 (mod 25)

Complete sets of metamorphoses



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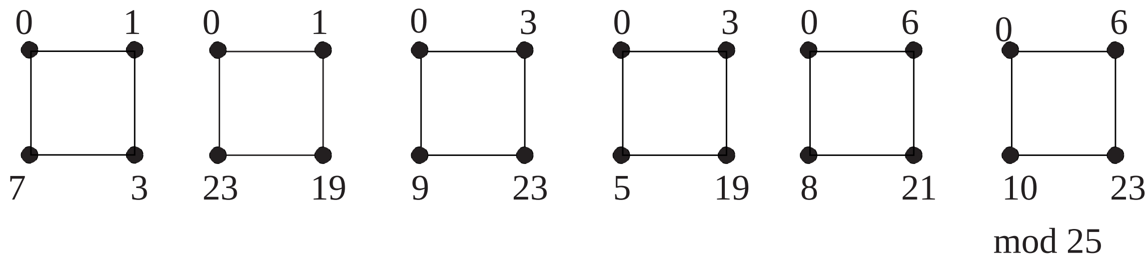
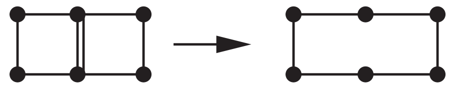


Complete sets of metamorphoses



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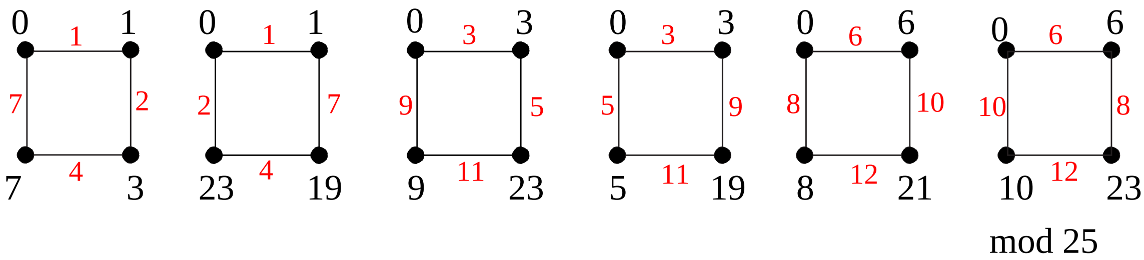
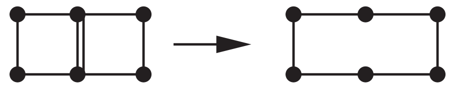


Complete sets of metamorphoses



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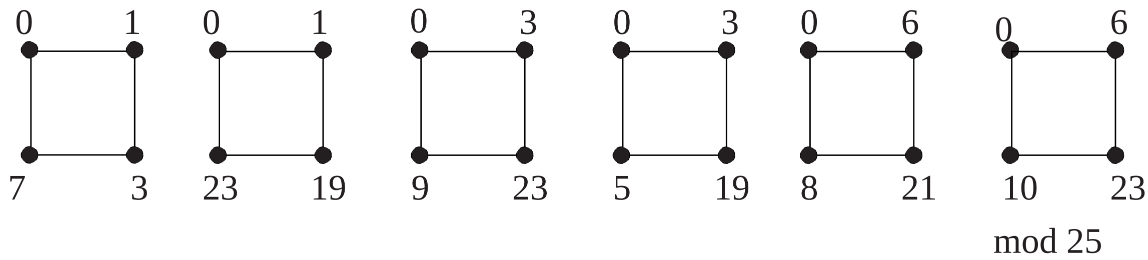

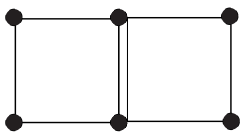

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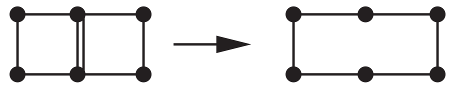

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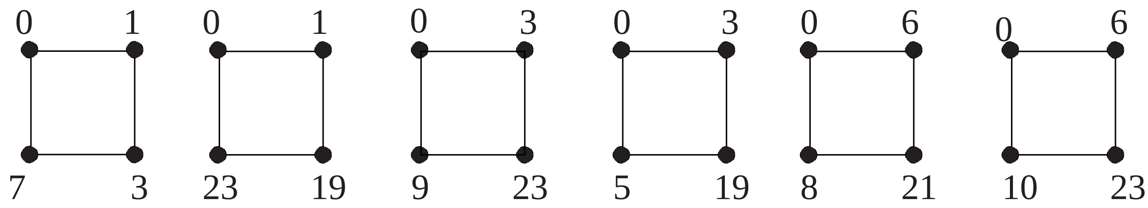
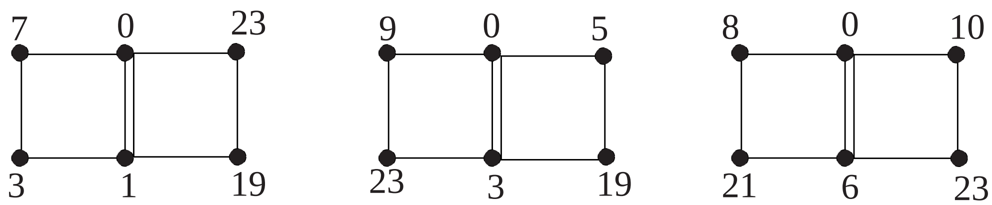
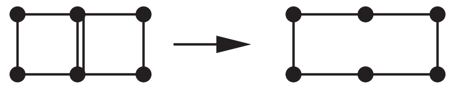


mod 25



Complete sets of metamorphoses



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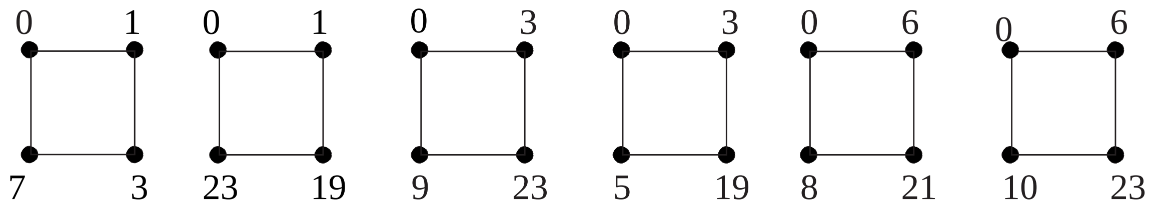

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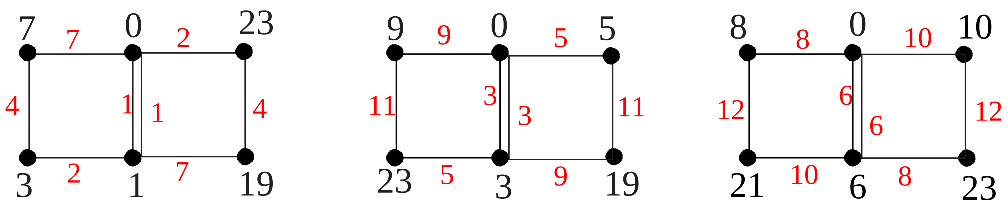
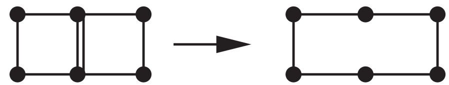


Complete sets of metamorphoses



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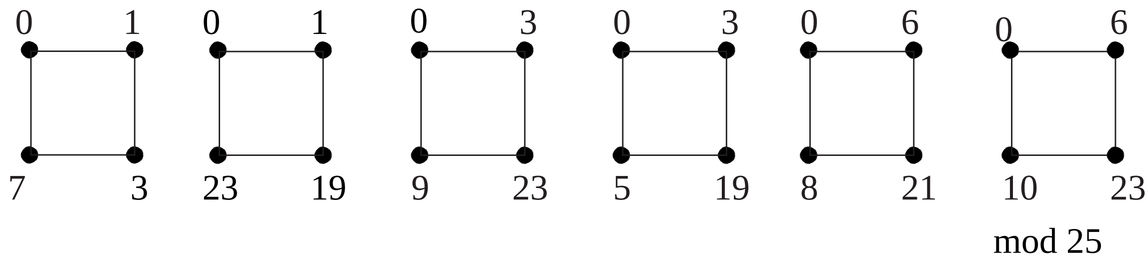
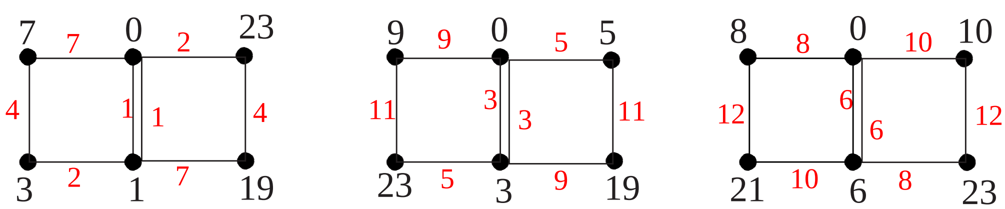
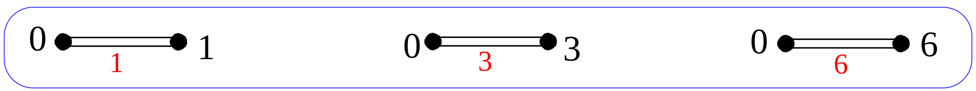
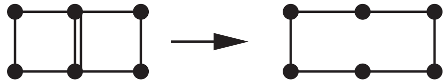






Complete sets of metamorphoses



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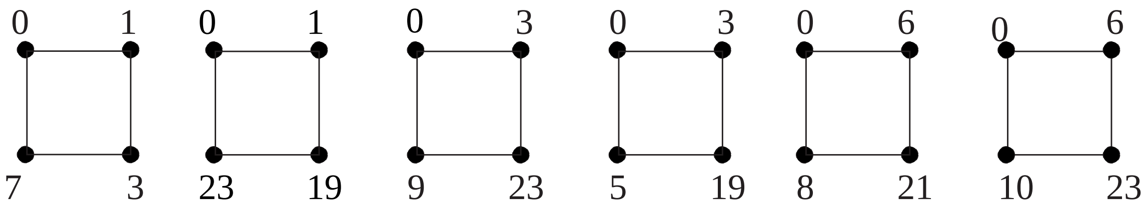

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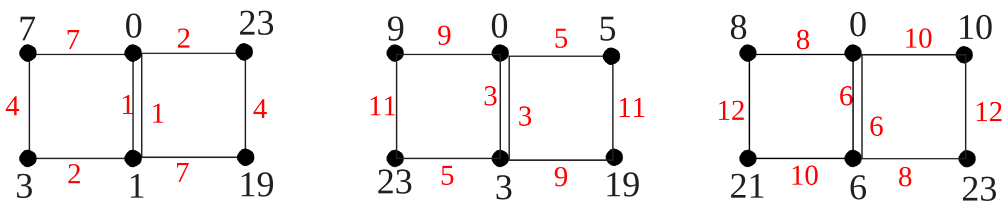
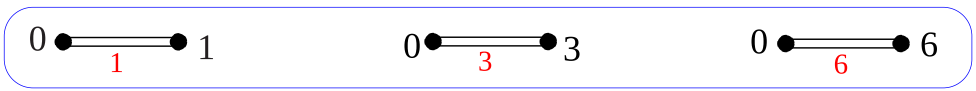
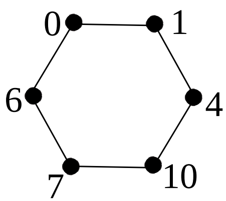
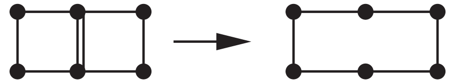






Complete sets of metamorphoses



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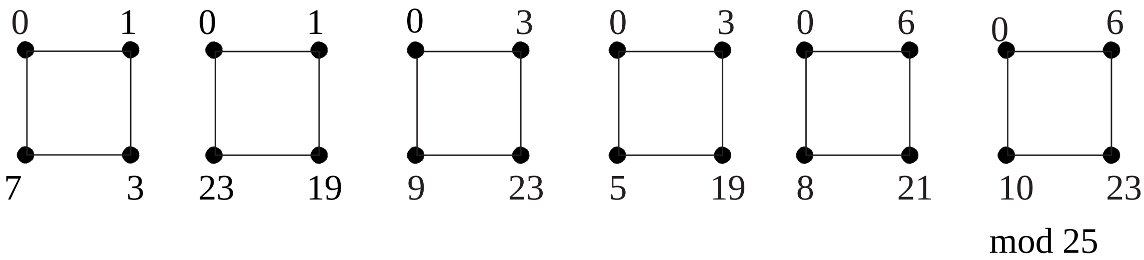
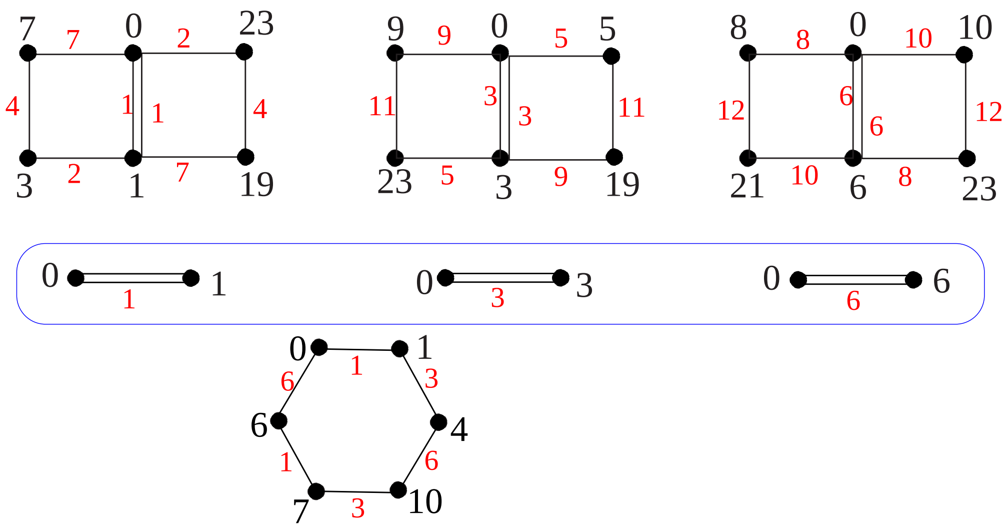
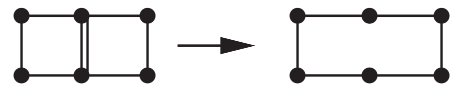




Complete sets of metamorphoses



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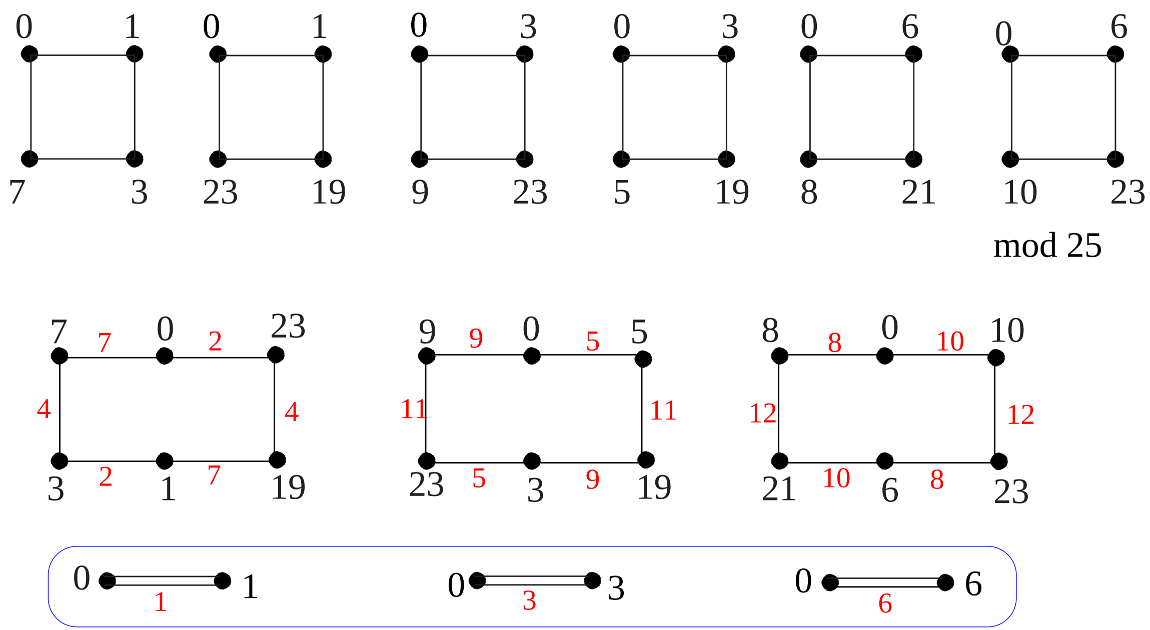
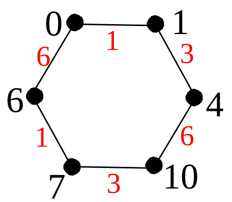
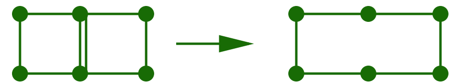




This is ONE metamorphosis, (A); need 3 more!

Metamorphosis: complete set, order 25

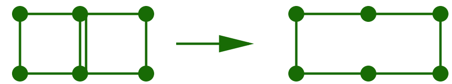


Metamorphosis (A):

(3,7,0,1), (0,1,19,23); (23,9,0,3),(0,3,19,5); (21,8,0,6),(0,6,23,10);

doubled edges form one 6-cycle $(0, 1, 4, 10, 7, 6)$ (all mod 25).

Metamorphosis: complete set, order 25



Metamorphosis (A):

(3,7,0,1), (0,1,19,23); (23,9,0,3),(0,3,19,5); (21,8,0,6),(0,6,23,10);
doubled edges form one 6-cycle $(0, 1, 4, 10, 7, 6)$ (all mod 25).

Metamorphosis (B):

(6,24,0,2),(0,2,3,21); (2,11,0,5),(0,5,19,3); (21,6,0,8),(0,8,2,12);
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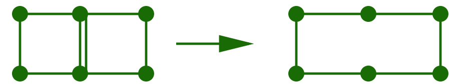
Metamorphosis: complete set, order 25



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Metamorphosis (C):

(22,23,0,4),(0,4,6,7); (23,3,0,9),(0,9,6,11); (4,12,0,10),(0,10,23,6);
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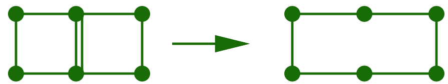

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Metamorphosis (D):

(3,1,0,7),(0,7,6,4); (2,5,0,11),(0,11,6,9); (4,10,0,12),(0,12,2,8);
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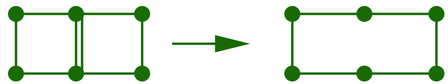

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doubled edges form one 6-cycle (0,7,18,5,19,12) (all mod 25).

Note: the collection of *all* doubled edges exactly covers $2K_{25}$;
uses differences (A) 1, 3, 6; (B) 2, 5, 8; (C) 4, 9, 10; (D) 7, 11, 12.

Metamorphosis: complete set

$2K_n$ for $n \equiv$ 0,1,9,16 (mod 24), *not* order 9.
Got smallest in each class: 24, 25, 33, 16.

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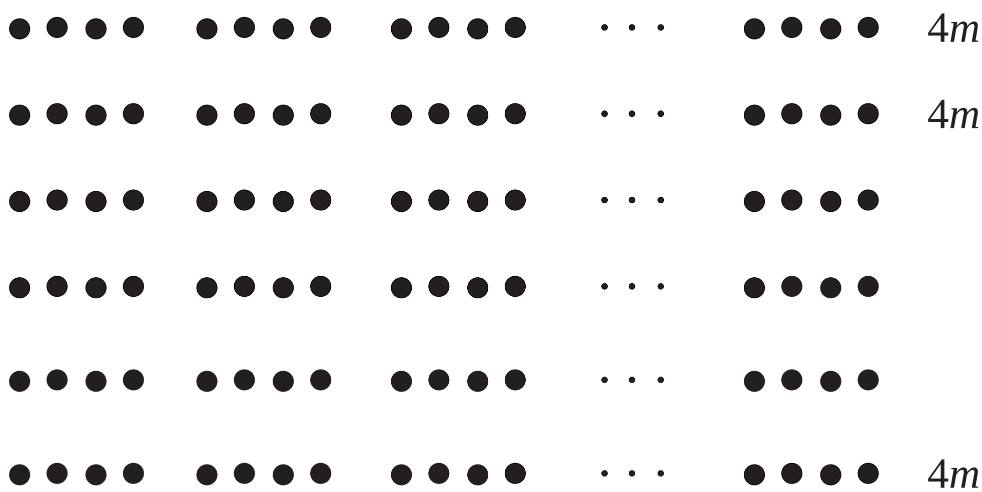

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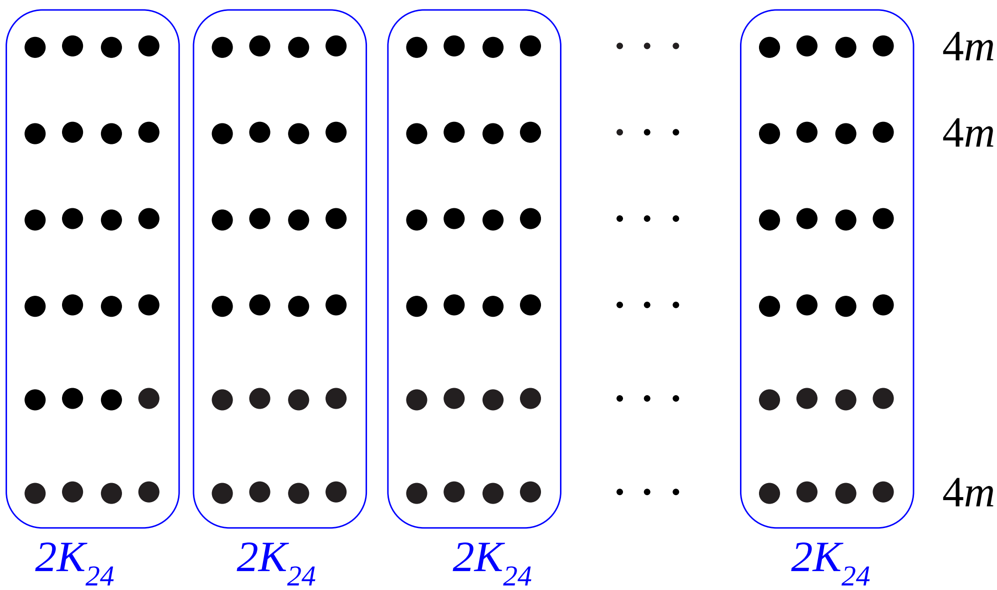

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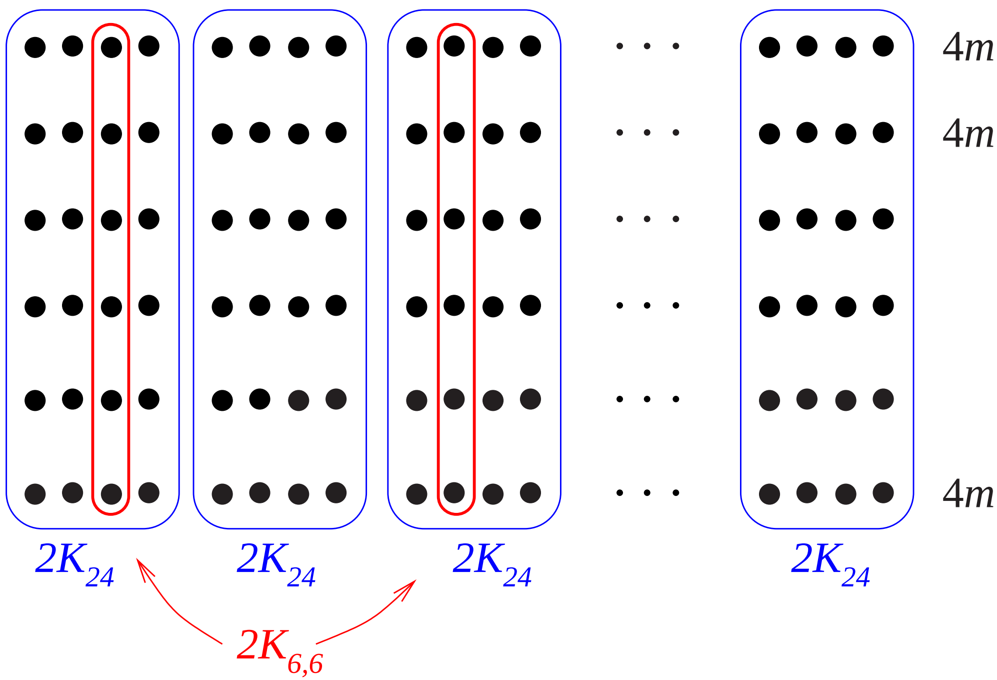

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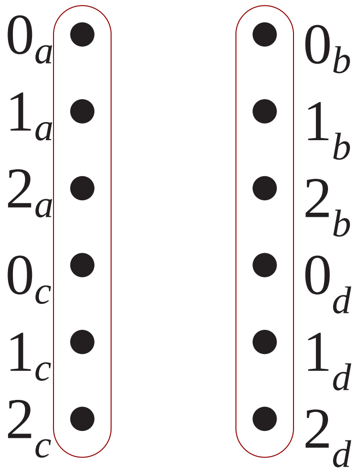

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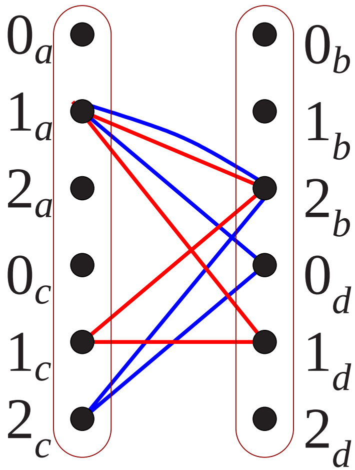

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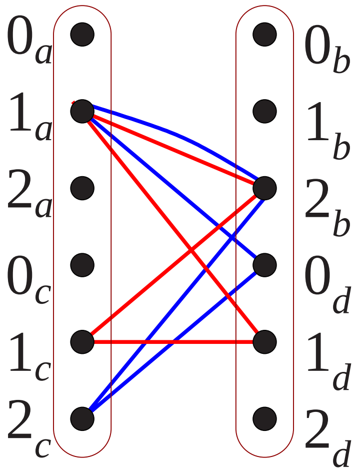

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(x_a, y_b, x_c, s_d) and $(x_a, y_b, (x+1)_c, (s+2)_d)$, addition mod 3.
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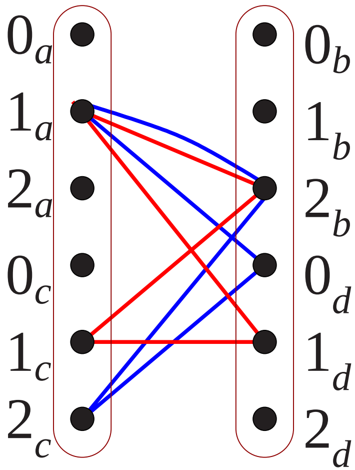

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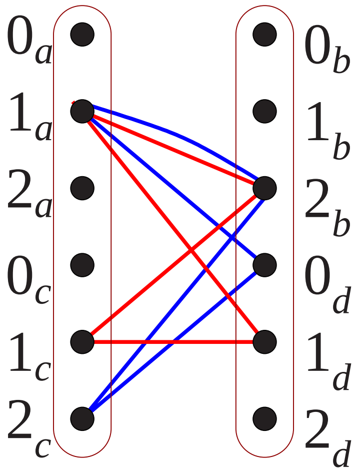

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Need *four* metamorphoses:

Metamorphosis: complete set 0 (mod 24), $K_{6,6}$

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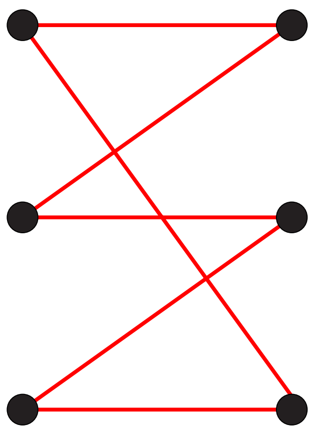

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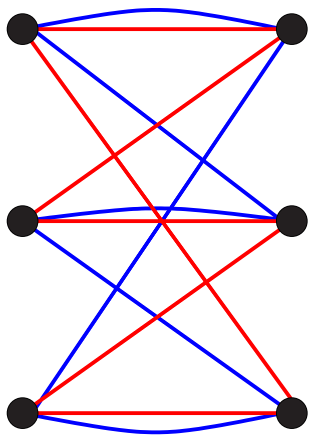

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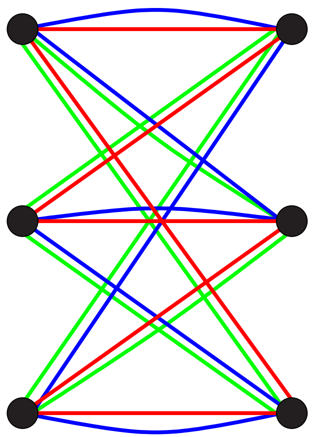

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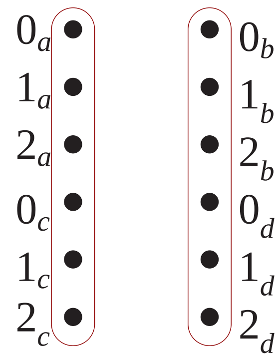

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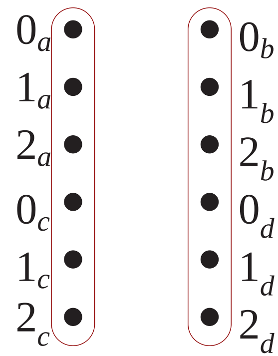

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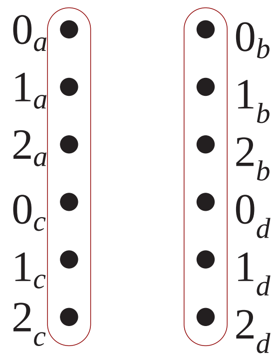

Metamorphosis (C):
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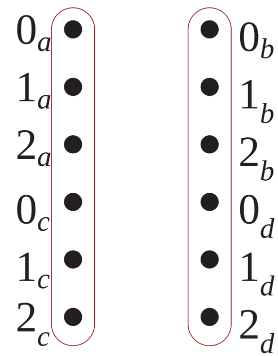

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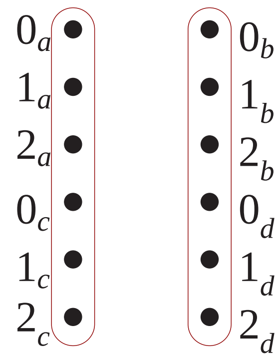

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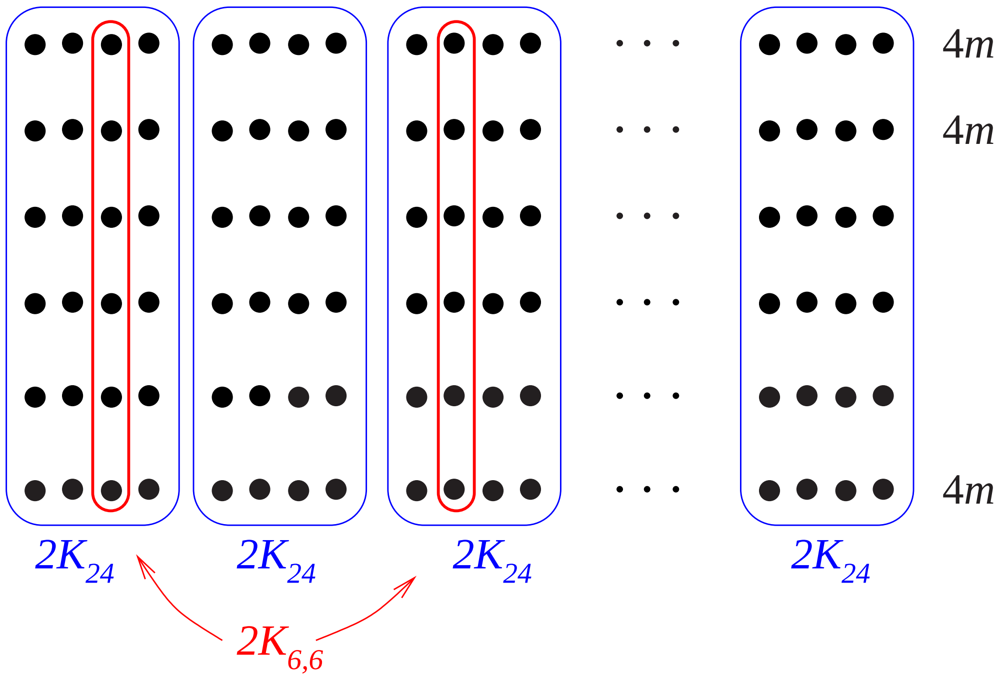
Use the pairs $x_c\, s_d$; then as above get 6-cycles.

So we have a complete set of (four) metamorphoses from this one twofold 4-cycle decomposition of $2K_{6,6}$.

Metamorphosis: complete set 0 (mod 24)

So using complete sets of $2K_{24}$ and $K_{6,6}$ we have $2K_{24m}$:



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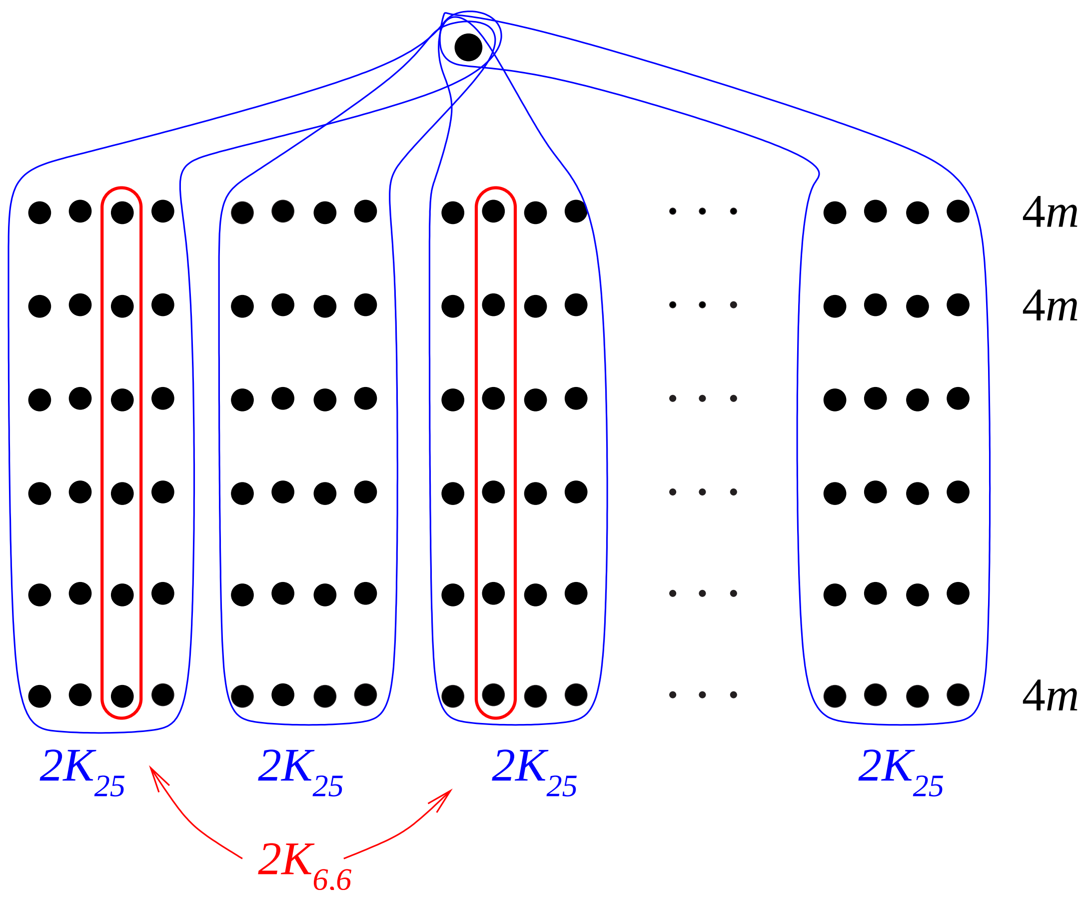

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Theorem There exists a twofold 4-cycle decomposition of $2K_n$ with *four* separate pairings to give metamorphoses into 6-cycle systems, so that the collection of 6-cycles formed from the repeated edges in ALL FOUR metamorphoses themselves form a decomposition of $2K_n$,
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In other words ...

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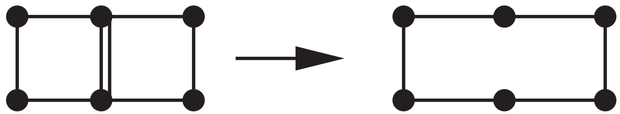

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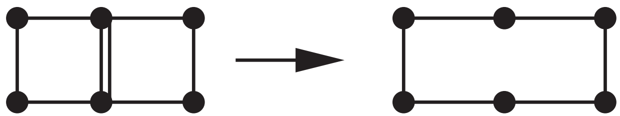

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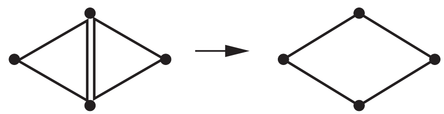

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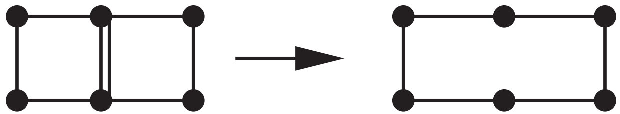

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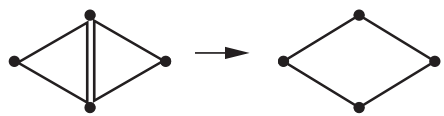
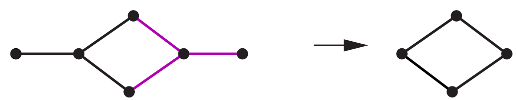
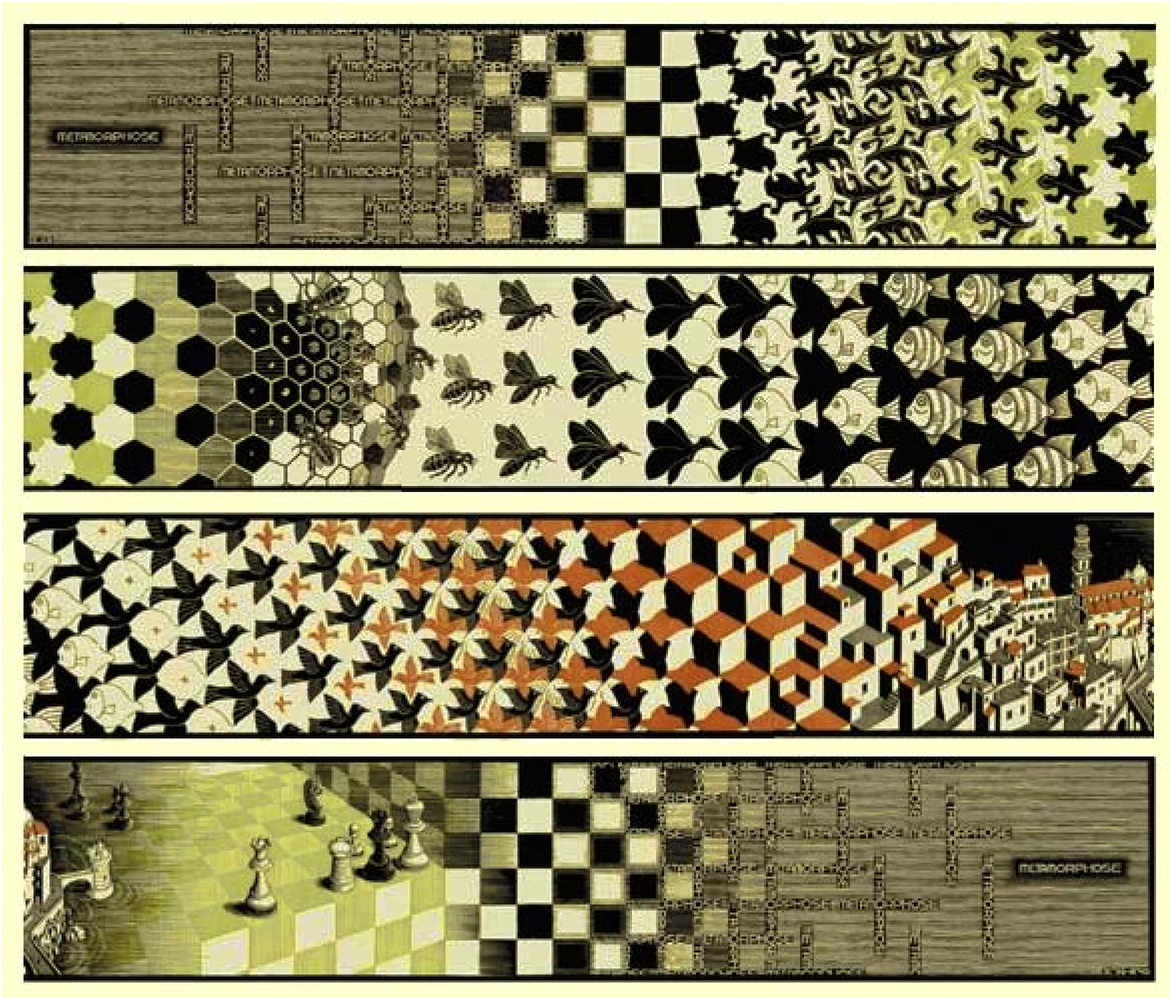


and paired $K_{1,3}$ into 4-cycles (EJB, Khodkar, Lindner).



Some open problems



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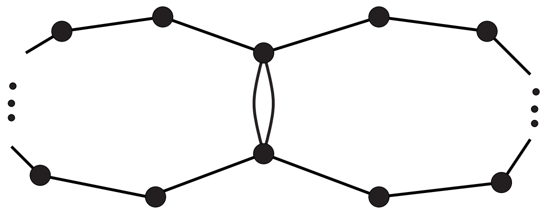

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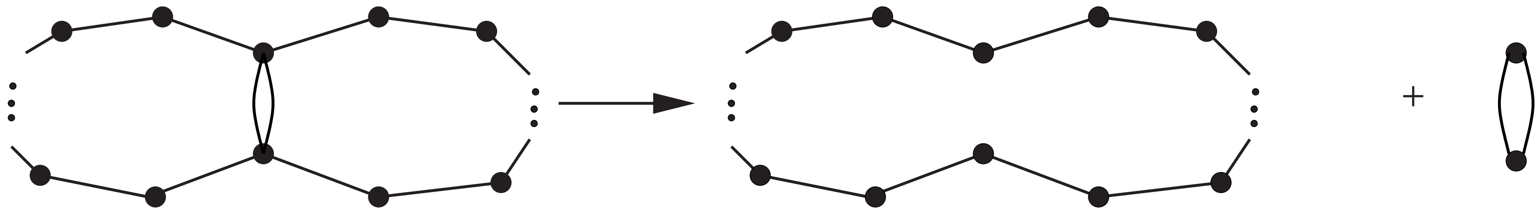

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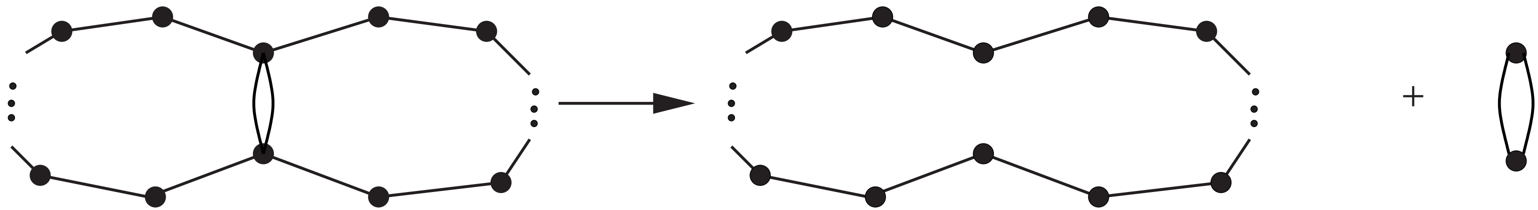

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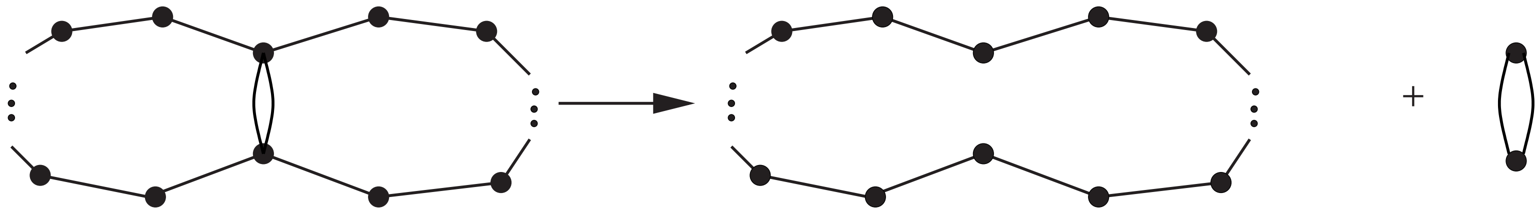

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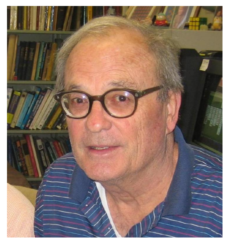
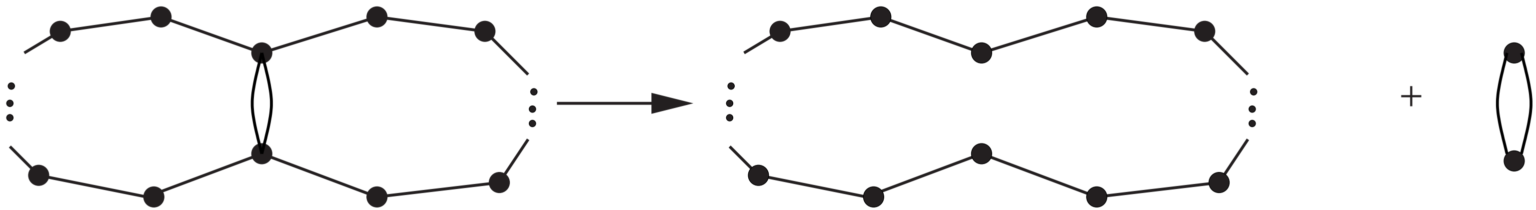


LINDNER

Some open problems



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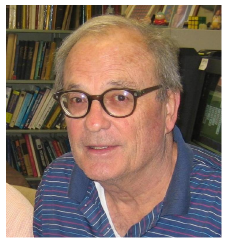
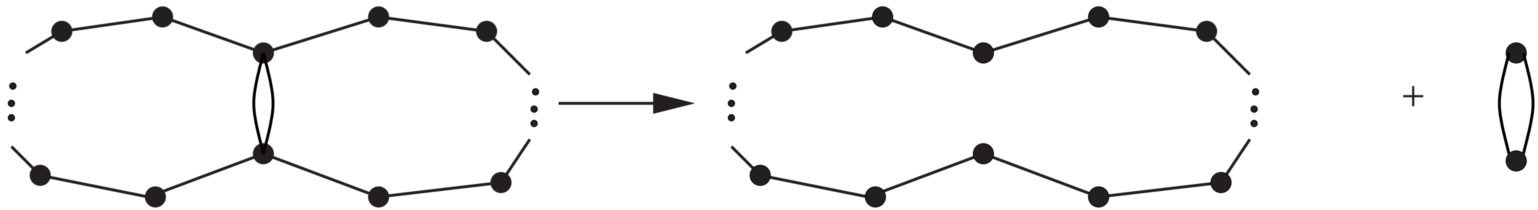


GIONFRIDDO LINDNER YAZICI

$k = 4$: Yazıcı, 2005

Some open problems



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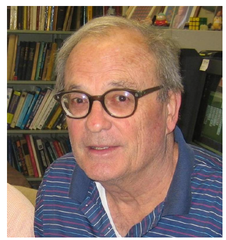

LINDNER

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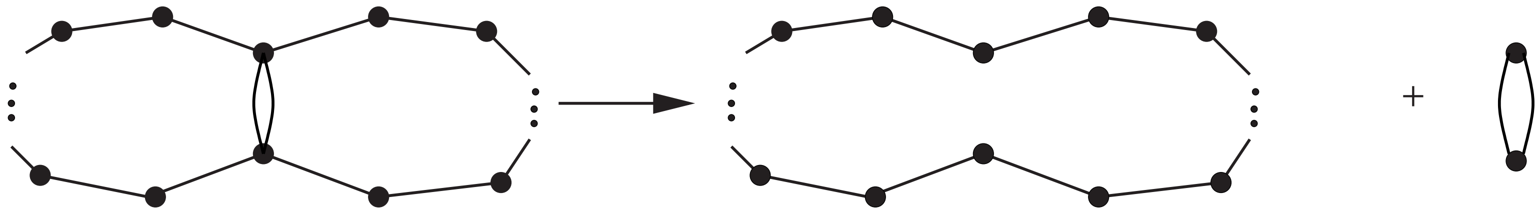
What about arbitrary k?

Some open problems



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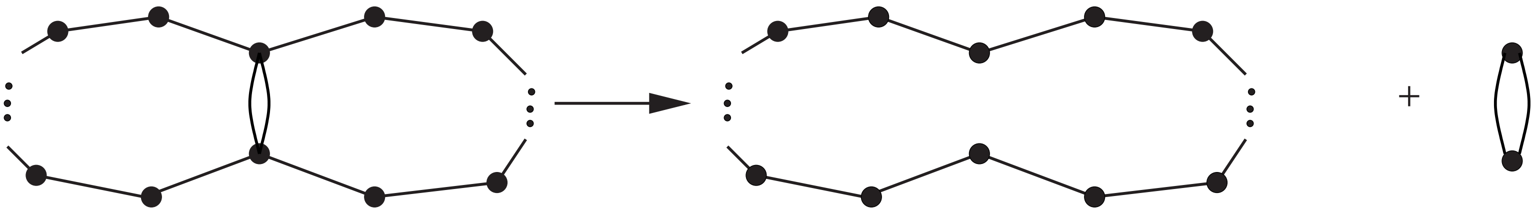

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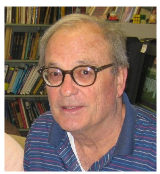
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MESZKA


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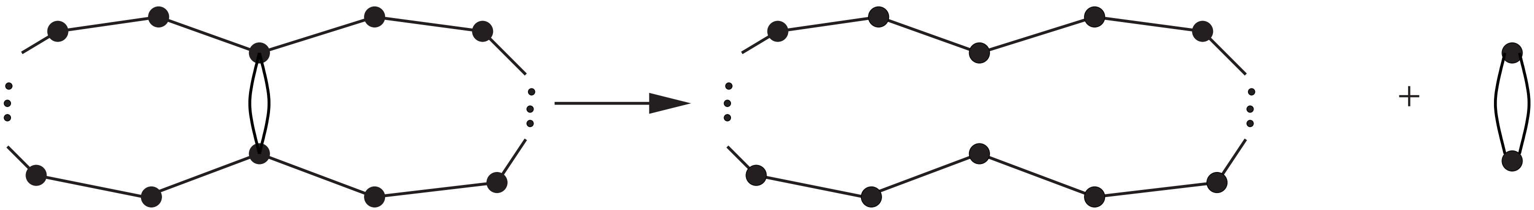

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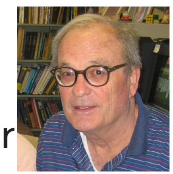

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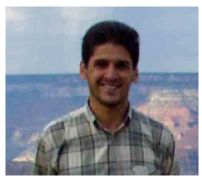

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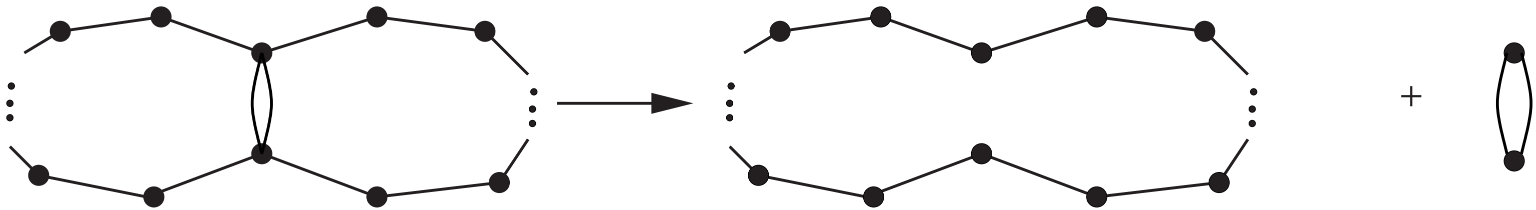

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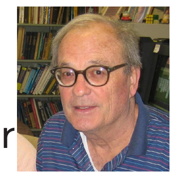
LINDNER


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Open Problem: metamorphosis from theta graph design to cycle system

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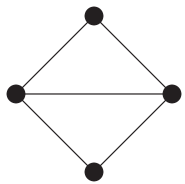
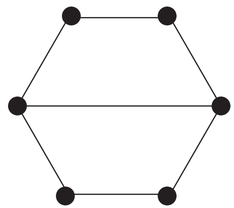


$\Theta(1, 2, 2) = K_4 - e$



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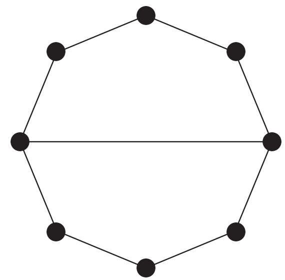

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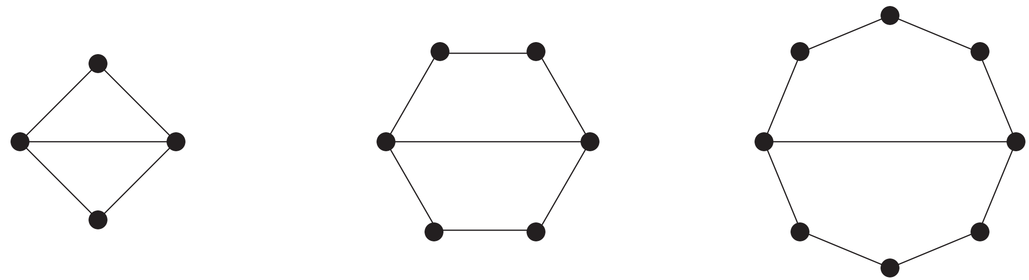

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Metamorphosis from K_4-e design of order n to a 4-cycle design (or packing)

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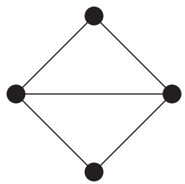

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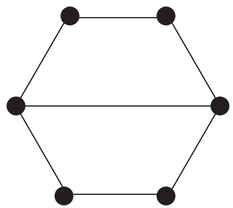

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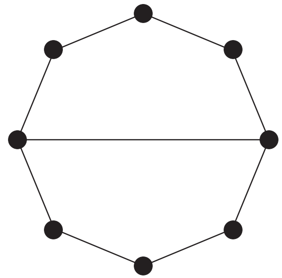

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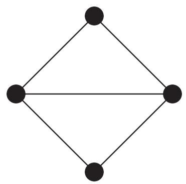

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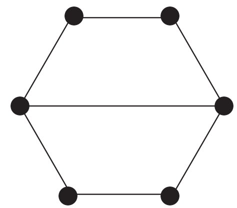

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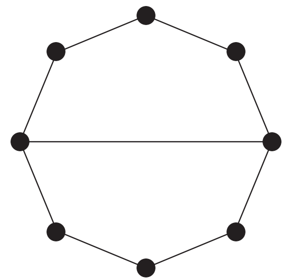

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Some results on existence of theta graphs $\Theta(1,k,k)$ of order n:
k odd and $n \equiv 0 \pmod{2k+1}$, but NOT $\Theta(1,3,3)$ of order 7;
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Blinco, 2001

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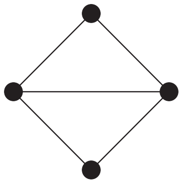

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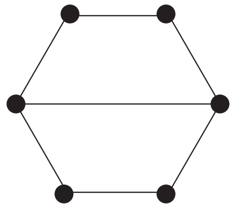

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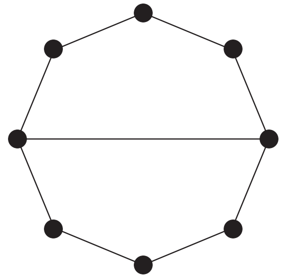

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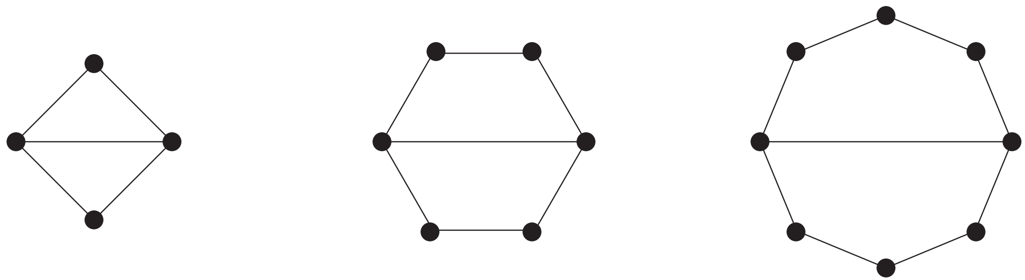

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Open Problem: resolvable metamorphosis

Example: resolvable K_4-design of order 16 (an affine plane of order 4, or a (16,20,5,4,1) BIBD); find a metamorphosis into a resolvable maximum packing with 4-cycles.

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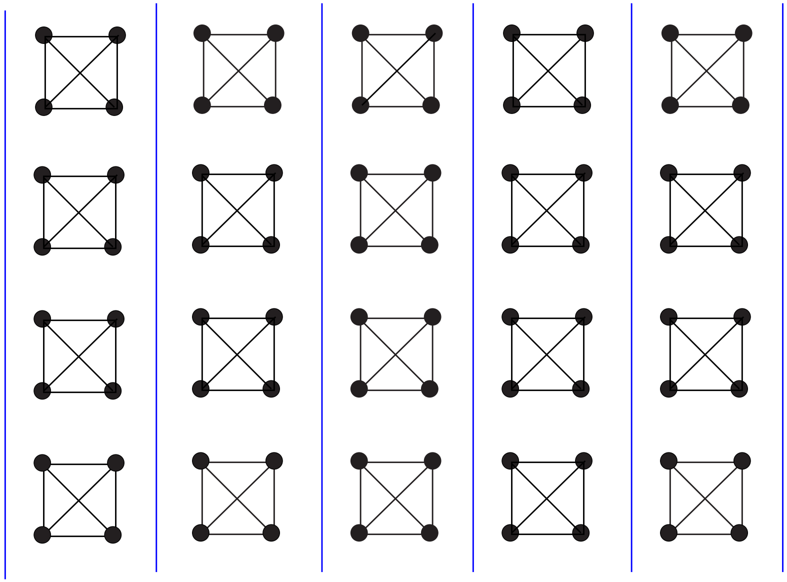

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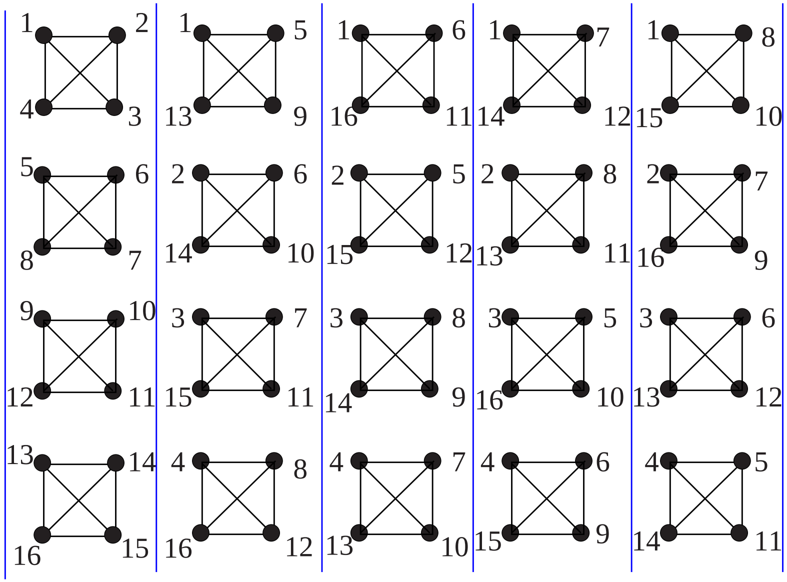

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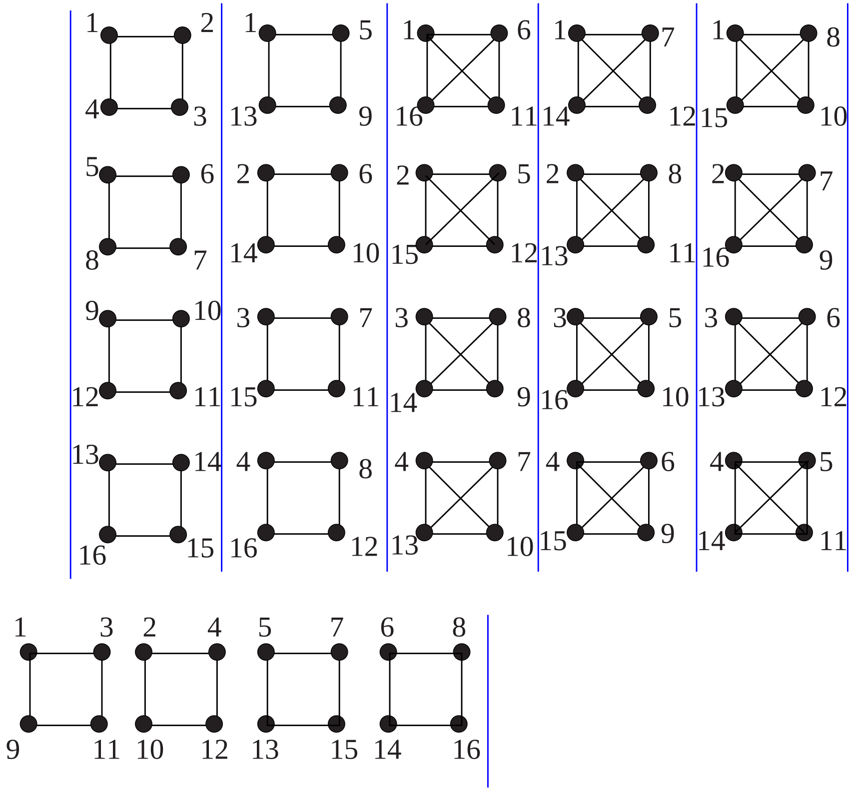

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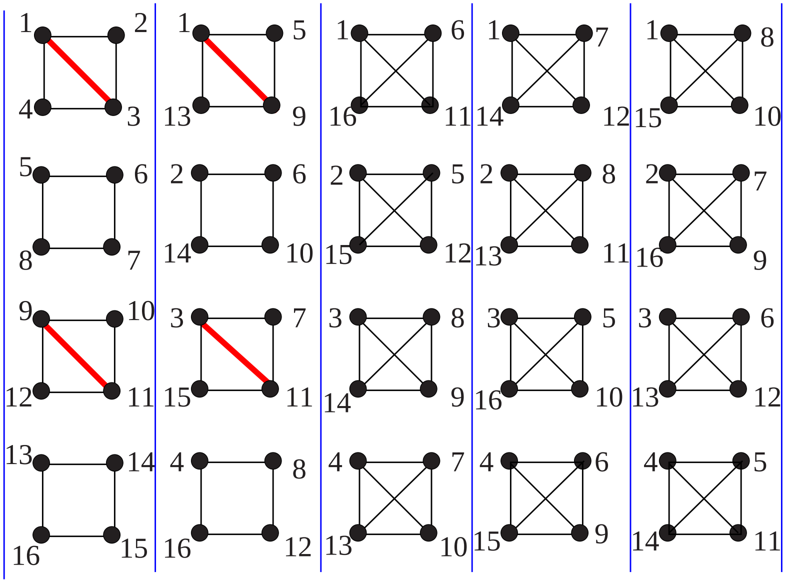

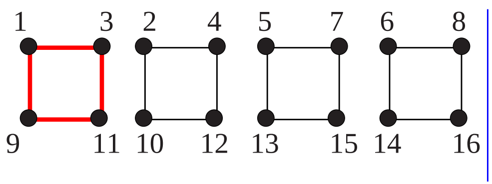

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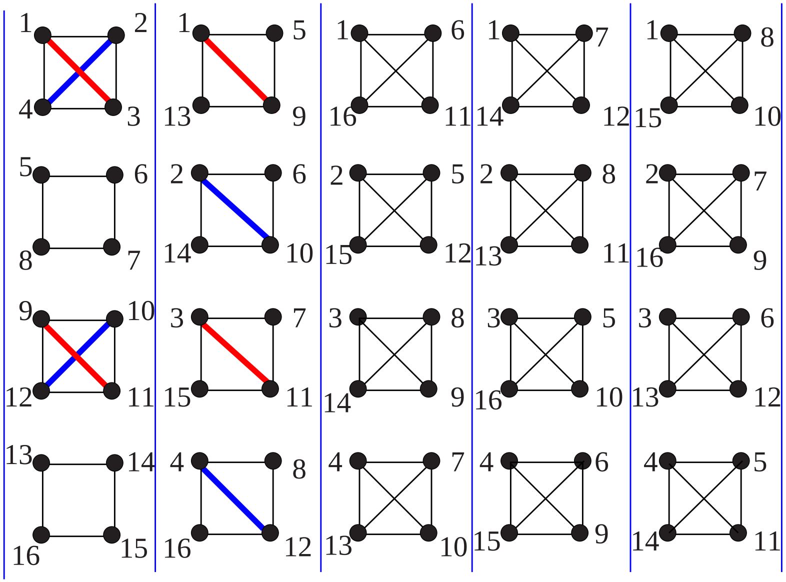

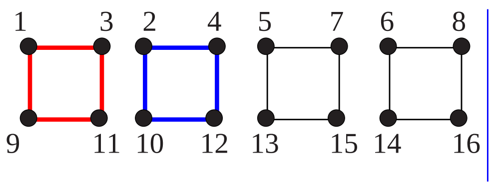

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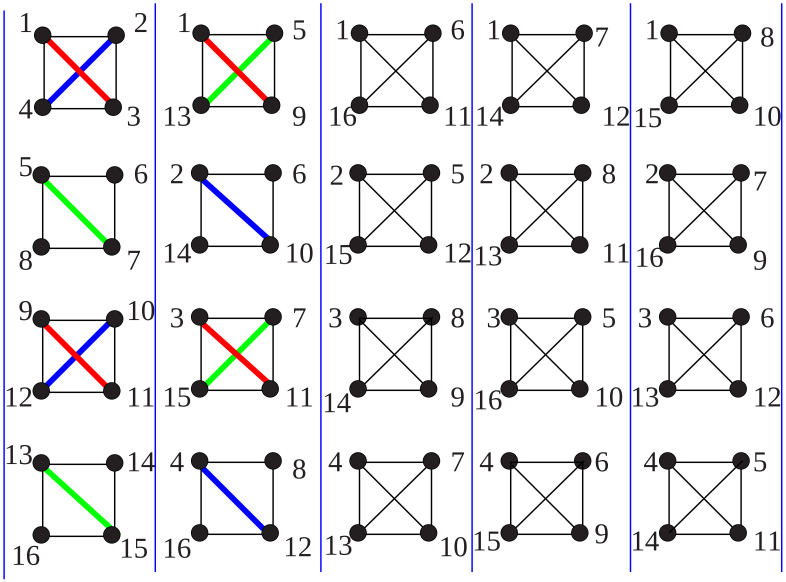

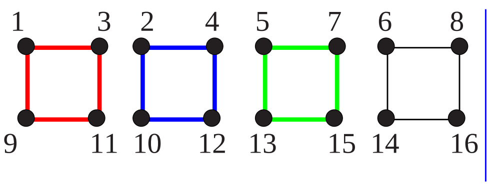

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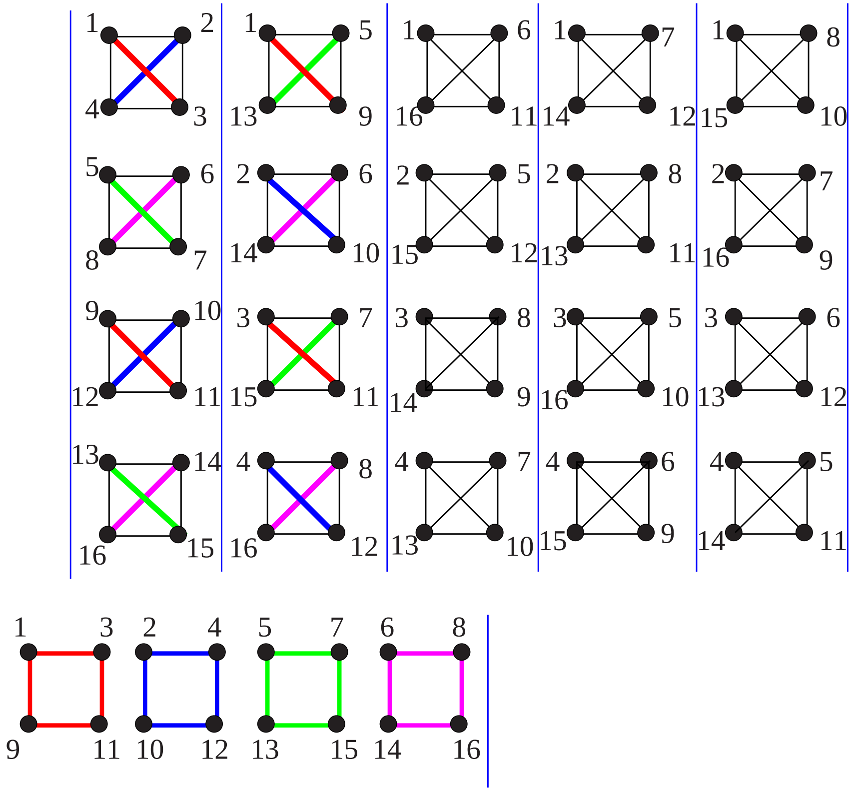

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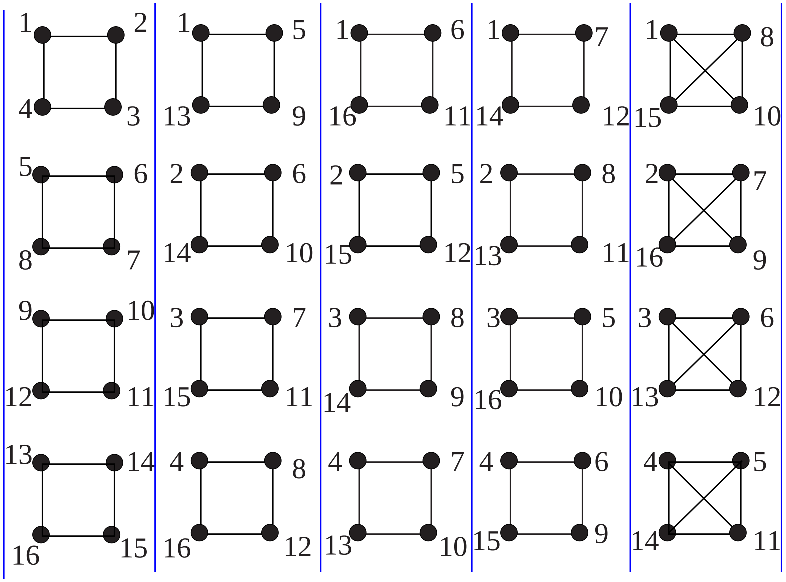

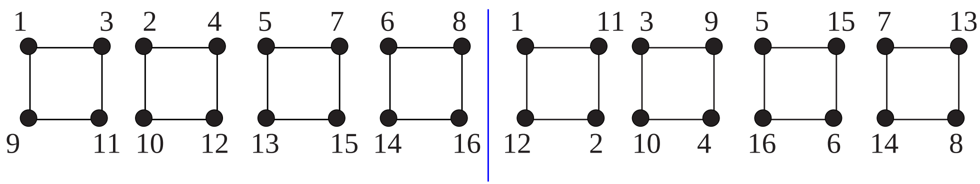

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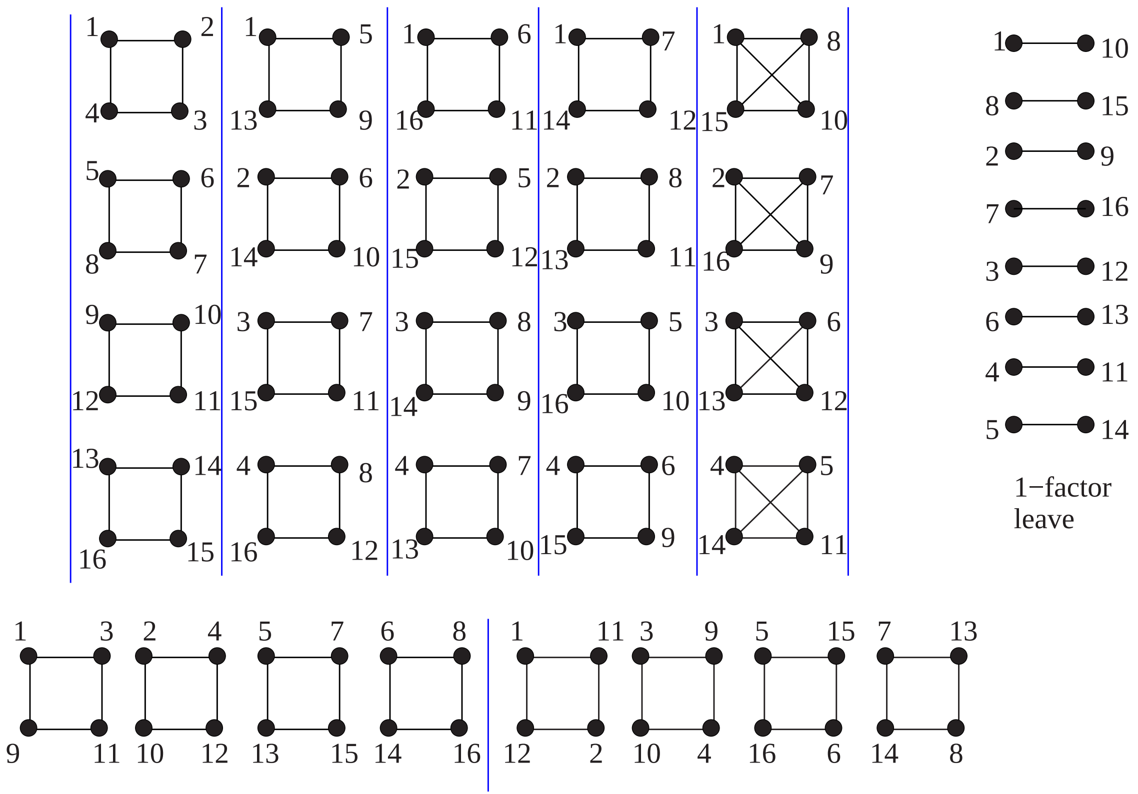

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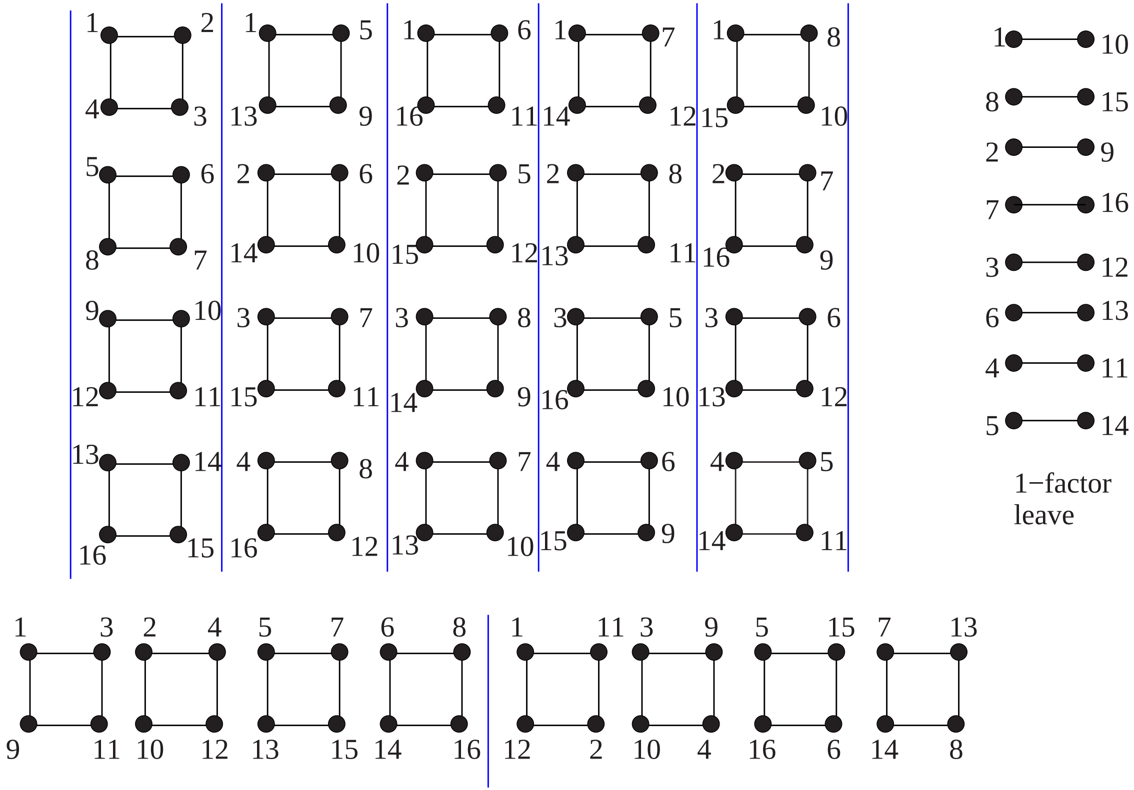

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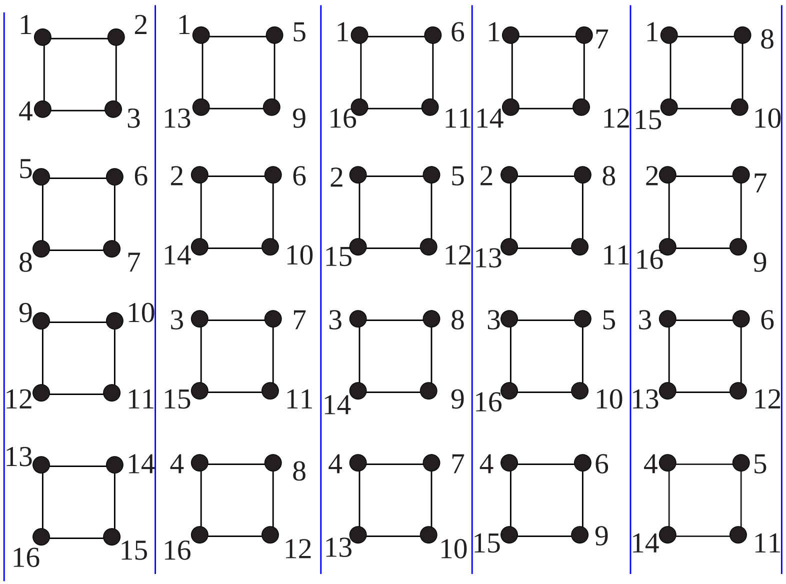
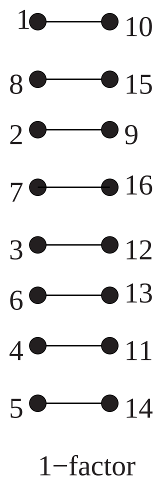
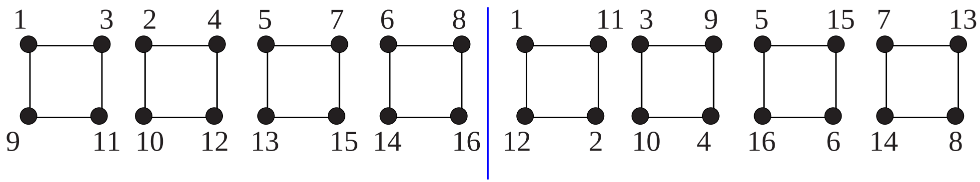


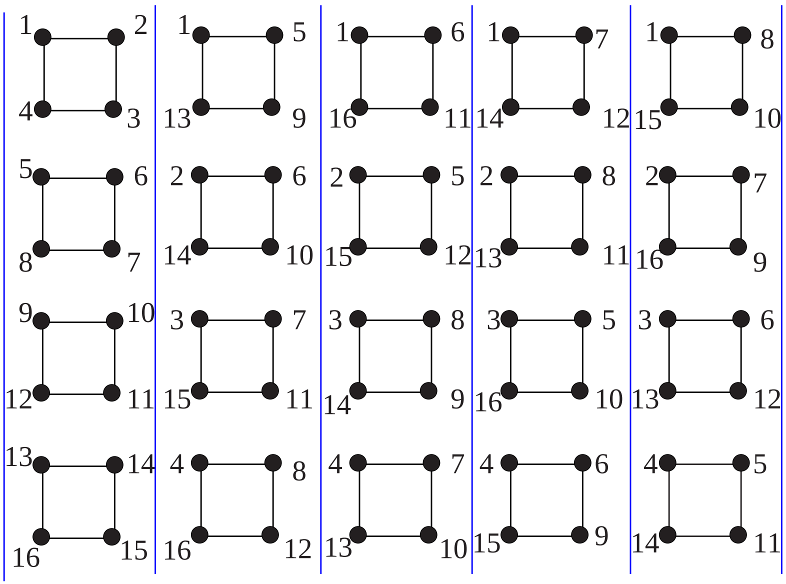


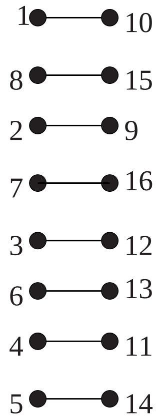
In general, take a resolvable K_4-design of order $12n+4$, and find a metamorphosis into a *resolvable* maximum packing with 4-cycles (of same order) and leave a 1-factor.



Open Problem: resolvable metamorphosis

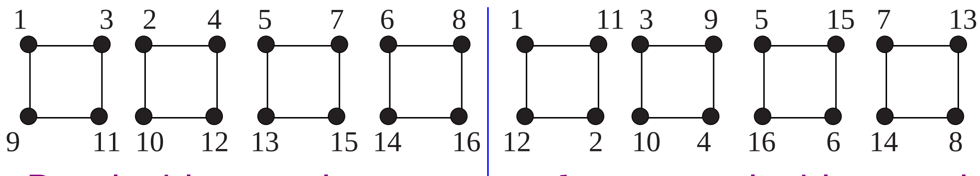
Example: resolvable K_4-design of order 16 (an affine plane of order 4, or a (16,20,5,4,1) BIBD); find a metamorphosis into a resolvable maximum packing with 4-cycles.





1−factor leave

In general, take a resolvable K_4-design of order $12n + 4$, and find a metamorphosis into a *resolvable* maximum packing with 4-cycles (of same order) and leave a 1-factor.



Resolvable 4-cycle systems, *almost* resolvable 4-cycle systems, and packings/coverings of these: existence has recently been dealt with. *But that's another story!*